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|  | **UNIT I PARTIAL DIFFERENTIAL EQUATIONS PART-A** | | |
| **1.** | **Eliminate the arbitrary function** ‘f **‘from *z***  ***f***  ***y***  **and form the Partial Differential Equation.**   ***x***      **(April/May 2019)** | | |
|  | Given *z*  *f*  *y*   (1)   *x*      Differentiating (1)partially with respect to x , we get *z*  *p*  *f* '  *y*    *y*  (2) Differentiating(1)  *x*  *x*   *x*2        partially with respect to y , we get *z*  *q*  *f* '  *y*   1  (3)  *y*  *x*   *x*        From (2) & (3) *p*   *y*  *px*  *qy*  0  *q x* | | |
| **2.** | **Find the complete integral for *p***  ***q***  **1. (April/May 2019)** | | |
|  | Given *p*  *q* 1    (1) This of the form *f* ( *p*, *q*)  0 To find Complete Integral:  Let the complete solution of (1) is *z*  *ax*  *by*  *c*    (2)  Let *p*  *a* & *q*  *b* in (1)  (1)  *a*  *b*  1  *b*  1 *a*  *b*  1 *a* 2  Sub *b* in (2) | | |
|  | *z*  *ax*  1 *a* 2 *y*  *c* |    (3) |
| **3.** | **Form a partial differential equation by eliminating arbitrary function ‘f’ from z = f(x+ay)eay .**  **(April/May 2017)** | | |
|  | Given z = f(x+by)eay . (i)  Diff (i) partially w.r.t x and y  We get p = f ‘ (x+by)eay ---(1) , q = f ‘ (x+by)eay .b+ f(x+by)eay. a (2)  Sub.(1) in (2) we get q =az+bp | | |
| **4.** | **Obtain partial differential equation by eliminating arbitrary constant ‘a’ and ‘b’ from**   **x – a****2**  **y – b****2**  **z (Nov/Dec 2019)** | | |
|  | Given x – a2 y – b2  z (1)  Diff (1) partially w.r.t x and y  2x – a  p  x  a  p  2  2y – b  q  y – b  q  2 | | |



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**QUESTION BANK**



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|  | Substituting in (1), we get *p*2  *q*2  4*z* |
| **5.** | **Form the partial differential equation by eliminating the arbitrary constants ‘a’ & ‘b’ from**  **z**  **ax3**  **by 3 (April/May 2014)** |
|  | Given z  ax3  by3 (1)  Diff (1) partially w.r.t x and y  *z*  *p*  3*ax*2 , *z*  *q*  3*by*2  *x* *y*  *a*  *p*      (2) *b*  *q*      (3) 3*x*2 3*y*2  Substituting (2)& (3) in (1) we get px +qy=3z |
| **6.** | **Form the PDE of all spheres whose center lie on the z –axis. (Nov/Dec 2016)** |
|  | The equation of the sphere whose center lie on the z –axis is x2  y2  z  c2  r2 ,where r is constant. Differentiating w.r.t ‘ x’ , 2x  2z cp  0 (1)  Differentiating w.r.t ‘ y ’ , 2y  2z cq  0 (2)  (1)  2x  2(z  c)p  (2)  2y  2(z  c)q which gives x  p  y q  we get qx= py |
| **7.** | **Obtain the partial differential equation by eliminating arbitrary constants ‘a’ and ‘ b’ from**   ***x***  ***a*** **2**   ***y***  ***b*****2**  ***z*2**  **1 . (Nov/Dec 2016)** |
|  |  *x*  *a*2   *y*  *b*2  *z*2  1 (1)  Differentiating (1) partially w.r. t x and y we get  2*x*  *a*  2*zp*  0  *x*  *a*  *zp* (2)  2 *y*  *b*  2*zq*  0  *y*  *b*  *zq* (3)  Substituting (2) & (3) in (1) we get  *z*2 *p*2  *z*2*q*2  *z*2  1 i.e., *z*2  *p*2  *q*2 1  1 |
| **8.** | **Find the complete integral of p+q = pq** |
|  | Given p+q = pq (1)  This is of the type f(p,q)=0  Let z= ax+by+c be the solution for (1)  Partially Diff w.r.t ‘x ‘ and ‘y’ we get p = a and q = b   a  b  ab   b  a  a 1  Therefore z  ax   a  y  c is the complete solution.   a 1     |
| **9.** | **Form a PDE by eliminating the arbitrary constant ‘a ’ & ‘ b ’ from the equation**  **(x**  **a)2**  **(y**  **b)2**  **z2 cot 2**  |
|  | (x  a)2  (y  b)2  z2 cot 2     (1) |

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|  | Differentiating partially with respect to x , we get 2(x  a)  2z p cot2   (2)  Differentiating partially with respect to y , we get 2(y  b)  2z q cot2   (3)  (x  a)  z p cot2  & (y  b)  z q cot2   Substituting in (1)  z p cot2  2   z q cot2  2  z2 cot2    p2  q2  tan 2 |
| **10.** | **Form the partial differential equation by eliminating ‘ g ’ from *g***  ***x*2**  ***y*2**  ***z*2 , *x***  ***y***  ***z***   **0 .** |
|  | We know that if *g*(u, v) = 0 then u = *f* (v)  *x*2  *y*2  *z*2  *f* *x*  *y*  *z* (1)  Differentiating (1) partially w.r. t x and y We get  2*x*  2*zp*  *f* ' *x*  *y*  *z*1 *p* (2)  2*y*  2*zq*  *f* ' *x*  *y*  *z*1 *q* (3)  Divide (2) & (3)  *x*  *zp*  1 *p*  *x*  *qx*  *zp*  *zpq*  *y*  *py*  *zq*  *zpq*  *y*  *zq* 1 *q*   *z*  *y* *p* *x*  *z**q*  *y*  *x* |
| **11.** | **Solve *p*****1**  ***q***  ***qz* . (Nov/Dec 2014)** |
|  | *p* 1 *q*  *qz* - (1)  This equation is of the form *f* *z*, *p*, *q*  0  *z*  *g*  *x*  *ay* be the solution Let *x*  *ay*  *u* and *z*  *g* *u*  *p*  *dz q*  *adz*  *du du*  (1) reduces to  *dz* 1 *a dz*   *a dz z*  1 *a dz*  *az*  *a dz*  *az* 1 *dz*  *z*  1  *dz*  *du du*  *du*  *du du du du a* 1    *z*   *a*  Integrating log  *z*  1   *u*  *b*   *a*      i.e., log *z*  1   *x*  *ay*  *b* is the complete solution.   *a*     |
| **12.** | **Solve z = px + qy + 2 *pq* (Nov/Dec 2013)** |
|  | This of Clairaut’s type and the complete solution is z= ax+by+2 ab  (1)  Diff (1) partially w.r.t ‘a ‘ and ‘b’ we get  *z*  *x*  2 *b*  0, *z*  *y*  2 *a*  0  *a* 2 *a* *b* 2 *b* |



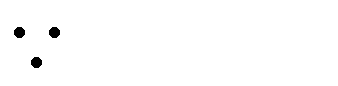


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|  |  x   b & y   a  xy  b a  1 a b a b  xy= 1 is singular integral. |
| **13.** | **Solve xp + yq = z . (Nov/Dec 2012)** |
|  | The equation is of the form *pP*  *qQ*  *R*  Here *P*  *x*, *Q*  *y* & *R*  *z*.  Auxiliary Equation is *dx*  *dy*  *dx*  *x y z*  Considering *dx*  *dy* and integrating w.r.t ‘ x ‘ and ‘ y’ we get   1. *y*   log *x*  log *y*  log *a*  log *a*  log x log y  *a*  *x*  *y*  Considering *dy*  *dz* and integrating w.r.t ‘ y’ and ‘z ‘ we get   1. *z*   log *y*  log *z*  log *b*  log *b*  log *y*  log *z*  *b*  *y*  *z*  The solution is  *x* , *y*   0   *y z*     |
| **14.** | **Solve y(p-2x) = logq. (Nov/Dec 2011)** |
|  | The equation can be reduced to the type F1 x, p  F2 y, q   yp – 2x  log q  p – 2x  log q  k  y   p  2x  k  p  2x  k & log q  k  q  eky  y  We know that z   pdx  qdy z   (2x  k) dx  ekydy  eky  z = (x2+kx)+ +c  k |
| **15.** | ***2z*** ***2z*** ***2z***  **Solve *2*** ***x2***  ***5*** ***x******y***  ***2*** ***y2***  ***0* (Nov/ Dec 2018)** |
|  | Auxiliary Equation is 2*m*2  5*m*  2  0 (Replace D by ‘m’ and D ’ by 1 )  *m*   1 ,2  2  Complimentary function is *z*    *y*  *x*     *y*  2*x*  1  2  2    |
| **16.** | **Find the particular integral of (*D*2**  **2*DD*'** ***D*' 2) *z***  ***ex*** ***y*** |
|  | P.I = 1 *ex* *y*  *D*2  2*DD* ' *D* '2 |

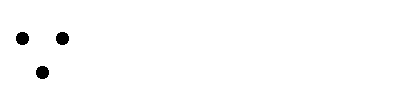
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|  | Replace D by 1 and *D*' by –1 ,we get  1 xy x xy x 2 xy  = e  e  e  1  2 1 2D  2D' 2 | | |
| **17.** |  **2z**  **2z**  **2z**  **Find the Particular Integral**  **2**   **sin(2x**  **3y)**  **x2** **x****y** **y2** | | |
|  | 2z  2z 2z 2 2  2 2  2  sin(2x  3y)  D  2DD' D' z  sin(2x  3y)  x xy y  PI  sin(2x  3y) Replace D2 =  4 , D' 2  9 & DD'  - 6  D2  2DD' D' 2   sin(2x  3y)  sin(2x  3y)   sin(2x  3y)  4  2(6)  9 1 | | |
| **18.** | **Solve**  ***D*3**  **3*DD*' 2** **2*D*' 3**  ***z***  **0 .** | | |
|  | Substituting D  m, & D'  1  The Auxiliary equation is m3  3m  2  0  m = 1, 1, –2  Complimentary function is z =1  *y*  *x*  *x*2  *y*  *x* 3  *y*  2*x* | | |
| **19.** | **Solve**  ***D***  ***D***  **1** ***D***  ***D***  **2** ***z***  ***e*2*x*** ***y* .** | | |
|  | This is of the form *D*  *m*1*D* *C*1 *D*  *m*2*D* *C*2  *D*  *mnD* *Cn*  *z*  0  Hence *m*1  1 *c*1  1 *m*2  1 *c*2  2  Hence the C.F. is *z*  *ex*  *y*  *x*  *e*2*x*  *y*  *x*  1 2  *e*2 *x* *y e*2 *x* *y* 1  *y*  P.I.          *e*2 *x*  *D*  *D* 1 *D*  *D*  2 2 11 2 1 2 2  Hence, the complete solution is *z*  *ex*  *y*  *x*  *e*2*x*  *y*  *x*  1 *e*2*x* *y*  1 2 2 | | |
| **20.** | **Solve (*D***  **1)(*D***  ***D*'**  **1)*z***  **0** | | **(Nov/Dec 2012)** |
|  | (*D* 1)(*D*  *D*' 1)*z*  0  Here c1  1, c2  1 , m1  0, m2  1  General solution *z*  *ex f* ( *y*)  *e**x f* ( *y*  *x*)  1 2 | | |
|  | **PART - B** | | |
| **1.** | **a)** | **Find the singular integral of *z***  ***px***  ***qy***  ***p*2**  ***pq***  ***q*2** | **(Nov/Dec 2019)** |
|  | **b)** | **Solve the Lagrange’s linear equation (x2 -yz)p+(y2 -zx)q=z2 -xy** | **(Nov/ Dec 2018)** |
| **2.** | **a)** | **Solve** **D2**  **DD '** **6D ' 2**  **z**  **x2y**  **y cos x** | |
|  | **b)** | **Solve z2(p2**  **q2 )**  **x**  **y** | **(Nov/Dec 2020)** |
| **3.** | **a)** | **Find the singular solution of the equation z=px**  **qy+p2q2** | **(April/May 2019)** |
|  | **b)** | **Solve 2(z**  **xp**  **yq)**  **yp2** | **(Nov/Dec 2020)** |

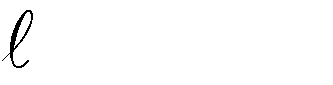
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| **4.** | **a)** | **Form the partial differential equation by eliminating the arbitrary function from**  **f** **x2**  **y2 , z**  **xy**   **0 (Nov/Dec 2018)** |
|  | **b)** | **2z** **2z 2**  **Solve**  **2**  **sin(x**  **2y)**  **3x y**  **2x** **x2****y (Nov/Dec 2020)** |
| **5.** | **a)** | **Find the singular solution of the equation z=px**  **qy+ 1**  **p2**  **q2 (Nov/Dec 2020)** |
|  | **b)** | **Solve the partial differential equation** **D2**  **2DD**  **D****2**  **2D**  **2D** **z**  **sin(x**  **2y)**  **(Nov/ Dec 2018)** |
|  | **UNIT II FOURIER SERIES PART-A** | |
| **1.** | **State the sufficient condition for a function f(x) to be expressed as a Fourier series. (Nov/Dec 2020)** | |
|  | A function f(x) can be expanded as a Fourier series in the interval (*c*, *c*  2*l*) if the following conditions are satisfied.   1. f(x) is periodic, single valued and finite in (*c*, *c*  2*l*) 2. f(x) has only finite number of finite discontinuities and no infinite discontinuities in (*c*, *c*  2*l*) . 3. f(x) has only finite number of maxima and minima in (*c*, *c*  2*l*) | |
| **2.** | **Does f (x)**  **tan x possess a Fourier expansion?** | |
|  | *f* ( *x*)  tan *x*  sin *x* does not possess a Fourier expansion because the function has an infinite discontinuity  cos *x*  at the point *x*   .  2 | |
| **3.** | **Determine the value of** *an* & *a*0 **in the Fourier series expansion of f(x)**  **x in** **2**  ***x***  **2**  **3** | |
|  | *f* (*x*)  *x*3  *f* (*x*)  (*x*)3  *x*3   *f* (*x*)  *f* (*x*) is an odd function  *a*  *a*  0  *n* 0 | |
| **4.** | **Find the Fourier constant bn for *x sin x* in**   ***x***   **, when expressed as a Fourier series.** | |
|  | *f* ( *x*)  *x* sin *x*,    *x*    *f* (*x*)  (*x*) sin( *x*)  *x* sin *x*  *f* ( *x*)   *f*  *x* is an even function *bn*  0 | |
| **5.** | **Give the expression for the Fourier series coefficient bn for the function f(x) defined in** **2**  ***x***  **2 .** | |
|  | *l*  *U*.*L*  *L*.*L*  2  (2)  4  2 2 2 2  1 *l*  *n*  1 2  *n* *x*   *bn*  *l*  *f* (*x*) sin  *l*  *xdx*  *bn*  2  *f*  *x*sin  2  *dx*  *l*   2   | |
| **6.** | **Find bn of the Fourier series for the function f(x) = x2 , –**  **< x <**  **(Apr/May 2018)** | |
|  | f **x**  **x2** , –   **x**    **f** **x**  **x****2**  **x2**  **f** **x**  *f(x)* is an even function  bn = 0 | |
| **7.** | **If the Fourier series of the function f(x) = x + x2, in the interval (–** **,** **) is**  **2**    ***n***  **4 2**  **,**  **1**  **2 cos *nx***  **sin *nx***  **3 *n*****1**  ***n n***   **then find the value of he infinite series 1**  **1**  **1**  **... (Nov/Dec 2017)**  **12 22 32** | |

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|  | *x*      2     2  2  (1)*n*  Put then   4 cos *n*  2 3 *n*1 *n*2   2   2   (1)*n* (1)*n*    4  3 *n*1 *n*2  2 2   (1)2*n*  3  4 *n*1 *n*2  1  1  1   2  12 22 32 ...  6 |
| **8.** | **Find the value of the Fourier series for *f* ( *x*)**  **0 *in* (*****c*, 0) at the point of discontinuity *x=0.***    **1 *in* (0, *c*)**  **(May/June 2016)** |
|  | f (0 )  f (0 )  x=0 is a point of discontinuity, sum of Fourier series of f(x) is .  2  f(0-) = lim *f* (0  *h*)  lim 0  0 .  *h*0 *h*0  f(0+) = lim *f* (0  *h*)  lim1  1 .  *h*0 *h*0  f(x) at x = 0 is 0 1  1 .  2 2 |
| **9.** | **Sketch the even extension of the function f(x) = sin x, 0**  ***x***   **(Nov/Dec 2019)** |
|  |    0    2 2 |
| **10.** | **What is the behavior of Fourier series of a function f(x) at the point of discontinuity? (Nov/Dec 2018)** |
|  | At the point of discontinuity , f(x) converges to *f* (*x*)  1  *f* *x*   *f* *x*   2 |
| **11.** | **The cosine series for f(x) = x sin x in 0**  ***x***   **is given as**  **x sin x = 1**  **(****1)*n* .Deduce that 1**   **1**  **1**  **1**  **...**   **.**    **1**  **cos *x***  **cos *nx***    **2 *n*****2 *n*2**  **1**  **1.3 3.5 5.7**  **2** |
|  |    1     (1)*n*  *n*  Put x = ,which is a point of continuity => sin  1 cos   2 2 cos  2 2 2 2  2  *n*2 *n* 1  2     (1)*n*  *n*    i.e.  1  2 cos  cos  0  2 2 *n* 1  2  2   (1)*n*  *n*      *n*2 1 cos 2  1 2  *n*2   |



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|  | 1 cos  1 cos 3   1 cos 2  1 cos 5  ..   1   3 8  2  15 24  2  . 2          1  1  1  ...  1    3 15 35 2  1  1  1  ...  1    1.3 3.5 5.7 2  i.e. 1  1  1  1        1.3 3.5 5.7 ... 2   |
| **12.** | **Find the value of** *an* **in the cosine series expansion of f(x) = K in the interval (0, 10), K is a constant.** |
|  | 2 *l n**x*  *an*  *l*  *f* (*x*) cos *l dx*  0    *n* *x* 10  2 10  *n* *x*  *K* sin  10  10 *K*  *an*   *K* cos  *dx*      sin *n*  sin 0  0 sin *n*  sin 0  0 10 0  10  5  *n*  5*n*   10 0 |
| **13.** | **If f(x) = x2+x is expressed as a Fourier series in the interval (-2,2) to which value this series converges at x = 2?** |
|  | f (2 )  f (2 ) [(2)2  2] [(2)2  2]  2  6   Since x = 2 is an end point then f(x) converges to 4  2 2 2 |
| **14.** | **State Parseval’s theorem in the interval (c, c+2l).** |
|  | If the Fourier series corresponding to f(x) converges uniformly to f(x) in (c,c+2l) then  *c*2 2   1   *f* (*x*) 2 *dx*  *a*0   *a* 2  *b* 2   2 *n n*  *c n*  1 |
| **15.** | **If the fourier series of the function f(x) = x for –**  **< x <**  **with period 2** **is given by**  ***f* ( *x*)**  **2** **sin *x***  **sin 2 *x***  **sin 3 *x***  **...** **then find the sum of the series 1**  **1**  **1**  **... (Apr/May 2015)**   **2 3**  **3 5** |
|  | Put x =  ,which is a point of continuity =>   2 sin   1 sin 2    1 sin 3   ..   2 2  2 2  2  3  2  .           1 1  1 1   i.e.  2 1 (0)  (1) ... => 1   ...   2  2 3  3 5 4 |
| **16.** | **What do you mean by Harmonic analysis?** |
|  | The process of finding the harmonics in the Fourier series expansion of a function numerically is known as harmonic analysis. |
| **17.** | **Obtain the constant term of the Fourier cosine series of y = f(x) in (0,6) using the following table x: 0 1 2 3 4 5**  **y: 9 18 24 28 26 20** |
|  |  *y*  125  *a*0  *a*0  2 *N*  2  6   41.67  Constant term = 2  20.83    |
| **18.** | **Find the root mean square value of the function f(x) = x in (0,*l*)** |





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| *b l l*  RMS value =  1  2  1  2  1  *x*3   *l*3  *l y b*  *a f* (*x*) *dx l*  0 *x dx l*  3  3*l*  *a* 0  0 3 | | |
| **19.** | **Find the constant term in the expression of cos2 *x* as a Fourier series in the interval (** **,** **) .** | |
| 1  1  1  cos 2*x*  1  sin 2*x*   *a*0    cos *xdx*     *dx*  *x*    2     2  2  2   *a*  1   sin 2   ( )  sin( 2 )   2  1  0 2  2   2  2       Constant term = *a*0  1  2 2 | | |
| **20.** | **π2**  **(**  **1)n 1 1 1 π2**  **If x2**   **4**  **cosnx,deducethat**    **...**  ***in* [** **,** **] 3 n****1 n2 12 22 32 6** | |
| Put *x*   then   2   2   (1)*n*  4 cos *n*  3 *n*1 *n*2   2   2   (1)*n* (1)*n*  4 | | |
| 3 *n*1 *n*2  2 2   (1)2*n* | | |
| 3  4 *n*1 *n*2  1  1  1   2  12 22 32 ...  6 | | |
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|  |  | **PART - B** |
| **1.** | **a)** | **Find the Fourier series *f* ( *x*)**  ***eax* in 0**  ***x***  **2** **. (Nov / Dec 2018)** |
|  | **b)** | **1 1 1**  **2**  **Find the Fourier series of f(x) = x2 in (0, 2l). Hence deduce that**    **...**     **12 22 32 6**  **(Nov/Dec 2019)** |
| **2.** | **a)** | **Find the complex form of the Fourier series of *f* ( *x*)**  **cos *ax* in**  **,**  **, where ‘a’ is neither zero nor an integer. (Nov/Dec 2019)** |
|  | **b)** | *x*,    *x*  0 1 1  2  **Obtain the Fourier series expansion of** *f* (*x*)   *x* , 0  *x*   **and evaluate**1 32  52 ...  8    **(Nov/Dec 2020)** |
| **3.** | **a)** | **Find the half range Fourier cosine series for *f* ( *x*)**  ***x*(**  ***x*) in the interval (0,** **)**  **(Apr/May 2018)** |
|  | **b)** | **Find the Fourier series expansions of** *f* (*x*)  *x*2  *x* **in** ( , ) **of periodicity** 2 **. (Nov/Dec 2020)** |
| **4.** | **a)** | **Find the first two harmonic of the Fourier series of f (x), given by (Apr/May 2018)**  ***x* 0**  2  4 5 2    3 3 3 3  ***f (x)* 1 1.4 1.9 1.7 1.5 1.2 1.0** |



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|  | **b)** | **Find the Fourier series expansion of *f* ( *x*)**  **sin *ax* in** ***l*, *l***  **(Nov/Dec 2019)** |
| **5.** | **a)** | **Obtain half range Fourier cosine series expansion of** *f* (*x*) (*x* 1)2 **in the interval** 0  *x* 1 **and**  1 1  2  **evaluate** 1   ...   22 32 6 **(Nov/Dec 2020)** |
|  | **b)** | **Obtain the constant term and the first three harmonics in the Fourier Cosine series of y = f(x) in (0,6) from the following table (Nov/Dec 2019)** |
|  | **UNIT- III - APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS** | |
|  | **PART - A** | |
| **1.** | **Solve 3 *x*** ***u***  **2 *y*** ***u***  **0 by the method of separation of variables. (Nov/Dec 2015)**  ***x*** ***y*** | |
|  | Let u = X(x) Y(y)  ux = X’(x) Y(y) and uy = X(x) Y’(y)  3*x*X Y  2 *y*X Y  0  3*x*X Y  2 *y*X Y  3*x* X  2y Y  *k*  X Y  X  *k and* Y  *k*  X 3*x* Y 2y  *Integrating*  *k k k k*  *X*  *c*1 *x* 3 , *Y*  *c*2 *y* 2  *u*  *cx* 3 *y* 2 | |
| **2.** | **State any two assumptions made in the derivation of the one dimensional wave equation.**  **(Nov/Dec 2016)** | |
|  | 1. The string is perfectly elastic and does not offer any resistance to bending. 2. The mass of the string per unit length is constant. | |
| **3.** | **Write down the partial differential equation governing one dimensional wave equation?** | |
|  | 2 *y*  2 2 *y*  *t* 2 *C* *x*2 where y(x,t) is the displacement of the string **.** | |
| **4.** | **2 *y*** **2 *y***  **In the wave equation**  ***C* 2 , what does *C2* stands for? (May/June2013)**  ***t* 2** ***x*2** | |
|  | *C* 2  *T* , where *T* is the tension and *m* is the mass of the string.  *m* | |
| **5.** | **Write all three possible solutions of one dimensional heat equation. (Apr/May2019)** | |
|  | 1. yx , t   Aepx  Be– px  Cepat  De– pat  2. *y(x,t)* = (A cos *px* + Bsin *px*) (C cos *pat* + *D*sin *pat*). 3. *y(x,t)* = (A*x* +B) (C*t* + D) | |
| **6.** | **Classify the two-dimensional steady state heat conduction equation. (Nov/Dec 2019)** | |
|  | *uxx*  *uyy*  0 is the steady state heat conduction equation. | |

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| **x** | **0** | **1** | **2** | **3** | **4** | **5** |
| **f (x)** | **4** | **8** | **15** | **7** | **6** | **2** |

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|  | Here, *A*  1, *B*  0, *C*  1  *B*2  4*AC*  4  0   Elliptic. |
| **7.** | **Classify the partial differential equation** **1**  ***x*2**  ***z***  **2 *xy z***  **1**  ***y*2**  ***z***  ***xz***  **3*x*2 *yz***  **2*z***  **0**  ***xx xy yy x y*** |
|  | Compare the given equation with *Az*  *Bz*  *Cz*  *f* *x*, *y*, *z*, *z* , *z*   0  *xx xy yy x y*  Where *A*  1 *x*2 , *B*  2*xy*, *c*  1 *y* 2    B2  4AC  (2xy)2  4(1 x2 )(1 y2 )  4x2y2  4  4x2  4y2  4x2y2  4x2  4y2  4  If   0  4*x*2  4 *y* 2  4  *x*2  *y* 2  1  then the given PDE is parabolic on the circle *x*2  *y* 2  1  If   0  4*x*2  4 *y* 2  4  *x*2  *y* 2  1  then the given PDE is elliptic inside the circle *x*2  *y* 2  1  If   0  4*x*2  4 *y* 2  4  *x*2  *y* 2  1  then the given PDE is Hyperbolic outside the circle *x*2  *y* 2  1 |
| **8.** | **Give the mathematical formulation of the problem of one-dimensional heat conduction in a rod of length**  *l* **with insulated ends and with initial temperature** *f* (*x*). **(Nov/Dec 2019)** |
|  | Mathematical formulation,  To solve *ut*  *c uxx* subject to,  2  *u*(*o*, *t*)  *u*(*l*, *t*)  0 *t*  0 and *u*(*x*, 0)  *f* (*x*), 0  *x*  *l* |
| **9.** | **Write the boundary conditions and initial conditions for solving the vibration of string equation, if the string is subjected to initial displacement *f(x)* and initial velocity *g(x)*. (Nov/Dec2006,07)** |
|  | The initial and Boundary conditions are:  i) y(0, t)  0, t  0 ii) y(*l*, t)  0, t  0 iii) y (x, 0)  g(x), 0  x  *l* iv) y(x, 0)  f (x), 0  x  *l*  t |
| **10.** | **What are the various possible solutions of one dimensional heat flow equation?**  **(Nov/Dec 2010,2015,2016) & (Apr/May 2018)** |
|  | 2 2   1. *u(x ,t)* = (*Aepx*  *Be* *px* ) C*e* *p t*   2 2   1. u(x ,t )= (*A*cos *px*  *B*sin *px*)*Ce* *p t* 2. u(*x ,t)* = *Ax* + *B* |
| **11.** | **Why can’t u(x ,t) = ( *Aepx***  ***Be*** ***px* )*Ce*****2 *p*2*t* be the correct solution in solving one dimension heat equation?** |
|  | As *t* *, u*  . It is not possible. |
| **12.** | **What is meant by steady state? In steady state condition derive the solution of one dimension heat flow equation. (Nov/Dec 2013, 2020)** |
|  | The state in which the temperature does not vary with respect to time ‘t’ is called steady state. Therefore when steady state condition exists, u(x, t) becomes u(x).  *u*   2  2*u*  The one dimension heat flow equation is *t* *x*2 .  *u*   2*u* *u*  In steady state condition, *t* 0 .  *x* 2  0  *x*  *A*  *u*  *Ax*  *B* . |
| **13.** | **2*u*** **2*u*** **2*u*** ***u*** ***u***  **Classify the partial differential equation 3** ***x*2**  **4** ***x******y***  **6** ***y*2**  **2** ***x***  ***y***  ***u***  **0 (Apr/May 2008)** |

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|  | 2*u* 2*u* 2*u* *u* *u*  Given 3 *x* 2  4 *x**y*  6 *y* 2  2 *x*  *y*  *u*  0  Compare the given equation with *Au*  *Bu*  *Cu*  *f* *x*, *y*,*u*,*u* ,*u*  0  *xx xy yy x y*  Where Where *A*  3, *B*  4, *C*  6    *B* 2  4 *AC*  16  72  56  0   The given PDE is Elliptic. |
| **14.** | **A plate is bounded by the lines *x=0,y=0, x=l* and *y=l*. Its faces are insulated. The edge coinciding with *x-* axis is kept at 100****C. The edge coinciding with y*-*axis is kept at 50****C. The other two edges are kept at 0****C. Write the boundary conditions that are needed for solving two dimensional heat flow equation.**  **(Nov/Dec2011,2012)** |
|  | **Boundary conditions are:**  *i*) *u*(*x*,0)  100∘ *C* ; 0  *x*  *l ii*) *u*(0, *y*)  50∘ *C* ; 0  *y*  *l iii*) *u*(*x*,*l*)  0∘ *C* ; 0  *x*  *l*  *iv*) *u*(*l*, *y*)  0∘ *C* ; 0  *y*  *l* |
| **15.** | **What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation with respect to the time? (May/June 2017)** |
|  | The solution of one dimensional wave equation is of periodic in nature. But the solution of one dimensional heat equation is not periodic in nature. |
| **16.** | **Given the boundary conditions on a square or rectangular plate, how will you identify the proper solution?** |
|  | If the non-zero temperature is prescribed either on x=0 or on x=a (vertical edges), the proper solution will be  *u(x ,y)* = (Ae*px* + Be *– px*) (C cospy + Dsin *py*)  If the non-zero temperature is prescribed either on y=0 or y=b (horizontal edges), the proper solution will be  *u(x ,y)*= (A cos *px* + Bsin *px*) (C e *py* + De – *py*) |
| **17.** | **An infinitely long plate is bounded by two parallel edges and an end at right angles to them. The breath of the edge y = 0 is**  **and it is maintained at constant temperature u0 at all points and the other edges are**  **kept at zero temperatures. Formulate the boundary value problem to determine the steady state temperature.** |
|  | 2*u*  0 subject to  *u*(0, *y*)  0 , *u*( , *y*)  0 *for y*  0  *u*(*x*, )  0 *and u* (*x*,0)  *u*0 *for* 0  *x*   |
| **18.** | **Write the possible solutions of the steady state two dimensional heat flow equation** 2*u*  2*u*  **.**  0  *x*2 *y* 2  **(Apr/May 2011,2012), (Nov/Dec 2020)** |
|  | The possible solutions of the steady state two dimensional heat flow equation 2*u*  2*u*  are  0  *x*2 *y* 2  *u*  *x* , *y*    *Aepx*  *Be*– *px*  *Ccos py*  *Dsinpy*   *u*  *x* , *y*   *A cospx*  *Bsinpx* *C epy*  *De*– *py*   *u*  *x*, *y*   *Ax*  *B* *Cy*  *D* |
| **19.** | **A rod 40 cm long with insulated sides has its ends *A* and *B* kept at 20****C and 60****C respectively. Find the steady state temperature at a location 15 cm from *A.* (Apr/May 2011 & Nov/Dec 2012)** |

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|  |  2*u*  When the steady state condition prevail the heat flow equation is *x* 2  0  The steady state temperature is *u*(*x*)  *Ax*  *B*       (1)  The boundary conditions are *u*(0)  20      (2) and *u*(40)  60      (3)  Applying (2) in (1)  *u*(0)  *A*(0)  *B*  20  *B*  20  Sub B in (1)  *u*( *x*)  *Ax*  20        (4)  Sub (3) in (4)  *u*(40)  40*A*  20  60  *A*  60  20  1  40  Sub *A* in (4)  *u*( *x*)  *x*  20  The steady state temperature at a location 15 cm from *A* is  *u*(15)  15  20  35∘ *C* | |
| **20.** | **The ends *A* and *B* of a rod 20cm long have the temperature at 30** ∘ **C and 80** ∘ **C until steady state prevails.**  **Find the steady state temperature. (Nov/Dec 2014)** | |
|  |  2*u*  When the steady state condition prevail the heat flow equation is *x* 2  0  The steady state temperature is *u*( *x*)  *Ax*  *B*        (1)  The boundary conditions are *u*(0)  30      (2) and *u*(20)  80      (3)  Applying (2) in (1)  *u*(0)  *A*(0)  *B*  30  *B*  30  Sub B in (1)  *u*( *x*)  *Ax*  30        (4)  Sub (3) in (4), we get *u*(20)  20*A*  30  80  *A*  80  30  5  20 2  Sub *A* in (4)   *u*(*x*)  5 *x*  30 which is the required temperature.  2 | |
|  | **PART B** | |
| **1.** | **a)** | **A uniform string is stretched and fastened to two points** '*l*' **apart. Motion is started by displacing the string into the form of the curve *y***  ***kx*(*l***  ***x*) and then releasing it from this position at time *t***  **0 . Find the displacement of the point of the string at a distance** *x* **from one end at time** *t* **.**  **(Nov/Dec 2018)** |
|  | **b)** | **Using the method of separation of variables solve** ***u***  **2** ***u***  ***u*** , **where *u*( *x*, 0)**  **6*e*****3*x***  ***x*** ***t***  **(Nov/Dec 2018)** |
|  | **c)** | **Solve *u***  ***a*2*u* by the method of separation of variables and obtain all possible solutions.**  ***t xx***  **(Nov/Dec 2019)** |
|  | **d)** | **Solve the problem of a tightly stretched string with fixed end points *x***  **0 & *x***  ***l* which is initially**  **in the position *y***  ***f* ( *x*) and which is initially set vibrating by giving to each of its points a velocity**  ***dy***  ***g*(x) *at t***  **0. (Nov/Dec 2019)**  ***dt*** |
|  | **e)** | **Classify the partial differential equation (1–x2)fxx – 2xyfxy + (1–y2)fyy=0 (Nov/Dec 2019)** |
| **2.** | **a)** | **A tightly stretched string of length *l* is fastened at both end A & C. The string is at rest, with the** |

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|  |  | **point B (x = b) drawn aside through a small distance ‘d’ and released to execute small transverse vibration. Find the transverse displacement of any point of the string at any subsequent time.**  **(Nov/Dec 2020)** |
|  | **b)** | **The ends A and B of a rod *l* cm long have their temperatures kept at** 20 *C* **and** 40 *C* **, until steady state conditions prevail. The temperature of the end B is suddenly reduced to** 10 *C* **and that of A is increased to** 50 *C* **. Find the steady state temperature distribution in the rod after time t.**  **(Apr/May,2018)** |
| **3.** | **a)** | **If a string of length** *l* **is initially at rest in its equilibrium position and each of its points is given a**  ***cx* ; 0**  ***x***  ***l***  **velocity** *v* **such that *v***   **2 , find the displacement at any time** *t* **.**   ***l***  ***c*(*l***  ***x*);**  ***x***  ***l***   **2** |
|  | **b)** | **2*u*** **2*u***  **Derive the general solutions for one dimensional wave equation**  ***c*2 using separation of**  ***t* 2** ***x*2**  **variables method (Nov/Dec 2018)** |
| **4.** | **a)** | **A square plate is bounded by the lines *x***  **0, *x***  **20, *y***  **0and *y***  **20 . Its faces are insulated. The**  **temperature along the upper horizontal edge is given by u(x,20) = x (20 – x) when 0 < x < 20, while the other three edges are kept at** 0 *C* **. Find steady state temperature distribution** *u*(*x*, *y*) **in the**  **plate. (Nov/Dec 2016)** |
|  | **b)** | **A rectangular plate with insulated surface is 8 cm wide and so long compared to its width that it may be considered as an infinite plate. If the temperature along the short edge y = 0 is**  ***u*( *x*,0)**  **100sin**   ***x***  **, 0 < x < 8 while two long edges x = 0 & x = 8 as well as the other short edge are**   **8**      **kept at** 00 *C* **, then find the steady state temperature at any point of the plate. (Nov/Dec 2019)** |
| **5.** | **a)** | **A rectangular plate with insulated surface is 20cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at**  **10 *y*, 0**  ***y***  **10**  **short edge *x***  **0 is given by *u***   **and the two long edges as well as the**  **10(20**  ***y*), 10**  ***y***  **20**  **other short edges are kept at** 0∘ *C* **. Find the steady state temperature distribution in the plate.**  **(Apr/May,2017)** |
|  | **b)** | **A uniform bar of length *l* through which heat flow is insulated at its sides. The ends are kept at zero temperature. If the initial temperature at the interior points of the bar is given by *k* (*l x***  ***x*2 ) ,**  **for 0 < x < *l* , find the temperature distribution in the bar after time *t*. (Nov/Dec 2020)** |
|  | **UNIT IV – FOURIER TRANSFORMS PART - A** | |
| **1.** | **Write the Fourier transform pair.** | |
|  | The Fourier transform pair is defined as  *F* *f* *x*  1  *eisx f* *x**dx*  *F**s* : *F* 1 *F*  *f*  *x*  1  *e**isxF* *s**ds*  *f*  *x*  2     2     | |
| **2.** | **State Fourier integral theorem**. | |
|  | If *f* *x*is piecewise continuous, differentiable and absolutely integrable in  ,  then  *f* *x*  1   *f* *t* *eis**x*  *t*  *dtds*  2      | |



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| **3.** | **1; for x**  **2**  **Find the Fourier transform of f(x)**    **0; for x**  **2** |
|  | The Fourier transform of the function *f* (*x*) is  *F* *f* (*x*) 1  *f* (*x*)*eisxdx* .    2    1 2 1  *eisx* 2 1  *ei*2*s e**i*2*s*    *F*  *f* (*x*)   1.*eisxdx*         2 2 2  *is* 2 2  *is is*    1  *ei*2*s*  *e**i*2*s*   1 1 (2*i* sin 2*s*)  *F* (*s*)  2 sin 2*s*     2  *is*  2 *is*  *s* |
| **4.** | **If F(s) is the Fourier transform of f (x), then show that *F*{ *f* ( *x***  ***a*)}**  ***eias F* (*s*)** |
|  | 1  1   F {f(x-a)}=  *f* (*x*  *a*)*eisxdx* =  *f* (*t*)*ei*(*a**t*)*s dt* where x - a = t  2  2   *ias* 1  *its ias*  = *e*  *f* (*t*)*e dt* = *e* F(s) = *eias* F{f(x)}  2  |
| **5.** | **Write the Fourier sine transform pair and Fourier Cosine transform pair.** |
|  | The Fourier sine transform pair is defined as  2  1   2   *Fs*  *f* *x*    *f* (*x*) sin *sxdx*  *Fs* *s* : *F* *F* *s*  *f*  *x*    sin *sx F* *s*  *ds*  *s s*  0 0  The Fourier cosine transform pair is defined as     *F*  *f* *x*  2 *f* (*x*) cos *sxdx*  *F* *s*: *F* 1 *F* *s*  *f*  *x*  2 cos *sx F* *s* *ds*  *c*   *c*  *c*    *c*  0 0 |
| **6.** | **0 , *x***  ***a***  **Find the Fourier transform of f(x) defined by *f* ( *x*)**   **, *a***  ***x***  ***b***  **1**  **0 , *x***  ***b* .**   |
|  | *F* *f* *x* 1  *isx f* *x**dx*   *e*  2     1  *a eisx*.0 *dx*  *beisx*.1*dx*   *eisx*.0 *dx*  2        *a b*   1 *b isx* 1  *eisx*  *a* 1      *e* .1*dx*     *eias*  *eibs*  2 *a* 2  *is*  *b is* 2 |
| **7.** | **State Convolution theorem in Fourier Transform. (Nov/Dec 2018 & Nov/Dec2019)** |
|  | The Fourier transform of the convolution of *f* *x* and *g**x* is the product of their Fourier transforms  i.e. *F**f* *x* *g**x* *F**f* *x**F**g**x* |

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| **8.** | **State Parseval’s identity on Fourier Transform.** |
|  |  2  2   *F* (*s*) *ds*   *f* (*x*) *dx*, *where F*[ *f* (*x*)]  *F* (*s*)      |
| **9.** | **If *F***  ***f* ( *x*)**  ***F*(*s*), then find *F*** ***eiax f* ( *x*)****.** |
|  | *F* *f* (*x*) *F*(*s*)  1  *f* (*x*)*eisxdx*    2    *F**eiax f* (*x*) 1  *iax f* (*x*)*eisxdx*  1  *f* (*x*)*ei*(*s*  *a*)*xdx*  *F* (*s*  *a*)   *e*   2   2   |
| **10.** | **Find the Fourier transform of e****a |x|, if a** > **0** |
|  |  *F*  *f* (*x*) 1  *e**a x eisxdx*  1  *e**a x* (cos *sx*  *i* sin *sx*)*dx*     2  2    1   *e**a x* cos *sxdx*  *i*  *e**a x* sin *sxdx*  1   *e**ax* cos *sxdx*  2  *a*       2    2 2   2    2  0    (*s*  *a* )     |
| **11.** | **Find the Fourier sine transform of e****ax , *a***  **0 . Hence find F** **xe**  **ax**  **.**  **s**   |
|  | 2   *Fs*  *f* (*x*)   *f* (*x*) sin *sxdx* .  0  *F* *e* *ax*   2  *e* *ax* sin *sxdx*  2 *s s*      *s* 2  *a*2  0  We know that by property *F* *xf* *x*   *d F*  *f* *x*  *S ds C*         *ax*  *d* 2 *s* 2 (*s* 2  *a* 2 ).1 *s*.(2*s*)  2  *s* 2  *a* 2    *F xe*           *s*   *ds*  *s* 2  *a* 2    2 2 2     2 2 2     *s*  *a*     *s*  *a*            |
| **12.** | **Define Self-reciprocal function under Fourier transform and give an example.** |
|  | If *f* *s* is the Fourier transform of *f* *x*, then *f* *x*is said to be self-reciprocal under Fourier    *x*2   *s*2  transform. *F*  *e* 2   *e* 2 .       |
| **13.** | **1**  **s**   **Prove that Fc** **f** **ax**  **Fc**   **,*a***  **0**  **a**  **a**  |
|  |   *Fc*  *f* *ax*  2  *f* (*ax*) cos *sxdx* , *Put ax*  *t*, *adx*  *dt*, *dx*  *dt*   0 *a* |

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|  | 2   *st*  *dt*  *when x*  0, *t*  0 *and x*  , *t*   *Fc*  *f* *ax*    *f* (*t*) cos *a*  *a*  0    1 2   *s*  1  *s*   *Fc*  *f* *ax*  *a*   *f* (*t*) cos *a*  *t dt* = *a Fc*  *a*  .  0     |
| **14.** | **State Parseval’s identity in Fourier sine and cosine Transform.** |
|  |       *Fc* (*s*) 2 *ds*  *f* (*x*) 2 *dx* &  *Fs* (*s*) 2 *ds*  *f* (*x*) 2 *dx*  0 0 0 0 |
| **15.** | **Find the Fourier Sine Transform of *f* ( *x*)**  ***k*, 0**  ***x***   **,exist? Justify your answer. (Nov/Dec 2018)** |
|  | No.Because sinx and cosx are bounded between -1 and 1.sin∞ and cos∞ are not defined. |
| **16.** | **Prove that *F***  ***f* ( *x*)cos *ax***  **1** ***F* (*s***  ***a*)**  ***F* (*s***  ***a*)** **where *F***  ***f* ( *x*)**  ***F* (*s*)is the Fourier Cosine**  ***c* 2**  ***c c***  ***c c***  **transform of f(x). (Nov/Dec 2019)** |
|  |   *F*  *f* (*x*) cos *ax*  2 *f* (*x*) cos *ax* cos *sx dx*  *c*    0  2   =   *f* (*x*) cos *sx* cos *ax dx*  0  2  1  =   *f* (*x*) 2 cos(*s*  *a*)*x*  cos(*s*  *a*)*x**dx*  0  1  2  2    **=**   *f* (*x*) cos(*s*  *a*) *x dx*   *f* (*x*) cos(*s*  *a*) *x dx*  2   0  0  |
| **17.** | **If F [f(x)] = F(s) then *F* ( *f* (*ax*))**  **1 *F***  ***s***  **, *a***  **0**  ***a***  ***a***     |
|  | **By definition**    *F*  *f* *ax* 1  *f* (*ax*) *eisx dx* ,  2   If a > 0 *Put ax*  *t*, *adx*  *dt*, *dx*  *dt*  *a*  1  *i* *s* *t dt* 1  *s*   *when x*    *t*   *and x*    *t*    *F*  *f* *ax*   *f* (*t*) *e*  *a*  = *F*   . –(1)       2  *a a*  *a*   If a < 0 *Put ax*  *t*, *adx*  *dt*, *dx*  *dt*  *a*  *when x*    *t*   *and x*   *t*    1  *i* *s* *t dt* 1  *i* *s* *t dt* 1     *F*  *f* *ax*   *f* (*t*) *e*  *a*    *f* (*t*) *e*  *a*  = *F*  *s*  . ---(2)         2  *a* 2  *a a*  *a*   From (1) & (2) we get *F* ( *f* (*ax*))  1 *F*  *s* , *a*  0  *a*  *a*     |

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| **18.** | ***1***  ***x 2 ; 0***  ***x***  ***1***  **Find the Fourier cosine transform of *f(x)***   **.**  ***0 , otherwise*** | | |
|  |   *FC*  *f* (*x*)  2  *f* (*x*) cos *sxdx*   0  1   *Fc*  *f* (*x*)  2  (1  *x* 2 ) cos *sxdx*   0  2  2 sin *sx*   cos *sx*    sin *sx* 1    (1  *x* ) *s*  (2*x*) *s* 2   (2) *s*3       0   2   2 cos *s*  2 sin *s*   2 2  sin *s*  *s* cos *s*     *s* 2 *s*3    *s*3       | | |
| **19.** | **1**  **Find the Fourier Sine transform of .**  **x** | | |
|  |  1  2  sin *sx* 2  sin *t* 2      *Fs*  *x*     *x dx*    *t dt*    2   2    0 0   | | |
| **20.** | **Find** *f* (*x*) **from the integral equation**   **s**   **f(x)cos *s* xdx**  **e**  **0** | | |
|  | Given  *f* (*x*) cos *sxdx*  *e**s* ,    0  2  *f* (*x*) cos *sx dx*  2 *e**s*      0  2  2  2  2  1   *f* (*x*)    *Fc* (*s*) cos *sx ds*     *e* cos *sx ds*    1 *x*2   *s*    0 0   | | |
|  | **PART - B** | | |
| **1.** | **a)** | **1**  ***x*2 ; *x***  **1**  **Find the Fourier transform of *f* ( *x*)**   **. Hence deduce that**  **0; *x***  **1**   ***x* cos *x***  **sin *x***  ***x***    ***x*3 cos**  **2** ***dx*.**  **0**   | **(Nov/Dec 2018)** |
|  | **b)** | **Find the Fourier transform of *f* ( *x*)**  **cos *x*, 0**  ***x***  **1** | **(Nov/Dec 2018)** |
| **2.** | **a)** | ***e*** ***ax***  **Find the Fourier sine transform of *x* where *a***  **0 .** | **(Nov/Dec 2018)** |
|  | **b)** | * ***x*2**   **2 2**  **Find the Fourier cosine transform of *e******a x* and hence show that *e* 2 is self-reciprocal with respect to**  **the Fourier cosine Transform.** | |

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| **3.** | **a)** | **1**  ***x* ; *if x***  **1**  **sin4 *t***  **Find the Fourier transform of *f* ( *x*)**   **. Hence Evaluate**  **4 *dt* .**  **0 ; *if x***  **1 0 *t*** |
|  | **b)** | 1 , 0  *x*    1 cos( )  **Find Fourier sine integral of** *f* ( *x* )  0, *x*   **and hence evaluate**   sin( *x*)*d* **.**   0 |
| **4.** | **a)** | **1 , *x***  **1**  **sin *x***  **Find the Fourier transforms of** *f* ( *x* ) **defined by *f* ( *x* )**   **and hence evaluate**  ***dx***  **0 , *x***  **1 0 *x***  **(Nov/Dec 2020)** |
|  | **b)** | **Find the Fourier sine and cosine transform of *e******ax* , *a***  **0 and hence deduce their inversion formulae.**  **(Nov/Dec 2019)** |
| **5.** | **a)** | **Using Parseval’s identity evaluate the following integrals.**   ***dx***  **1)**  **( *x*2**  ***a*2 )2**  **0**   ***x*2**  **2)**  **( *x*2**  ***a*2 )2 *dx*, where *a***  **0. (Nov/Dec 2019)**  **0** |
|  | **b)** | 2  **Verify Convolution theorem for Fourier transform, if** *f* (*x*)  *g*(*x*)  *e**x* **(Nov/Dec 2020)** |
|  | **UNIT V Z – TRANSFORMS AND DIFFERENCE EQUATIONS** | |
|  | **PART - A** | |
| **1.** | **What is the Z- transform of discrete unit step function** | |
|  | Discrete Unit step function is  *u* *n*  1, *n*  0  0, *n*  0      *Z* *u* *n*  1.*z**n*  1 *z*1  *z*2  ..  1  *n*0 1 *z*1  = *z* if *z*  1  *z* 1 | |
| **2.** | **Find *Z***  **1**  **.**   **2*n***     | |
|  | *Z*  1    1 *z**n*  1 1 *z*1  1 *z*2  1 *z*3    2*n*   2*n* 2 22 23 …    0   1 1  1  1  ...  2*z* 4*z*2 8*z*3   *z*  *z*  1  2   2*z* , *z*  2 2*z* 1 | |
| **3.** | **Find *Z*** ***u*** ***n***  **1** | |

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|  |    *Z* *u* *n* 1  *u* *n* 1 *z**n*  *u* *n* 1 *z**n*  *n*0 *n*1      *z**n*  1  1  1  ....  1 1 1  1    *n*1 *z z*2 *z*3 *z*  *z z*2   1  1 1 1  *z* 1 1   *z* 1 *z*   *z*  *z*         1  *z*   1 if *z*  1  *z*  *z* 1  *z* 1    | |
| **4.** | **Show that *Z*** ***an f* (*z*)**  ***F***   ***z***  **where *Z***  ***f* (*n*)**  ***F*** ***z*** **is the Z-transform of** *f* (*x*).   ***a***     | **(Nov/Dec 2019)** |
|  |   *Z* *an f* (*z*)   *an f* (*n*)*z**n*  *n*0    *z*  *n*    *a*  *f* (*n*)  *n*0     *F*  *z*    *a*     | |
| **5.** | **Form the difference equation by eliminating arbitrary constant A from *yn***  ***A*.2**  ***n*** | **(Nov/Dec 2017)** |
|  | Given *y*  *A*.2*n* , *y*  *A*.2*n*1  *A*.22*n*  2 *A*2*n*  2 *y*  *y*  2 *y*  0  *n n*1 *n n*1 *n* | |
| **6.** | **If *Z***  ***f*** ***n***  ***U***  ***z***  **, then show that *Z***  ***f*** ***n***  ***k***   ***zkU***  ***z***  | |
|  |   *Z*  *f* *n*  *k*    *f* *n*  *k*  *z**n*  0      *zk*  *f* *n*  *k*  *z**n**k*   *zk*  *f* *r*  *z**r*  *zkU*  *z*   *n*0 *r* 0 | |
| **7.** |  ***f*** ***n***  **1**  **If *Z***  ***f*** ***n***  ***U***  ***z***  **, then *Z***  ***n***    ***z U***  ***z***  ***dz* .**    | |
|  |  *f* *n*  *f* *n* *n*  *n*1 *z**n* *n*1  *Z*  *n*    *n z*   *f* (*n*) *z dz* , since *n*   *z dz*    *n*0 *n*0      *f* *n* *z**n*1*dz*  *z*1 *f* *n* *z**n* *dz*   *z*1 *U*  *z*  *dz* .  *n*0 | |
| **8.** | **If *Z***  ***f*** ***n***  ***F***  ***z***  **then find *Z*** ***n f*** ***n*** | **(April/May 2018)** |
|  | *d F*  *z*    *d Z*  *f* *n*   *d*  *f* (*n*)*z**n*   *n f* (*n*)*z**n*1  *dz dz*   *dz n*0 *n*0   *n*    *n f* (*n*) *z*   1 *n f* (*n*)*z**n n*0 *z z n*0   *Z* *n f* *n*  *z d F* (*z*) .  *dz* | |

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| **9.** | **Find *Z*** ***an*****3**  **.** |
|  |   *Z* *an*3   *an*3*z**n*  *a*3*Z* *an*  = *a*3 *z*  *n*0 *z*  *a* |
| **10.** | **Find *Z*** **1**  ***z***    ***z***  **1**  ***z***  **2**     |
|  |  *z*   Let    *U*  *z*    *z* 1 *z*  2   *U*  *z*  1 *A B*  Then     *z*  *z* 1 *z*  2 *z* 1 *z*  2  *z*  2, 1  *B z*  1, 1   *A*  *U*  *z*   1  1  *z z* 1 *z*  2  *U*  *z*   *z*  *z* *u* *n*  1 2*n*  2*n* 1  *z* 1 *z*  2 |
| **11.** | **Find the Z-transforms of**  ***n***  **. (April/May 2018)** |
|  |     *n* 1  1 2  1 3  *Z n*   *n z*  1   2    3   ....  *n*  1 *z*  *z*   *z*   1  2 1  1  1 2 1  *z* 2 *z*   1  3   1      .  *z*  *z z*2  *z*  *z*  *z*  *z* 1  *z* 12 |
| **12.** | **Find Z-Transform of 1 . (Nov/Dec 2017)**  ***n*** |
|  |  1   1 1 1  1 2 1  1 3  1   *z* 1   *z*   *Z*     *z**n*         ....  *log* 1   log   log    *n*  *n*1 *n z* 2  *z*  3  *z*   *z*   *z*   *z* 1  |
| **13.** | **Find Z-Transform of *an* . (Nov/Dec 2020)** |
|  |  *n*   *n* *n*   *a* *n z Z a*   *a z*    *z*   *z*  *a*  *n* 0 *n* 0   |
| **14.** | **Find Z-Transform of *Z***  **1**  **.**   ***n*!**    |
|  |  1   1 *n* 1 1 1/*z*  *Z*  *n*!   *n*! *z*  1  1! *z*  2  *e*    *n*0 2! *z* |
| **15.** | **Solve *y***  ***y***  **2*n* , given that *y*** **0**  **1.**  ***n*****1 *n*** |
|  | *Z* *y*  *y*   *Z* 2*n*  *n*1 *n* |

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|  | *zY*  *z*   *y* 0  *Y*  *z*   *z* , *Y* *z*  *Z* *y*   *z*  2 *n*  *Y*  *z*  *z* 1  1 *z*  *z*  2   *Y*  *z*    *z*  2  *z*  *Y*  *z*   2  *z* 1   *z*  2 *z* 1  *z*  2 *z* 1  *y*  *Z* 1  2   2*n*  *n*  *z*  2     |
| **16.** | **1** ***z* 2**   **Using Convolution theorem , evaluate *Z*** **( *z***  **1)( *z***  **3)**    |
|  | We know that *Z* 1  *z*   1and *Z* 1  *z*   3*n*   *z* 1  *z*  3  1  *z*2   1  *z z*   Now *Z*  *z* 1 *z*  3  *Z*  *z* 1. *z*  3     *n n*   13*n*  1.3*n**m*  3*n* 3*m*  *m*0 *m*0   1 *n*1  *n*  1 *m*  3  1 3*n* 1 3*n*1  / 3*n*1   3*n*    3*n*     *m*0  3   1  1 1 3 / 3   3       1 3*n*1  3*n*1 1    2 2 . |
| **17.** | **Find *Z*** ***e*****2*t* sin 2*t*** |
|  | *Z* *e*2*t* sin 2*t* *Z* sin 2*t*   *z* sin 2*T*   *z*  *ze*2*T*  2    *z*  2*z* cos 2*T* 1 *z* 2*T*     *ze*   *ze*2*T* sin 2*T*  *z*2*e*4*T*  2*ze*2*T* cos 2*T* 1 |
| **18.** | **Find *Z*** **sin *at*** |
|  |   *Z* sin *at*  sin *anT* *z**n*  *n*0   *Z* sin *n*    *z* sin   *z* sin *aT*    *aT*  2  2   *z*  2*z* cos 1 *aT z*  2*z* cos *aT* 1 |
| **19.** | **State initial and final value theorems of Z - transform. (Nov/Dec 2019, 2020)** |
|  | Initial value Theorem |

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|  | *limit limit*  *If Z*  *f* *n*  *U*  *z* , *n*  0 then n  0 *f* *n*  *f* 0  *z*  *U*  *z*   Final value Theorem  *limit limit*  *If Z*  *f* *n*  *U*  *z* , *n*  0 then *n*   *f* *n*  *z*  1 *z* 1*U*  *z*  | | |
| **20.** | **Form the difference equation from *yn***  ***A*.2**  ***B*.3**  ***n n*** | | |
|  | Given *yn*  *A*.2  *B*.3 , *y*  2 *A*.2  3*B*.3 , *y*  4*A*.2  9*B*.3  *n n n n n n*  *n*1 *n*2  Elimination of A and B forms the difference equation  *yn* 1 1  *yn*1 2 3  0  *yn*2 4 9  *yn*2  5*yn*1  6 *yn*  0 . | | |
| **PART B** | | | |
| **1.** | **a)** | **Solve *yn*****2**  **4 *yn*****1**  **4 *yn***  **0, *y*** **0**  **1, *y*** **1**  **0 using Z-transform.** | **(Apr/May 2018)** |
|  | **b)** | **State and Prove final value theorem of Z-Transform** | |
| **2.** | **a)** | **Solve *y*** ***n***  **3**  **3 *y*** ***n***  **1**  **2 *y*** ***n***  **0, given that *y*** **0**  **4, *y*** **1**  **0, *y*** **2**  **8 Using Z- transform.** | |
|  | **b)** | **2*z*2**  **3*z***  **12**  **If *U***  ***z***    ***z***  **1****4 , find *u*2 and *u*3 .** | |
| **3.** | **a)** |   **8*z*2**   **Find *Z* 1**   **by convolution theorem.**   **(2*z***  **1)(4*z***  **1)**  | **(Apr/May 2018,Nov/Dec 2019)** |
|  | **b)** | **Find *Z***  **2*n***  **3**    **(*n***  **1)(*n***  **2)**     | **(Nov/Dec 2017)** |
| **4.** | **a)** | **Find *Z*** ***rn* cos *n***  **and hence deduce *Z***  ***cos n***    **2**     | |
|  | **b)** | ***z*2**  **3*z***  **Find the inverse Z-Transform of**  ***z***  **5****(*z***  **2) by residue method.** | **(Apr/May 2018)** |
|  | **c)** | **Find Z-transform of 2n**  **5 sin *n***  **3*a*4 .**  **4** | **(Nov/Dec 2020)** |
|  | **d)** | ***z*2**  **Using Convolution theorem, find the inverse Z-transform of**  ***z***  **2** **( *z***  **3) . (Nov/Dec 2020)** | |
|  | **e)** | **Using Z-transformation, solve *Un*** **2**  **4*Un*****1**  **3*Un***  **3 given that *U***  **0, *U***  **1**. **(Nov/Dec 2020)**  ***n***  **0 1** | |
| **5.** | **a)** | **Form the difference equation corresponding to the family of curves *yn***  ***an***  ***b*2 .**  ***n*** | |
|  | **b)** |  ***z***   **Find *Z*** **1**   **using the method of partial fraction.**    ***z***  **1** ***z***  **1****2**  | |
|  | **c)** | **Find *Z*** **sin *bt*** **and hence find *Z*** **e*****at* sin *bt***  | **(Nov/Dec 2019)** |

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|  | **d)** | **Solve using Z-transforms technique the difference equation *yn*** **2**  **7 *yn*****1**  **12 *yn***  **2 with**  ***n***  ***y*0**  **0**  ***y*1 . (Nov/Dec 2019)** |
|  | **e)** | **Using residue method, find *Z*** **1**  ***z***  **(Nov/Dec 2019)**   ***z*2**  **2*z***  **2**  |