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|  | **UNIT I PARTIAL DIFFERENTIAL EQUATIONS PART-A** |
| **1.** | **Eliminate the arbitrary function** ‘f **‘from *z***  ***f***  ***y***  **and form the Partial Differential Equation.** ***x***  **(April/May 2019)** |
|  | Given *z*  *f*  *y*   (1) *x*  Differentiating (1)partially with respect to x , we get *z*  *p*  *f* '  *y*    *y*  (2) Differentiating(1)*x*  *x*   *x*2    partially with respect to y , we get *z*  *q*  *f* '  *y*   1  (3)*y*  *x*   *x*    From (2) & (3) *p*   *y*  *px*  *qy*  0*q x* |
| **2.** | **Find the complete integral for *p***  ***q***  **1. (April/May 2019)** |
|  | Given *p*  *q* 1    (1) This of the form *f* ( *p*, *q*)  0 To find Complete Integral:Let the complete solution of (1) is *z*  *ax*  *by*  *c*    (2)Let *p*  *a* & *q*  *b* in (1)(1)  *a*  *b*  1  *b*  1 *a*  *b*  1 *a* 2Sub *b* in (2) |
|  | *z*  *ax*  1 *a* 2 *y*  *c* |    (3) |
| **3.** | **Form a partial differential equation by eliminating arbitrary function ‘f’ from z = f(x+ay)eay .****(April/May 2017)** |
|  | Given z = f(x+by)eay . (i)Diff (i) partially w.r.t x and yWe get p = f ‘ (x+by)eay ---(1) , q = f ‘ (x+by)eay .b+ f(x+by)eay. a (2)Sub.(1) in (2) we get q =az+bp |
| **4.** | **Obtain partial differential equation by eliminating arbitrary constant ‘a’ and ‘b’ from** **x – a****2**  **y – b****2**  **z (Nov/Dec 2019)** |
|  | Given x – a2 y – b2  z (1)Diff (1) partially w.r.t x and y2x – a  p  x  a  p22y – b  q  y – b  q2 |



 **DHANALAKSHMI SRINIVASAN COLLEGE OF ENGINEERING AND TECHNOLOGY**

**Mamallapuram, Chennai-603104.**

 **DEPARTMENT OF METHAMATICS**

**QUESTION BANK**



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|  | Substituting in (1), we get *p*2  *q*2  4*z* |
| **5.** | **Form the partial differential equation by eliminating the arbitrary constants ‘a’ & ‘b’ from****z**  **ax3**  **by 3 (April/May 2014)** |
|  | Given z  ax3  by3 (1)Diff (1) partially w.r.t x and y*z*  *p*  3*ax*2 , *z*  *q*  3*by*2*x* *y**a*  *p*      (2) *b*  *q*      (3) 3*x*2 3*y*2Substituting (2)& (3) in (1) we get px +qy=3z |
| **6.** | **Form the PDE of all spheres whose center lie on the z –axis. (Nov/Dec 2016)** |
|  | The equation of the sphere whose center lie on the z –axis is x2  y2  z  c2  r2 ,where r is constant. Differentiating w.r.t ‘ x’ , 2x  2z cp  0 (1)Differentiating w.r.t ‘ y ’ , 2y  2z cq  0 (2)(1)  2x  2(z  c)p(2)  2y  2(z  c)q which gives x  py qwe get qx= py |
| **7.** | **Obtain the partial differential equation by eliminating arbitrary constants ‘a’ and ‘ b’ from** ***x***  ***a*** **2**   ***y***  ***b*****2**  ***z*2**  **1 . (Nov/Dec 2016)** |
|  |  *x*  *a*2   *y*  *b*2  *z*2  1 (1)Differentiating (1) partially w.r. t x and y we get2*x*  *a*  2*zp*  0  *x*  *a*  *zp* (2)2 *y*  *b*  2*zq*  0  *y*  *b*  *zq* (3)Substituting (2) & (3) in (1) we get*z*2 *p*2  *z*2*q*2  *z*2  1 i.e., *z*2  *p*2  *q*2 1  1 |
| **8.** | **Find the complete integral of p+q = pq** |
|  | Given p+q = pq (1)This is of the type f(p,q)=0Let z= ax+by+c be the solution for (1)Partially Diff w.r.t ‘x ‘ and ‘y’ we get p = a and q = b a  b  ab b  aa 1Therefore z  ax   a  y  c is the complete solution. a 1   |
| **9.** | **Form a PDE by eliminating the arbitrary constant ‘a ’ & ‘ b ’ from the equation****(x**  **a)2**  **(y**  **b)2**  **z2 cot 2**  |
|  | (x  a)2  (y  b)2  z2 cot 2     (1) |

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|  | Differentiating partially with respect to x , we get 2(x  a)  2z p cot2   (2)Differentiating partially with respect to y , we get 2(y  b)  2z q cot2   (3)(x  a)  z p cot2  & (y  b)  z q cot2 Substituting in (1)  z p cot2  2   z q cot2  2  z2 cot2  p2  q2  tan 2 |
| **10.** | **Form the partial differential equation by eliminating ‘ g ’ from *g***  ***x*2**  ***y*2**  ***z*2 , *x***  ***y***  ***z***   **0 .** |
|  | We know that if *g*(u, v) = 0 then u = *f* (v)*x*2  *y*2  *z*2  *f* *x*  *y*  *z* (1)Differentiating (1) partially w.r. t x and y We get2*x*  2*zp*  *f* ' *x*  *y*  *z*1 *p* (2)2*y*  2*zq*  *f* ' *x*  *y*  *z*1 *q* (3)Divide (2) & (3)*x*  *zp*  1 *p*  *x*  *qx*  *zp*  *zpq*  *y*  *py*  *zq*  *zpq**y*  *zq* 1 *q* *z*  *y* *p* *x*  *z**q*  *y*  *x* |
| **11.** | **Solve *p*****1**  ***q***  ***qz* . (Nov/Dec 2014)** |
|  | *p* 1 *q*  *qz* - (1)This equation is of the form *f* *z*, *p*, *q*  0*z*  *g*  *x*  *ay* be the solution Let *x*  *ay*  *u* and *z*  *g* *u**p*  *dz q*  *adz**du du*(1) reduces to*dz* 1 *a dz*   *a dz z*  1 *a dz*  *az*  *a dz*  *az* 1 *dz*  *z*  1  *dz*  *du du*  *du*  *du du du du a* 1  *z* *a*Integrating log  *z*  1   *u*  *b* *a*  i.e., log *z*  1   *x*  *ay*  *b* is the complete solution. *a*   |
| **12.** | **Solve z = px + qy + 2 *pq* (Nov/Dec 2013)** |
|  | This of Clairaut’s type and the complete solution is z= ax+by+2 ab  (1)Diff (1) partially w.r.t ‘a ‘ and ‘b’ we get*z*  *x*  2 *b*  0, *z*  *y*  2 *a*  0*a* 2 *a* *b* 2 *b* |





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|  |  x   b & y   a  xy  b a  1 a b a bxy= 1 is singular integral. |
| **13.** | **Solve xp + yq = z . (Nov/Dec 2012)** |
|  | The equation is of the form *pP*  *qQ*  *R*Here *P*  *x*, *Q*  *y* & *R*  *z*.Auxiliary Equation is *dx*  *dy*  *dx**x y z*Considering *dx*  *dy* and integrating w.r.t ‘ x ‘ and ‘ y’ we get1. *y*

log *x*  log *y*  log *a*  log *a*  log x log y  *a*  *x**y*Considering *dy*  *dz* and integrating w.r.t ‘ y’ and ‘z ‘ we get1. *z*

log *y*  log *z*  log *b*  log *b*  log *y*  log *z*  *b*  *y**z*The solution is  *x* , *y*   0 *y z*   |
| **14.** | **Solve y(p-2x) = logq. (Nov/Dec 2011)** |
|  | The equation can be reduced to the type F1 x, p  F2 y, q yp – 2x  log q  p – 2x  log q  ky p  2x  k  p  2x  k & log q  k  q  ekyyWe know that z   pdx  qdy z   (2x  k) dx  ekydyekyz = (x2+kx)+ +ck |
| **15.** | ***2z*** ***2z*** ***2z*****Solve *2*** ***x2***  ***5*** ***x******y***  ***2*** ***y2***  ***0* (Nov/ Dec 2018)** |
|  | Auxiliary Equation is 2*m*2  5*m*  2  0 (Replace D by ‘m’ and D ’ by 1 )*m*   1 ,22Complimentary function is *z*    *y*  *x*     *y*  2*x*1  2  2  |
| **16.** | **Find the particular integral of (*D*2**  **2*DD*'** ***D*' 2) *z***  ***ex*** ***y*** |
|  | P.I = 1 *ex* *y**D*2  2*DD* ' *D* '2 |

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|  | Replace D by 1 and *D*' by –1 ,we get1 xy x xy x 2 xy= e  e  e1  2 1 2D  2D' 2 |
| **17.** |  **2z**  **2z**  **2z****Find the Particular Integral**  **2**   **sin(2x**  **3y)****x2** **x****y** **y2** |
|  | 2z  2z 2z 2 22 2  2  sin(2x  3y)  D  2DD' D' z  sin(2x  3y)x xy yPI  sin(2x  3y) Replace D2 =  4 , D' 2  9 & DD'  - 6D2  2DD' D' 2 sin(2x  3y)  sin(2x  3y)   sin(2x  3y)4  2(6)  9 1 |
| **18.** | **Solve**  ***D*3**  **3*DD*' 2** **2*D*' 3**  ***z***  **0 .** |
|  | Substituting D  m, & D'  1The Auxiliary equation is m3  3m  2  0m = 1, 1, –2Complimentary function is z =1  *y*  *x*  *x*2  *y*  *x* 3  *y*  2*x* |
| **19.** | **Solve**  ***D***  ***D***  **1** ***D***  ***D***  **2** ***z***  ***e*2*x*** ***y* .** |
|  | This is of the form *D*  *m*1*D* *C*1 *D*  *m*2*D* *C*2  *D*  *mnD* *Cn*  *z*  0Hence *m*1  1 *c*1  1 *m*2  1 *c*2  2Hence the C.F. is *z*  *ex*  *y*  *x*  *e*2*x*  *y*  *x*1 2*e*2 *x* *y e*2 *x* *y* 1  *y*P.I.          *e*2 *x**D*  *D* 1 *D*  *D*  2 2 11 2 1 2 2Hence, the complete solution is *z*  *ex*  *y*  *x*  *e*2*x*  *y*  *x*  1 *e*2*x* *y*1 2 2 |
| **20.** | **Solve (*D***  **1)(*D***  ***D*'**  **1)*z***  **0** | **(Nov/Dec 2012)** |
|  | (*D* 1)(*D*  *D*' 1)*z*  0Here c1  1, c2  1 , m1  0, m2  1General solution *z*  *ex f* ( *y*)  *e**x f* ( *y*  *x*)1 2 |
|  | **PART - B** |
| **1.** | **a)** | **Find the singular integral of *z***  ***px***  ***qy***  ***p*2**  ***pq***  ***q*2** | **(Nov/Dec 2019)** |
|  | **b)** | **Solve the Lagrange’s linear equation (x2 -yz)p+(y2 -zx)q=z2 -xy** | **(Nov/ Dec 2018)** |
| **2.** | **a)** | **Solve** **D2**  **DD '** **6D ' 2**  **z**  **x2y**  **y cos x** |
|  | **b)** | **Solve z2(p2**  **q2 )**  **x**  **y** | **(Nov/Dec 2020)** |
| **3.** | **a)** | **Find the singular solution of the equation z=px**  **qy+p2q2** | **(April/May 2019)** |
|  | **b)** | **Solve 2(z**  **xp**  **yq)**  **yp2** | **(Nov/Dec 2020)** |

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| **4.** | **a)** | **Form the partial differential equation by eliminating the arbitrary function from****f** **x2**  **y2 , z**  **xy**   **0 (Nov/Dec 2018)** |
|  | **b)** | **2z** **2z 2****Solve**  **2**  **sin(x**  **2y)**  **3x y****2x** **x2****y (Nov/Dec 2020)** |
| **5.** | **a)** | **Find the singular solution of the equation z=px**  **qy+ 1**  **p2**  **q2 (Nov/Dec 2020)** |
|  | **b)** | **Solve the partial differential equation** **D2**  **2DD**  **D****2**  **2D**  **2D** **z**  **sin(x**  **2y)****(Nov/ Dec 2018)** |
|  | **UNIT II FOURIER SERIES PART-A** |
| **1.** | **State the sufficient condition for a function f(x) to be expressed as a Fourier series. (Nov/Dec 2020)** |
|  | A function f(x) can be expanded as a Fourier series in the interval (*c*, *c*  2*l*) if the following conditions are satisfied.1. f(x) is periodic, single valued and finite in (*c*, *c*  2*l*)
2. f(x) has only finite number of finite discontinuities and no infinite discontinuities in (*c*, *c*  2*l*) .
3. f(x) has only finite number of maxima and minima in (*c*, *c*  2*l*)
 |
| **2.** | **Does f (x)**  **tan x possess a Fourier expansion?** |
|  | *f* ( *x*)  tan *x*  sin *x* does not possess a Fourier expansion because the function has an infinite discontinuitycos *x*at the point *x*   .2 |
| **3.** | **Determine the value of** *an* & *a*0 **in the Fourier series expansion of f(x)**  **x in** **2**  ***x***  **2****3** |
|  | *f* (*x*)  *x*3  *f* (*x*)  (*x*)3  *x*3   *f* (*x*)  *f* (*x*) is an odd function  *a*  *a*  0*n* 0 |
| **4.** | **Find the Fourier constant bn for *x sin x* in**   ***x***   **, when expressed as a Fourier series.** |
|  | *f* ( *x*)  *x* sin *x*,    *x*  *f* (*x*)  (*x*) sin( *x*)  *x* sin *x*  *f* ( *x*) *f*  *x* is an even function *bn*  0 |
| **5.** | **Give the expression for the Fourier series coefficient bn for the function f(x) defined in** **2**  ***x***  **2 .** |
|  | *l*  *U*.*L*  *L*.*L*  2  (2)  4  2 2 2 21 *l*  *n*  1 2  *n* *x* *bn*  *l*  *f* (*x*) sin  *l*  *xdx*  *bn*  2  *f*  *x*sin  2  *dx**l*   2   |
| **6.** | **Find bn of the Fourier series for the function f(x) = x2 , –**  **< x <**  **(Apr/May 2018)** |
|  | f **x**  **x2** , –   **x**  **f** **x**  **x****2**  **x2**  **f** **x***f(x)* is an even function  bn = 0 |
| **7.** | **If the Fourier series of the function f(x) = x + x2, in the interval (–** **,** **) is**  **2**    ***n***  **4 2**  **,****1**  **2 cos *nx***  **sin *nx*****3 *n*****1**  ***n n*** **then find the value of he infinite series 1**  **1**  **1**  **... (Nov/Dec 2017)****12 22 32** |

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|  | *x*      2     2  2  (1)*n*Put then   4 cos *n*2 3 *n*1 *n*2 2   2   (1)*n* (1)*n* 43 *n*1 *n*22 2   (1)2*n*3  4 *n*1 *n*21  1  1   212 22 32 ...  6 |
| **8.** | **Find the value of the Fourier series for *f* ( *x*)**  **0 *in* (*****c*, 0) at the point of discontinuity *x=0.*****1 *in* (0, *c*)****(May/June 2016)** |
|  | f (0 )  f (0 )x=0 is a point of discontinuity, sum of Fourier series of f(x) is .2f(0-) = lim *f* (0  *h*)  lim 0  0 .*h*0 *h*0f(0+) = lim *f* (0  *h*)  lim1  1 .*h*0 *h*0f(x) at x = 0 is 0 1  1 .2 2 |
| **9.** | **Sketch the even extension of the function f(x) = sin x, 0**  ***x***   **(Nov/Dec 2019)** |
|  |    0  2 2 |
| **10.** | **What is the behavior of Fourier series of a function f(x) at the point of discontinuity? (Nov/Dec 2018)** |
|  | At the point of discontinuity , f(x) converges to *f* (*x*)  1  *f* *x*   *f* *x* 2 |
| **11.** | **The cosine series for f(x) = x sin x in 0**  ***x***   **is given as****x sin x = 1**  **(****1)*n* .Deduce that 1**   **1**  **1**  **1**  **...**   **.** **1**  **cos *x***  **cos *nx***  **2 *n*****2 *n*2**  **1**  **1.3 3.5 5.7**  **2** |
|  |    1     (1)*n*  *n*  Put x = ,which is a point of continuity => sin  1 cos   2 2 cos  2 2 2 2  2  *n*2 *n* 1  2   (1)*n*  *n*  i.e.  1  2 cos  cos  02 2 *n* 1  2  2 (1)*n*  *n*    *n*2 1 cos 2  1 2*n*2   |



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|  | 1 cos  1 cos 3   1 cos 2  1 cos 5  ..   1 3 8  2  15 24  2  . 2      1  1  1  ...  1  3 15 35 21  1  1  ...  1  1.3 3.5 5.7 2i.e. 1  1  1  1     1.3 3.5 5.7 ... 2 |
| **12.** | **Find the value of** *an* **in the cosine series expansion of f(x) = K in the interval (0, 10), K is a constant.** |
|  | 2 *l n**x**an*  *l*  *f* (*x*) cos *l dx*0  *n* *x* 102 10  *n* *x*  *K* sin  10  10 *K**an*   *K* cos  *dx*      sin *n*  sin 0  0 sin *n*  sin 0  0 10 0  10  5  *n*  5*n* 10 0 |
| **13.** | **If f(x) = x2+x is expressed as a Fourier series in the interval (-2,2) to which value this series converges at x = 2?** |
|  | f (2 )  f (2 ) [(2)2  2] [(2)2  2]  2  6 Since x = 2 is an end point then f(x) converges to 42 2 2 |
| **14.** | **State Parseval’s theorem in the interval (c, c+2l).** |
|  | If the Fourier series corresponding to f(x) converges uniformly to f(x) in (c,c+2l) then*c*2 2 1   *f* (*x*) 2 *dx*  *a*0   *a* 2  *b* 2 2 *n n**c n*  1 |
| **15.** | **If the fourier series of the function f(x) = x for –**  **< x <**  **with period 2** **is given by*****f* ( *x*)**  **2** **sin *x***  **sin 2 *x***  **sin 3 *x***  **...** **then find the sum of the series 1**  **1**  **1**  **... (Apr/May 2015)** **2 3**  **3 5** |
|  | Put x =  ,which is a point of continuity =>   2 sin   1 sin 2    1 sin 3   .. 2 2  2 2  2  3  2  .       1 1  1 1 i.e.  2 1 (0)  (1) ... => 1   ... 2  2 3  3 5 4 |
| **16.** | **What do you mean by Harmonic analysis?** |
|  | The process of finding the harmonics in the Fourier series expansion of a function numerically is known as harmonic analysis. |
| **17.** | **Obtain the constant term of the Fourier cosine series of y = f(x) in (0,6) using the following table x: 0 1 2 3 4 5****y: 9 18 24 28 26 20** |
|  |  *y*  125  *a*0*a*0  2 *N*  2  6   41.67  Constant term = 2  20.83  |
| **18.** | **Find the root mean square value of the function f(x) = x in (0,*l*)** |





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| *b l l*RMS value =  1  2  1  2  1  *x*3   *l*3  *l y b*  *a f* (*x*) *dx l*  0 *x dx l*  3  3*l**a* 0  0 3 |
| **19.** | **Find the constant term in the expression of cos2 *x* as a Fourier series in the interval (** **,** **) .** |
| 1  1  1  cos 2*x*  1  sin 2*x* *a*0    cos *xdx*     *dx*  *x*  2  2  2  2 *a*  1   sin 2   ( )  sin( 2 )   2  10 2  2   2  2   Constant term = *a*0  12 2 |
| **20.** | **π2**  **(**  **1)n 1 1 1 π2****If x2**   **4**  **cosnx,deducethat**    **...**  ***in* [** **,** **] 3 n****1 n2 12 22 32 6** |
| Put *x*   then 2   2   (1)*n*4 cos *n*3 *n*1 *n*2 2   2   (1)*n* (1)*n*4 |
| 3 *n*1 *n*22 2   (1)2*n* |
| 3  4 *n*1 *n*21  1  1   212 22 32 ...  6 |
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|  |  | **PART - B** |
| **1.** | **a)** | **Find the Fourier series *f* ( *x*)**  ***eax* in 0**  ***x***  **2** **. (Nov / Dec 2018)** |
|  | **b)** | **1 1 1**  **2****Find the Fourier series of f(x) = x2 in (0, 2l). Hence deduce that**    **...**   **12 22 32 6****(Nov/Dec 2019)** |
| **2.** | **a)** | **Find the complex form of the Fourier series of *f* ( *x*)**  **cos *ax* in**  **,**  **, where ‘a’ is neither zero nor an integer. (Nov/Dec 2019)** |
|  | **b)** | *x*,    *x*  0 1 1  2**Obtain the Fourier series expansion of** *f* (*x*)   *x* , 0  *x*   **and evaluate**1 32  52 ...  8**(Nov/Dec 2020)** |
| **3.** | **a)** | **Find the half range Fourier cosine series for *f* ( *x*)**  ***x*(**  ***x*) in the interval (0,** **)****(Apr/May 2018)** |
|  | **b)** | **Find the Fourier series expansions of** *f* (*x*)  *x*2  *x* **in** ( , ) **of periodicity** 2 **. (Nov/Dec 2020)** |
| **4.** | **a)** | **Find the first two harmonic of the Fourier series of f (x), given by (Apr/May 2018)*****x* 0**  2  4 5 2 3 3 3 3***f (x)* 1 1.4 1.9 1.7 1.5 1.2 1.0** |



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|  | **b)** | **Find the Fourier series expansion of *f* ( *x*)**  **sin *ax* in** ***l*, *l***  **(Nov/Dec 2019)** |
| **5.** | **a)** | **Obtain half range Fourier cosine series expansion of** *f* (*x*) (*x* 1)2 **in the interval** 0  *x* 1 **and**1 1  2**evaluate** 1   ... 22 32 6 **(Nov/Dec 2020)** |
|  | **b)** | **Obtain the constant term and the first three harmonics in the Fourier Cosine series of y = f(x) in (0,6) from the following table (Nov/Dec 2019)** |
|  | **UNIT- III - APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS** |
|  | **PART - A** |
| **1.** | **Solve 3 *x*** ***u***  **2 *y*** ***u***  **0 by the method of separation of variables. (Nov/Dec 2015)*****x*** ***y*** |
|  | Let u = X(x) Y(y)ux = X’(x) Y(y) and uy = X(x) Y’(y)3*x*X Y  2 *y*X Y  0  3*x*X Y  2 *y*X Y3*x* X  2y Y  *k*X YX  *k and* Y  *k*X 3*x* Y 2y*Integrating**k k k k**X*  *c*1 *x* 3 , *Y*  *c*2 *y* 2  *u*  *cx* 3 *y* 2 |
| **2.** | **State any two assumptions made in the derivation of the one dimensional wave equation.****(Nov/Dec 2016)** |
|  | 1. The string is perfectly elastic and does not offer any resistance to bending.
2. The mass of the string per unit length is constant.
 |
| **3.** | **Write down the partial differential equation governing one dimensional wave equation?** |
|  | 2 *y*  2 2 *y**t* 2 *C* *x*2 where y(x,t) is the displacement of the string **.** |
| **4.** | **2 *y*** **2 *y*****In the wave equation**  ***C* 2 , what does *C2* stands for? (May/June2013)*****t* 2** ***x*2** |
|  | *C* 2  *T* , where *T* is the tension and *m* is the mass of the string.*m* |
| **5.** | **Write all three possible solutions of one dimensional heat equation. (Apr/May2019)** |
|  | 1. yx , t   Aepx  Be– px  Cepat  De– pat 
2. *y(x,t)* = (A cos *px* + Bsin *px*) (C cos *pat* + *D*sin *pat*).
3. *y(x,t)* = (A*x* +B) (C*t* + D)
 |
| **6.** | **Classify the two-dimensional steady state heat conduction equation. (Nov/Dec 2019)** |
|  | *uxx*  *uyy*  0 is the steady state heat conduction equation. |

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| **x** | **0** | **1** | **2** | **3** | **4** | **5** |
| **f (x)** | **4** | **8** | **15** | **7** | **6** | **2** |

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|  | Here, *A*  1, *B*  0, *C*  1*B*2  4*AC*  4  0 Elliptic. |
| **7.** | **Classify the partial differential equation** **1**  ***x*2**  ***z***  **2 *xy z***  **1**  ***y*2**  ***z***  ***xz***  **3*x*2 *yz***  **2*z***  **0*****xx xy yy x y*** |
|  | Compare the given equation with *Az*  *Bz*  *Cz*  *f* *x*, *y*, *z*, *z* , *z*   0*xx xy yy x y*Where *A*  1 *x*2 , *B*  2*xy*, *c*  1 *y* 2  B2  4AC  (2xy)2  4(1 x2 )(1 y2 )  4x2y2  4  4x2  4y2  4x2y2  4x2  4y2  4If   0  4*x*2  4 *y* 2  4  *x*2  *y* 2  1then the given PDE is parabolic on the circle *x*2  *y* 2  1If   0  4*x*2  4 *y* 2  4  *x*2  *y* 2  1then the given PDE is elliptic inside the circle *x*2  *y* 2  1If   0  4*x*2  4 *y* 2  4  *x*2  *y* 2  1then the given PDE is Hyperbolic outside the circle *x*2  *y* 2  1 |
| **8.** | **Give the mathematical formulation of the problem of one-dimensional heat conduction in a rod of length***l* **with insulated ends and with initial temperature** *f* (*x*). **(Nov/Dec 2019)** |
|  | Mathematical formulation,To solve *ut*  *c uxx* subject to,2*u*(*o*, *t*)  *u*(*l*, *t*)  0 *t*  0 and *u*(*x*, 0)  *f* (*x*), 0  *x*  *l* |
| **9.** | **Write the boundary conditions and initial conditions for solving the vibration of string equation, if the string is subjected to initial displacement *f(x)* and initial velocity *g(x)*. (Nov/Dec2006,07)** |
|  | The initial and Boundary conditions are:i) y(0, t)  0, t  0 ii) y(*l*, t)  0, t  0 iii) y (x, 0)  g(x), 0  x  *l* iv) y(x, 0)  f (x), 0  x  *l*t |
| **10.** | **What are the various possible solutions of one dimensional heat flow equation?****(Nov/Dec 2010,2015,2016) & (Apr/May 2018)** |
|  | 2 21. *u(x ,t)* = (*Aepx*  *Be* *px* ) C*e* *p t*

2 21. u(x ,t )= (*A*cos *px*  *B*sin *px*)*Ce* *p t*
2. u(*x ,t)* = *Ax* + *B*
 |
| **11.** | **Why can’t u(x ,t) = ( *Aepx***  ***Be*** ***px* )*Ce*****2 *p*2*t* be the correct solution in solving one dimension heat equation?** |
|  | As *t* *, u*  . It is not possible. |
| **12.** | **What is meant by steady state? In steady state condition derive the solution of one dimension heat flow equation. (Nov/Dec 2013, 2020)** |
|  | The state in which the temperature does not vary with respect to time ‘t’ is called steady state. Therefore when steady state condition exists, u(x, t) becomes u(x).*u*   2  2*u*The one dimension heat flow equation is *t* *x*2 .*u*   2*u* *u*In steady state condition, *t* 0 .  *x* 2  0  *x*  *A*  *u*  *Ax*  *B* . |
| **13.** | **2*u*** **2*u*** **2*u*** ***u*** ***u*****Classify the partial differential equation 3** ***x*2**  **4** ***x******y***  **6** ***y*2**  **2** ***x***  ***y***  ***u***  **0 (Apr/May 2008)** |

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|  | 2*u* 2*u* 2*u* *u* *u*Given 3 *x* 2  4 *x**y*  6 *y* 2  2 *x*  *y*  *u*  0Compare the given equation with *Au*  *Bu*  *Cu*  *f* *x*, *y*,*u*,*u* ,*u*  0*xx xy yy x y*Where Where *A*  3, *B*  4, *C*  6  *B* 2  4 *AC*  16  72  56  0 The given PDE is Elliptic. |
| **14.** | **A plate is bounded by the lines *x=0,y=0, x=l* and *y=l*. Its faces are insulated. The edge coinciding with *x-* axis is kept at 100****C. The edge coinciding with y*-*axis is kept at 50****C. The other two edges are kept at 0****C. Write the boundary conditions that are needed for solving two dimensional heat flow equation.****(Nov/Dec2011,2012)** |
|  | **Boundary conditions are:***i*) *u*(*x*,0)  100∘ *C* ; 0  *x*  *l ii*) *u*(0, *y*)  50∘ *C* ; 0  *y*  *l iii*) *u*(*x*,*l*)  0∘ *C* ; 0  *x*  *l**iv*) *u*(*l*, *y*)  0∘ *C* ; 0  *y*  *l* |
| **15.** | **What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation with respect to the time? (May/June 2017)** |
|  | The solution of one dimensional wave equation is of periodic in nature. But the solution of one dimensional heat equation is not periodic in nature. |
| **16.** | **Given the boundary conditions on a square or rectangular plate, how will you identify the proper solution?** |
|  | If the non-zero temperature is prescribed either on x=0 or on x=a (vertical edges), the proper solution will be*u(x ,y)* = (Ae*px* + Be *– px*) (C cospy + Dsin *py*)If the non-zero temperature is prescribed either on y=0 or y=b (horizontal edges), the proper solution will be*u(x ,y)*= (A cos *px* + Bsin *px*) (C e *py* + De – *py*) |
| **17.** | **An infinitely long plate is bounded by two parallel edges and an end at right angles to them. The breath of the edge y = 0 is**  **and it is maintained at constant temperature u0 at all points and the other edges are****kept at zero temperatures. Formulate the boundary value problem to determine the steady state temperature.** |
|  | 2*u*  0 subject to*u*(0, *y*)  0 , *u*( , *y*)  0 *for y*  0*u*(*x*, )  0 *and u* (*x*,0)  *u*0 *for* 0  *x*   |
| **18.** | **Write the possible solutions of the steady state two dimensional heat flow equation** 2*u*  2*u*  **.**0*x*2 *y* 2**(Apr/May 2011,2012), (Nov/Dec 2020)** |
|  | The possible solutions of the steady state two dimensional heat flow equation 2*u*  2*u*  are0*x*2 *y* 2*u*  *x* , *y*    *Aepx*  *Be*– *px*  *Ccos py*  *Dsinpy* *u*  *x* , *y*   *A cospx*  *Bsinpx* *C epy*  *De*– *py* *u*  *x*, *y*   *Ax*  *B* *Cy*  *D* |
| **19.** | **A rod 40 cm long with insulated sides has its ends *A* and *B* kept at 20****C and 60****C respectively. Find the steady state temperature at a location 15 cm from *A.* (Apr/May 2011 & Nov/Dec 2012)** |

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|  |  2*u*When the steady state condition prevail the heat flow equation is *x* 2  0The steady state temperature is *u*(*x*)  *Ax*  *B*       (1)The boundary conditions are *u*(0)  20      (2) and *u*(40)  60      (3)Applying (2) in (1)*u*(0)  *A*(0)  *B*  20  *B*  20Sub B in (1)  *u*( *x*)  *Ax*  20        (4)Sub (3) in (4)*u*(40)  40*A*  20  60  *A*  60  20  140Sub *A* in (4)  *u*( *x*)  *x*  20The steady state temperature at a location 15 cm from *A* is*u*(15)  15  20  35∘ *C* |
| **20.** | **The ends *A* and *B* of a rod 20cm long have the temperature at 30** ∘ **C and 80** ∘ **C until steady state prevails.****Find the steady state temperature. (Nov/Dec 2014)** |
|  |  2*u*When the steady state condition prevail the heat flow equation is *x* 2  0The steady state temperature is *u*( *x*)  *Ax*  *B*        (1)The boundary conditions are *u*(0)  30      (2) and *u*(20)  80      (3)Applying (2) in (1)*u*(0)  *A*(0)  *B*  30  *B*  30Sub B in (1)  *u*( *x*)  *Ax*  30        (4)Sub (3) in (4), we get *u*(20)  20*A*  30  80  *A*  80  30  520 2Sub *A* in (4) *u*(*x*)  5 *x*  30 which is the required temperature.2 |
|  | **PART B** |
| **1.** | **a)** | **A uniform string is stretched and fastened to two points** '*l*' **apart. Motion is started by displacing the string into the form of the curve *y***  ***kx*(*l***  ***x*) and then releasing it from this position at time *t***  **0 . Find the displacement of the point of the string at a distance** *x* **from one end at time** *t* **.****(Nov/Dec 2018)** |
|  | **b)** | **Using the method of separation of variables solve** ***u***  **2** ***u***  ***u*** , **where *u*( *x*, 0)**  **6*e*****3*x******x*** ***t*****(Nov/Dec 2018)** |
|  | **c)** | **Solve *u***  ***a*2*u* by the method of separation of variables and obtain all possible solutions.*****t xx*****(Nov/Dec 2019)** |
|  | **d)** | **Solve the problem of a tightly stretched string with fixed end points *x***  **0 & *x***  ***l* which is initially****in the position *y***  ***f* ( *x*) and which is initially set vibrating by giving to each of its points a velocity*****dy***  ***g*(x) *at t***  **0. (Nov/Dec 2019)*****dt*** |
|  | **e)** | **Classify the partial differential equation (1–x2)fxx – 2xyfxy + (1–y2)fyy=0 (Nov/Dec 2019)** |
| **2.** | **a)** | **A tightly stretched string of length *l* is fastened at both end A & C. The string is at rest, with the** |

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|  |  | **point B (x = b) drawn aside through a small distance ‘d’ and released to execute small transverse vibration. Find the transverse displacement of any point of the string at any subsequent time.****(Nov/Dec 2020)** |
|  | **b)** | **The ends A and B of a rod *l* cm long have their temperatures kept at** 20 *C* **and** 40 *C* **, until steady state conditions prevail. The temperature of the end B is suddenly reduced to** 10 *C* **and that of A is increased to** 50 *C* **. Find the steady state temperature distribution in the rod after time t.****(Apr/May,2018)** |
| **3.** | **a)** | **If a string of length** *l* **is initially at rest in its equilibrium position and each of its points is given a*****cx* ; 0**  ***x***  ***l*****velocity** *v* **such that *v***   **2 , find the displacement at any time** *t* **.** ***l******c*(*l***  ***x*);**  ***x***  ***l*** **2** |
|  | **b)** | **2*u*** **2*u*****Derive the general solutions for one dimensional wave equation**  ***c*2 using separation of*****t* 2** ***x*2****variables method (Nov/Dec 2018)** |
| **4.** | **a)** | **A square plate is bounded by the lines *x***  **0, *x***  **20, *y***  **0and *y***  **20 . Its faces are insulated. The****temperature along the upper horizontal edge is given by u(x,20) = x (20 – x) when 0 < x < 20, while the other three edges are kept at** 0 *C* **. Find steady state temperature distribution** *u*(*x*, *y*) **in the****plate. (Nov/Dec 2016)** |
|  | **b)** | **A rectangular plate with insulated surface is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along the short edge y = 0 is*****u*( *x*,0)**  **100sin**   ***x***  **, 0 < x < 8 while two long edges x = 0 & x = 8 as well as the other short edge are** **8**  **kept at** 00 *C* **, then find the steady state temperature at any point of the plate. (Nov/Dec 2019)** |
| **5.** | **a)** | **A rectangular plate with insulated surface is 20cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at****10 *y*, 0**  ***y***  **10****short edge *x***  **0 is given by *u***   **and the two long edges as well as the****10(20**  ***y*), 10**  ***y***  **20****other short edges are kept at** 0∘ *C* **. Find the steady state temperature distribution in the plate.****(Apr/May,2017)** |
|  | **b)** | **A uniform bar of length *l* through which heat flow is insulated at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given by *k* (*l x***  ***x*2 ) ,****for 0 < x < *l* , find the temperature distribution in the bar after time *t*. (Nov/Dec 2020)** |
|  | **UNIT IV – FOURIER TRANSFORMS PART - A** |
| **1.** | **Write the Fourier transform pair.** |
|  | The Fourier transform pair is defined as*F* *f* *x*  1  *eisx f* *x**dx*  *F**s* : *F* 1 *F*  *f*  *x*  1  *e**isxF* *s**ds*  *f*  *x*2     2   |
| **2.** | **State Fourier integral theorem**. |
|  | If *f* *x*is piecewise continuous, differentiable and absolutely integrable in  ,  then*f* *x*  1   *f* *t* *eis**x*  *t*  *dtds*2    |



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| **3.** | **1; for x**  **2****Find the Fourier transform of f(x)**  **0; for x**  **2** |
|  | The Fourier transform of the function *f* (*x*) is*F* *f* (*x*) 1  *f* (*x*)*eisxdx* .2  1 2 1  *eisx* 2 1  *ei*2*s e**i*2*s*  *F*  *f* (*x*)   1.*eisxdx*       2 2 2  *is* 2 2  *is is*  1  *ei*2*s*  *e**i*2*s*   1 1 (2*i* sin 2*s*)  *F* (*s*)  2 sin 2*s*   2  *is*  2 *is*  *s* |
| **4.** | **If F(s) is the Fourier transform of f (x), then show that *F*{ *f* ( *x***  ***a*)}**  ***eias F* (*s*)** |
|  | 1  1 F {f(x-a)}=  *f* (*x*  *a*)*eisxdx* =  *f* (*t*)*ei*(*a**t*)*s dt* where x - a = t2  2 *ias* 1  *its ias*= *e*  *f* (*t*)*e dt* = *e* F(s) = *eias* F{f(x)}2  |
| **5.** | **Write the Fourier sine transform pair and Fourier Cosine transform pair.** |
|  | The Fourier sine transform pair is defined as2  1   2 *Fs*  *f* *x*    *f* (*x*) sin *sxdx*  *Fs* *s* : *F* *F* *s*  *f*  *x*    sin *sx F* *s*  *ds**s s*0 0The Fourier cosine transform pair is defined as *F*  *f* *x*  2 *f* (*x*) cos *sxdx*  *F* *s*: *F* 1 *F* *s*  *f*  *x*  2 cos *sx F* *s* *ds**c*   *c*  *c*    *c*0 0 |
| **6.** | **0 , *x***  ***a*****Find the Fourier transform of f(x) defined by *f* ( *x*)**   **, *a***  ***x***  ***b*****1****0 , *x***  ***b* .** |
|  | *F* *f* *x* 1  *isx f* *x**dx* *e*2   1  *a eisx*.0 *dx*  *beisx*.1*dx*   *eisx*.0 *dx*2      *a b*  1 *b isx* 1  *eisx*  *a* 1    *e* .1*dx*     *eias*  *eibs*2 *a* 2  *is*  *b is* 2 |
| **7.** | **State Convolution theorem in Fourier Transform. (Nov/Dec 2018 & Nov/Dec2019)** |
|  | The Fourier transform of the convolution of *f* *x* and *g**x* is the product of their Fourier transformsi.e. *F**f* *x* *g**x* *F**f* *x**F**g**x* |

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| **8.** | **State Parseval’s identity on Fourier Transform.** |
|  |  2  2 *F* (*s*) *ds*   *f* (*x*) *dx*, *where F*[ *f* (*x*)]  *F* (*s*)    |
| **9.** | **If *F***  ***f* ( *x*)**  ***F*(*s*), then find *F*** ***eiax f* ( *x*)****.** |
|  | *F* *f* (*x*) *F*(*s*)  1  *f* (*x*)*eisxdx*2  *F**eiax f* (*x*) 1  *iax f* (*x*)*eisxdx*  1  *f* (*x*)*ei*(*s*  *a*)*xdx*  *F* (*s*  *a*) *e* 2   2   |
| **10.** | **Find the Fourier transform of e****a |x|, if a** > **0** |
|  |  *F*  *f* (*x*) 1  *e**a x eisxdx*  1  *e**a x* (cos *sx*  *i* sin *sx*)*dx* 2  2  1   *e**a x* cos *sxdx*  *i*  *e**a x* sin *sxdx*  1   *e**ax* cos *sxdx*  2  *a*      2    2 2 2    2  0    (*s*  *a* )   |
| **11.** | **Find the Fourier sine transform of e****ax , *a***  **0 . Hence find F** **xe**  **ax**  **.****s**   |
|  | 2 *Fs*  *f* (*x*)   *f* (*x*) sin *sxdx* .0*F* *e* *ax*   2  *e* *ax* sin *sxdx*  2 *s s*      *s* 2  *a*20We know that by property *F* *xf* *x*   *d F*  *f* *x**S ds C*     *ax*  *d* 2 *s* 2 (*s* 2  *a* 2 ).1 *s*.(2*s*)  2  *s* 2  *a* 2  *F xe*         *s*   *ds*  *s* 2  *a* 2    2 2 2     2 2 2   *s*  *a*     *s*  *a*          |
| **12.** | **Define Self-reciprocal function under Fourier transform and give an example.** |
|  | If *f* *s* is the Fourier transform of *f* *x*, then *f* *x*is said to be self-reciprocal under Fourier  *x*2   *s*2transform. *F*  *e* 2   *e* 2 .   |
| **13.** | **1**  **s** **Prove that Fc** **f** **ax**  **Fc**   **,*a***  **0****a**  **a**  |
|  | *Fc*  *f* *ax*  2  *f* (*ax*) cos *sxdx* , *Put ax*  *t*, *adx*  *dt*, *dx*  *dt* 0 *a* |

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|  | 2   *st*  *dt**when x*  0, *t*  0 *and x*  , *t*   *Fc*  *f* *ax*    *f* (*t*) cos *a*  *a*0  1 2   *s*  1  *s* *Fc*  *f* *ax*  *a*   *f* (*t*) cos *a*  *t dt* = *a Fc*  *a*  .0     |
| **14.** | **State Parseval’s identity in Fourier sine and cosine Transform.** |
|  |     *Fc* (*s*) 2 *ds*  *f* (*x*) 2 *dx* &  *Fs* (*s*) 2 *ds*  *f* (*x*) 2 *dx*0 0 0 0 |
| **15.** | **Find the Fourier Sine Transform of *f* ( *x*)**  ***k*, 0**  ***x***   **,exist? Justify your answer. (Nov/Dec 2018)** |
|  | No.Because sinx and cosx are bounded between -1 and 1.sin∞ and cos∞ are not defined. |
| **16.** | **Prove that *F***  ***f* ( *x*)cos *ax***  **1** ***F* (*s***  ***a*)**  ***F* (*s***  ***a*)** **where *F***  ***f* ( *x*)**  ***F* (*s*)is the Fourier Cosine*****c* 2**  ***c c***  ***c c*****transform of f(x). (Nov/Dec 2019)** |
|  | *F*  *f* (*x*) cos *ax*  2 *f* (*x*) cos *ax* cos *sx dx**c*  02 =   *f* (*x*) cos *sx* cos *ax dx*02  1=   *f* (*x*) 2 cos(*s*  *a*)*x*  cos(*s*  *a*)*x**dx*01  2  2  **=**   *f* (*x*) cos(*s*  *a*) *x dx*   *f* (*x*) cos(*s*  *a*) *x dx*2   0  0  |
| **17.** | **If F [f(x)] = F(s) then *F* ( *f* (*ax*))**  **1 *F***  ***s***  **, *a***  **0*****a***  ***a***   |
|  | **By definition***F*  *f* *ax* 1  *f* (*ax*) *eisx dx* ,2 If a > 0 *Put ax*  *t*, *adx*  *dt*, *dx*  *dt**a*1  *i* *s* *t dt* 1  *s* *when x*    *t*   *and x*    *t*    *F*  *f* *ax*   *f* (*t*) *e*  *a*  = *F*   . –(1)   2  *a a*  *a* If a < 0 *Put ax*  *t*, *adx*  *dt*, *dx*  *dt**a**when x*    *t*   *and x*   *t*  1  *i* *s* *t dt* 1  *i* *s* *t dt* 1   *F*  *f* *ax*   *f* (*t*) *e*  *a*    *f* (*t*) *e*  *a*  = *F*  *s*  . ---(2)     2  *a* 2  *a a*  *a* From (1) & (2) we get *F* ( *f* (*ax*))  1 *F*  *s* , *a*  0*a*  *a*   |

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| **18.** | ***1***  ***x 2 ; 0***  ***x***  ***1*****Find the Fourier cosine transform of *f(x)***   **.*****0 , otherwise*** |
|  | *FC*  *f* (*x*)  2  *f* (*x*) cos *sxdx* 01 *Fc*  *f* (*x*)  2  (1  *x* 2 ) cos *sxdx* 02  2 sin *sx*   cos *sx*    sin *sx* 1  (1  *x* ) *s*  (2*x*) *s* 2   (2) *s*3     0 2   2 cos *s*  2 sin *s*   2 2  sin *s*  *s* cos *s*   *s* 2 *s*3    *s*3     |
| **19.** | **1****Find the Fourier Sine transform of .****x** |
|  |  1  2  sin *sx* 2  sin *t* 2    *Fs*  *x*     *x dx*    *t dt*    2   2  0 0   |
| **20.** | **Find** *f* (*x*) **from the integral equation**   **s** **f(x)cos *s* xdx**  **e****0** |
|  | Given  *f* (*x*) cos *sxdx*  *e**s* ,02  *f* (*x*) cos *sx dx*  2 *e**s*  02  2  2  2  1 *f* (*x*)    *Fc* (*s*) cos *sx ds*     *e* cos *sx ds*    1 *x*2 *s* 0 0   |
|  | **PART - B** |
| **1.** | **a)** | **1**  ***x*2 ; *x***  **1****Find the Fourier transform of *f* ( *x*)**   **. Hence deduce that****0; *x***  **1** ***x* cos *x***  **sin *x***  ***x***  ***x*3 cos**  **2** ***dx*.****0**   | **(Nov/Dec 2018)** |
|  | **b)** | **Find the Fourier transform of *f* ( *x*)**  **cos *x*, 0**  ***x***  **1** | **(Nov/Dec 2018)** |
| **2.** | **a)** | ***e*** ***ax*** **Find the Fourier sine transform of *x* where *a***  **0 .** | **(Nov/Dec 2018)** |
|  | **b)** | * ***x*2**

**2 2****Find the Fourier cosine transform of *e******a x* and hence show that *e* 2 is self-reciprocal with respect to****the Fourier cosine Transform.** |

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| **3.** | **a)** | **1**  ***x* ; *if x***  **1**  **sin4 *t*****Find the Fourier transform of *f* ( *x*)**   **. Hence Evaluate**  **4 *dt* .****0 ; *if x***  **1 0 *t*** |
|  | **b)** | 1 , 0  *x*    1 cos( )**Find Fourier sine integral of** *f* ( *x* )  0, *x*   **and hence evaluate**   sin( *x*)*d* **.** 0 |
| **4.** | **a)** | **1 , *x***  **1**  **sin *x*****Find the Fourier transforms of** *f* ( *x* ) **defined by *f* ( *x* )**   **and hence evaluate**  ***dx*****0 , *x***  **1 0 *x*****(Nov/Dec 2020)** |
|  | **b)** | **Find the Fourier sine and cosine transform of *e******ax* , *a***  **0 and hence deduce their inversion formulae.****(Nov/Dec 2019)** |
| **5.** | **a)** | **Using Parseval’s identity evaluate the following integrals.** ***dx*****1)**  **( *x*2**  ***a*2 )2****0** ***x*2****2)**  **( *x*2**  ***a*2 )2 *dx*, where *a***  **0. (Nov/Dec 2019)****0** |
|  | **b)** | 2**Verify Convolution theorem for Fourier transform, if** *f* (*x*)  *g*(*x*)  *e**x* **(Nov/Dec 2020)** |
|  | **UNIT V Z – TRANSFORMS AND DIFFERENCE EQUATIONS** |
|  | **PART - A** |
| **1.** | **What is the Z- transform of discrete unit step function** |
|  | Discrete Unit step function is*u* *n*  1, *n*  00, *n*  0*Z* *u* *n*  1.*z**n*  1 *z*1  *z*2  ..  1 *n*0 1 *z*1= *z* if *z*  1*z* 1 |
| **2.** | **Find *Z***  **1**  **.** **2*n***   |
|  | *Z*  1    1 *z**n*  1 1 *z*1  1 *z*2  1 *z*3  2*n*   2*n* 2 22 23 …  0 1 1  1  1  ...2*z* 4*z*2 8*z*3 *z**z*  12 2*z* , *z*  2 2*z* 1 |
| **3.** | **Find *Z*** ***u*** ***n***  **1** |

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|  |  *Z* *u* *n* 1  *u* *n* 1 *z**n*  *u* *n* 1 *z**n**n*0 *n*1  *z**n*  1  1  1  ....  1 1 1  1  *n*1 *z z*2 *z*3 *z*  *z z*2 1  1 1 1  *z* 1 1 *z* 1 *z*   *z*  *z*     1  *z*   1 if *z*  1*z*  *z* 1  *z* 1  |
| **4.** | **Show that *Z*** ***an f* (*z*)**  ***F***   ***z***  **where *Z***  ***f* (*n*)**  ***F*** ***z*** **is the Z-transform of** *f* (*x*). ***a***   | **(Nov/Dec 2019)** |
|  | *Z* *an f* (*z*)   *an f* (*n*)*z**n**n*0  *z*  *n*  *a*  *f* (*n*)*n*0   *F*  *z*  *a*   |
| **5.** | **Form the difference equation by eliminating arbitrary constant A from *yn***  ***A*.2*****n*** | **(Nov/Dec 2017)** |
|  | Given *y*  *A*.2*n* , *y*  *A*.2*n*1  *A*.22*n*  2 *A*2*n*  2 *y*  *y*  2 *y*  0*n n*1 *n n*1 *n* |
| **6.** | **If *Z***  ***f*** ***n***  ***U***  ***z***  **, then show that *Z***  ***f*** ***n***  ***k***   ***zkU***  ***z***  |
|  | *Z*  *f* *n*  *k*    *f* *n*  *k*  *z**n*0  *zk*  *f* *n*  *k*  *z**n**k*   *zk*  *f* *r*  *z**r*  *zkU*  *z* *n*0 *r* 0 |
| **7.** |  ***f*** ***n***  **1****If *Z***  ***f*** ***n***  ***U***  ***z***  **, then *Z***  ***n***    ***z U***  ***z***  ***dz* .**  |
|  |  *f* *n*  *f* *n* *n*  *n*1 *z**n* *n*1*Z*  *n*    *n z*   *f* (*n*) *z dz* , since *n*   *z dz*  *n*0 *n*0  *f* *n* *z**n*1*dz*  *z*1 *f* *n* *z**n* *dz*   *z*1 *U*  *z*  *dz* .*n*0 |
| **8.** | **If *Z***  ***f*** ***n***  ***F***  ***z***  **then find *Z*** ***n f*** ***n*** | **(April/May 2018)** |
|  | *d F*  *z*    *d Z*  *f* *n*   *d*  *f* (*n*)*z**n*   *n f* (*n*)*z**n*1*dz dz*   *dz n*0 *n*0 *n*  *n f* (*n*) *z*   1 *n f* (*n*)*z**n n*0 *z z n*0 *Z* *n f* *n*  *z d F* (*z*) .*dz* |

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| **9.** | **Find *Z*** ***an*****3**  **.** |
|  | *Z* *an*3   *an*3*z**n*  *a*3*Z* *an*  = *a*3 *z**n*0 *z*  *a* |
| **10.** | **Find *Z*** **1**  ***z***  ***z***  **1**  ***z***  **2**   |
|  |  *z* Let    *U*  *z*  *z* 1 *z*  2 *U*  *z*  1 *A B*Then   *z*  *z* 1 *z*  2 *z* 1 *z*  2*z*  2, 1  *B z*  1, 1   *A**U*  *z*   1  1 *z z* 1 *z*  2*U*  *z*   *z*  *z* *u* *n*  1 2*n*  2*n* 1*z* 1 *z*  2 |
| **11.** | **Find the Z-transforms of**  ***n***  **. (April/May 2018)** |
|  |     *n* 1  1 2  1 3*Z n*   *n z*  1   2    3   ....*n*  1 *z*  *z*   *z* 1  2 1  1  1 2 1  *z* 2 *z* 1  3   1      .*z*  *z z*2  *z*  *z*  *z*  *z* 1  *z* 12 |
| **12.** | **Find Z-Transform of 1 . (Nov/Dec 2017)*****n*** |
|  |  1   1 1 1  1 2 1  1 3  1   *z* 1   *z* *Z*     *z**n*         ....  *log* 1   log   log  *n*  *n*1 *n z* 2  *z*  3  *z*   *z*   *z*   *z* 1  |
| **13.** | **Find Z-Transform of *an* . (Nov/Dec 2020)** |
|  |  *n*   *n* *n*   *a* *n z Z a*   *a z*    *z*   *z*  *a**n* 0 *n* 0   |
| **14.** | **Find Z-Transform of *Z***  **1**  **.** ***n*!**  |
|  |  1   1 *n* 1 1 1/*z**Z*  *n*!   *n*! *z*  1  1! *z*  2  *e*  *n*0 2! *z* |
| **15.** | **Solve *y***  ***y***  **2*n* , given that *y*** **0**  **1.*****n*****1 *n*** |
|  | *Z* *y*  *y*   *Z* 2*n**n*1 *n* |

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|  | *zY*  *z*   *y* 0  *Y*  *z*   *z* , *Y* *z*  *Z* *y* *z*  2 *n**Y*  *z*  *z* 1  1 *z**z*  2 *Y*  *z*    *z*  2  *z*  *Y*  *z*   2  *z* 1  *z*  2 *z* 1  *z*  2 *z* 1*y*  *Z* 1  2   2*n**n*  *z*  2   |
| **16.** | **1** ***z* 2** **Using Convolution theorem , evaluate *Z*** **( *z***  **1)( *z***  **3)**  |
|  | We know that *Z* 1  *z*   1and *Z* 1  *z*   3*n* *z* 1  *z*  31  *z*2   1  *z z* Now *Z*  *z* 1 *z*  3  *Z*  *z* 1. *z*  3 *n n* 13*n*  1.3*n**m*  3*n* 3*m**m*0 *m*0 1 *n*1*n*  1 *m*  3  1 3*n* 1 3*n*1  / 3*n*1 3*n*    3*n*   *m*0  3   1  1 1 3 / 3 3   1 3*n*1  3*n*1 1 2 2 . |
| **17.** | **Find *Z*** ***e*****2*t* sin 2*t*** |
|  | *Z* *e*2*t* sin 2*t* *Z* sin 2*t*   *z* sin 2*T* *z*  *ze*2*T*  2  *z*  2*z* cos 2*T* 1 *z* 2*T*   *ze* *ze*2*T* sin 2*T**z*2*e*4*T*  2*ze*2*T* cos 2*T* 1 |
| **18.** | **Find *Z*** **sin *at*** |
|  | *Z* sin *at*  sin *anT* *z**n**n*0 *Z* sin *n*    *z* sin   *z* sin *aT*  *aT*  2  2 *z*  2*z* cos 1 *aT z*  2*z* cos *aT* 1 |
| **19.** | **State initial and final value theorems of Z - transform. (Nov/Dec 2019, 2020)** |
|  | Initial value Theorem |

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|  | *limit limit**If Z*  *f* *n*  *U*  *z* , *n*  0 then n  0 *f* *n*  *f* 0  *z*  *U*  *z* Final value Theorem*limit limit**If Z*  *f* *n*  *U*  *z* , *n*  0 then *n*   *f* *n*  *z*  1 *z* 1*U*  *z*  |
| **20.** | **Form the difference equation from *yn***  ***A*.2**  ***B*.3*****n n*** |
|  | Given *yn*  *A*.2  *B*.3 , *y*  2 *A*.2  3*B*.3 , *y*  4*A*.2  9*B*.3*n n n n n n**n*1 *n*2Elimination of A and B forms the difference equation*yn* 1 1*yn*1 2 3  0*yn*2 4 9*yn*2  5*yn*1  6 *yn*  0 . |
| **PART B** |
| **1.** | **a)** | **Solve *yn*****2**  **4 *yn*****1**  **4 *yn***  **0, *y*** **0**  **1, *y*** **1**  **0 using Z-transform.** | **(Apr/May 2018)** |
|  | **b)** | **State and Prove final value theorem of Z-Transform** |
| **2.** | **a)** | **Solve *y*** ***n***  **3**  **3 *y*** ***n***  **1**  **2 *y*** ***n***  **0, given that *y*** **0**  **4, *y*** **1**  **0, *y*** **2**  **8 Using Z- transform.** |
|  | **b)** | **2*z*2**  **3*z***  **12****If *U***  ***z***    ***z***  **1****4 , find *u*2 and *u*3 .** |
| **3.** | **a)** |   **8*z*2** **Find *Z* 1**   **by convolution theorem.** **(2*z***  **1)(4*z***  **1)**  | **(Apr/May 2018,Nov/Dec 2019)** |
|  | **b)** | **Find *Z***  **2*n***  **3**  **(*n***  **1)(*n***  **2)**   | **(Nov/Dec 2017)** |
| **4.** | **a)** | **Find *Z*** ***rn* cos *n***  **and hence deduce *Z***  ***cos n***  **2**   |
|  | **b)** | ***z*2**  **3*z*****Find the inverse Z-Transform of**  ***z***  **5****(*z***  **2) by residue method.** | **(Apr/May 2018)** |
|  | **c)** | **Find Z-transform of 2n**  **5 sin *n***  **3*a*4 .****4** | **(Nov/Dec 2020)** |
|  | **d)** | ***z*2****Using Convolution theorem, find the inverse Z-transform of**  ***z***  **2** **( *z***  **3) . (Nov/Dec 2020)** |
|  | **e)** | **Using Z-transformation, solve *Un*** **2**  **4*Un*****1**  **3*Un***  **3 given that *U***  **0, *U***  **1**. **(Nov/Dec 2020)*****n*****0 1** |
| **5.** | **a)** | **Form the difference equation corresponding to the family of curves *yn***  ***an***  ***b*2 .*****n*** |
|  | **b)** |  ***z*** **Find *Z*** **1**   **using the method of partial fraction.**  ***z***  **1** ***z***  **1****2**  |
|  | **c)** | **Find *Z*** **sin *bt*** **and hence find *Z*** **e*****at* sin *bt***  | **(Nov/Dec 2019)** |

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|  | **d)** | **Solve using Z-transforms technique the difference equation *yn*** **2**  **7 *yn*****1**  **12 *yn***  **2 with*****n******y*0**  **0**  ***y*1 . (Nov/Dec 2019)** |
|  | **e)** | **Using residue method, find *Z*** **1**  ***z***  **(Nov/Dec 2019)** ***z*2**  **2*z***  **2**  |