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| **MA8452 - STATISTICS AND NUMERICAL METHODS - QUESTION BANK** | |
| **UNIT I - TESTING OF HYPOTHESIS** | |
| **PART A** | |
| **1.** | **Define Population, Sample and Sample Size.** |
|  | **Population:** The group of individuals under study is called population. The population may be finite or infinite.  **Sample:** A finite subset of statistical individuals in a population is called Sample.  **Sample Size:** The number of individuals in a sample is called Sample Size (*n*). |
| **2.** | **Define Parameters, Statistics, Standard Error and Random Sampling. (APR/ MAY 2018)** |
|  | **Parameter:**  The statistical constants in population namely mean µ and variance **2** which are called parameters.  **Statistic:**  Statistical measures computed from sample observations alone, (i.e.) mean *x* and variance *s2*  are known as statistics.  **Standard Error:**  The standard deviation of the sampling distribution of a statistic is known as standard error.  **Random Sampling:**  Random Sampling is one in which each unit of the population has an equal chance of being included in it. |
| **3.** | **Define critical region and acceptance region? (NOV/DEC 2017)** |
|  | **Critical region:**  A region corresponding to a statistic, in the sample space *S* which amounts to rejection of null hypothesis is called as critical region or region of rejection.  **Acceptance region:**  The region of the sample space *S* which amounts to the acceptance of null hypothesis is called acceptance region. |
| **4.** | **Define one-tailed and two-tailed test.** |
|  | A test of any statistical hypothesis where the alternative hypothesis is one tailed (right or left tailed) is called a one tailed test. (i.e.) Ho :  o vs H1 :  o (right tailed) or H1 :  o (Left tailed).  A test of statistical hypothesis whose alternative hypothesis is two tailed, such as  Ho :  o vs H1 :  o is known as two tailed test. |
| **5.** | **Explain Null Hypothesis and Alternative Hypothesis.** |
|  | **Null Hypothesis:**  For applying the tests of significance, we first set up a hypothesis which is a definite statement about the population parameter. Usually, such a hypothesis is a hypothesis of no difference and it is denoted by *H* 0 .  **Alternate Hypothesis:**  Any hypothesis which is complementary to the null hypothesis is called an alternative  hypothesis, denoted by *H*1 . |

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| **6.** | **What are type I and type II errors? (APR/MAY 2019) (NOV/DEC 2015, 2017)** |
|  | **Type I error:** Reject Null hypothesis when it is true. Example:  A test that shows a patient to have a disease when in fact the patient does not have the disease.  **Type II error:** Accept Null hypothesis when it is wrong. Example:  A blood test failing to detect the disease it was designed to detect, in a patient who  really has the disease. |
| **7.** | **Find the standard error of sample mean from the following data *n***  **16*,***  **16*.*5*,***  ***x***  **16*.*25*, S***  **1*.*65** |
|  | The Standard Error= *S*  1.65  0.426  *n*  1 16 1 |
| **8.** | **Define Level of Significance.** |
|  | The probability that the value of the statistic lies in the critical region is called the level of  significance and is denoted by α . |
| **9.** | **When do we use large sample tests and small sample tests? (APR/MAY 2021)** |
|  | If sample size is *n*  30, then we use large sample tests. If sample size is *n*<30, then we use small sample tests. |
| **10.** | **Write 95% confidence interval of the population mean for the small samples** |
|  | 95% confidence interval of the population mean is *x*  *t S*    *x*  *t S*  0.05 *n* 1 0.05 *n* 1 |
| **11.** | **For the following cases, specify which probability distribution to use in a hypothesis test**  ***a***  ***H*0 *:***   **25*, H*1 *:***   **25*, X***  **19*.*1*,***  **6*,n***  **13**  ***b*** ***H*0 *:***   **96*.*6*, H*1 *:***   **96*.*6*, X***  **60*,s***  **13*,n***  **40** |
|  | a) student *t*-distribution b) Normal distribution |
| **12.** | **Write down the formula to test statistic ‘*t*’ to test the significance of difference between**  **the sample mean and population mean for small sample.** |
|  | The test of significance of difference between the sample mean and population mean for small sample is given by  *t*  *x*    *s* / *n* 1  where *n*  Samplesize *x*  Sample mean   Population mean  *s* Sample Standard deviation. |
| **13.** | **What are the assumptions of *t*-test?** |
|  | 1. Parent populations, from which the samples have been drawn are normally distributed. 2. The population variances are equal and unknown. 3. The two samples are random and independent of each other. |

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| **14.** | **What are the uses of *t-* test?** | | | | | | |
|  | 1. To test the significance of difference between mean of a random sample and the mean of the population. 2. To test the significance of the difference between two sample means. | | | | | | |
| **15.** | **What are the assumptions of *F*-test?** | | | | | | |
|  | 1. Normality: The values in each group should be normally distributed. 2. Independence of error: The variations of each value around its own group mean. (i.e.) error should be independent of each value. 3. Homogeneity: The variances within each group should be equal for all groups. | | | | | | |
| **16.** | **What are the uses of *F*-test?** | | | | | | |
|  | 1. To test the equality of two sample variances. 2. To test more than two population mean. 3. To test the sample observation coming from normal population. | | | | | | |
| **17.** | **What is the formula for testing the ratio of variances? (NOV/DEC 2020) (APR/MAY 2021)** | | | | | | |
|  | Test for the Equality of variances of two Populations or Equality of variances  2 *n s*2 2 *n s*2  Let *S*1  1 1 and *S*2  2 2 ,  *n*1 1 *n*2 1  *S* 2  If *S* 2  *S* 2 then *F*  1 and Snedecor’s *F*-Distribution with (*v* , *v* ) degrees of freedom where  1 2 *S* 2 1 2  2  *v*1  *n*1 1 and *v*2  *n*2 1.  *S* 2  If *S* 2  *S* 2 then *F*  2 and Snedecor’s *F*-Distribution with (*v* , *v* ) degrees of freedom where  2 1 *S* 2 1 2  1  *v*1  *n*2 1 and *v*2  *n*1 1 | | | | | | |
| **18.** | **State the assumptions of Chi-square test.** | | | | | | |
|  | 1. The sample observations should be independent. 2. Constraints on the cell frequencies, if any must be linear. 3. The total frequency should be at least 50 4. No theoretical cell frequency should be less than 5 | | | | | | |
| **19.** | **Write down the expected frequencies of** 2  2 **contingency table.** | | | | | | |
|  | | ***a*** | ***b*** |  | | |
| ***c*** | ***d*** |
|  | Expected frequency table: | | | | | | |
|  | (*a*  *b*)(*a*  *c*)  *N* | | | | (*a*  *b*)(*b*  *d* )  *N* |  |
| (*a*  *c*)(*c*  *d* )  *N* | | | | (*c*  *d* )(*b*  *d* )  *N* |

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|  | **State *A*** | **State *B*** |
| **Average Sales** | **Rs. 2,500** | **Rs. 2,200** |
| **S.D.** | **Rs. 400** | **Rs. 550** |

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| **Sample** | **Size** | **Mean** | **Standard Deviation** |
| **I** | **32** | **72** | **8** |
| **II** | **36** | **74** | **6** |

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|  | where *N*  *a*  *b*  *c*  *d* |
| **20.** | **State any two applications of** **2 -test. (NOV/DEC 2016 & 2017) (APR/MAY 2019)** |
|  | 1. To test the homogeneity of independent estimates of the population variances. 2. To test the goodness of fit. 3. To test for independence of attributes. |
| **PART B** | |
| **1.** | **Random samples of 400 men and 600 women were asked whether they would like to have a flyover near their residence. 200 men and 325 women were in favour of the proposal. Test the hypothesis that proportions of men and women in favour of the proposal are**  **same against that they are not, at 5% level. (May/June 2016)** |
| **2.** | **In a random of 1000 people from city *A*, 400 are found to be consumers of rice. In a sample of 800 from city *B*, 400 are found to be consumers of rice. Does this data give a significant difference between the two cities as far as the proportion of rice consumers is**  **concerned? (NOV/DEC 2017)** |
| **3.** | **A salesman in a departmental store claim that at most 60 percent of the shoppers entering the store leaves without making a purchase. A random sample of 50 shoppers showed that 35 of them left without making a purchase. Are these sample results consistent with the**  **claim of the salesman?** |
| **4.** | **A random sample of 100 bulbs in the United States during the past year showed an average life span of 1570 hours with a standard deviation of 120 hours. Does this seem to indicate that the mean life span is greater than 1600 hours. Use a level of significance of**  **0.05. (NOV/DEC 2020) (APR/MAY 2021)** |
| **5.** | **A random sample of 100 bulbs from a company *P* shows a mean life 1300 hours and standard deviation of 82 hours. Another random sample of 100 bulbs from company *Q* showed a mean life 1248 hours and standard deviation of 93 hours. Are the bulbs of company *P* superior to bulbs of company *Q* at 5% level of significance?**  **(NOV/DEC 2017)** |
| **6.** | **The sales manager of a large company conducted a sample survey in states *A* and *B***  **taking 400 samples in each case. The results were (APR/MAY 2017)**  **Test whether the average sales is the same in the 2 states at 1% level of significance.** |
| **7.** | **Samples of two types tested gives the following results:**  **Test if the means are significant.** |
| **8.** | **A test of the breaking strengths of 6 ropes manufactured by a company showed a mean breaking strength of 3515 kg and a standard deviation of 60 kg, whereas the manufacturer claimed a mean breaking strength of 3630 kg. Can we support the manufacturer’s claim at a level of significance of 0.05?**  **(APR/MAY 2019) (NOV/DEC 2020) (APR/MAY 2021)** |

9. A random sample of 10 boys has the following I.Q’s: 70, 120, 110, 101, 88, 83, 95, 98, 107 and 100. Do these data support the assumption of a population mean I.Q of 100 at 5% level of significance? (MAY/JUNE 2016) & (NOV/DEC 2017)

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| **The nicotine cont** | **ent in** | **milligra** | **ms of 2** | **samples of** | **tobacc** | **o were found to be as follows** |
| **Sample A** | **24** | **27** | **26** | **21** | **25** | **(APR/MAY 2018)** |
| **Sample B** | **27** | **30** | **28** | **31** | **22** | **36** |

Can it be concluded that 2 samples come from normal population with the same mean.

1. **A group of 10 rats fed on diet *A* and another group of 8 rats fed on diet *B* recorded the following increase in weight**

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| **Diet *A*** | **5** | **6** | **8** | **1** | **12** | **4** | **3** | **9** | **6** | **10** |
| **Diet *B*** | **2** | **3** | **6** | **8** | **10** | **1** | **2** | **8** |  |  |

Does it show superiority of diet *A* over diet *B*.

1. **The following data relate to the marks obtained by 11 students in two tests, one held at the beginning of the year and other at the end of the year, after intensive coaching.**

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| **Test I:** | **19** | **23** | **16** | **24** | **17** | **18** | **20** | **18** | **21** | **19** | **20** |
| **Test II:** | **17** | **24** | **20** | **24** | **20** | **22** | **20** | **20** | **18** | **22** | **19** |

Do the data indicate that the students have benefited by coaching?

1. **An instructor has two classes *A* and *B*, in a particular subject. Class *A* has 16 students while class *B* has 25 students. On the same examination, although there was no significant difference in mean grade class *A* has standard deviation of 9, while class *B* had a standard deviation level of 12. Can we conclude at the 0.01 level of significance that the variability of class *B* is greater than that of class *A*. (NOV/DEC 2020) (APR/MAY 2021)**
2. **Two independent samples of sizes 9 and 7 from a normal population had the following values of the variables. Do the estimates of the population variance differ significantly at 5% level of significance? (APR/MAY 2017)**

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| **Sample I:** | **18** | **13** | **12** | **15** | **12** | **14** | **16** | **14** | **15** |
| **Sample II:** | **16** | **19** | **13** | **16** | **18** | **13** | **15** |  |  |

1. **Two random**

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| **samples gav** | **e the fo** | **llowing results:** |  |
| **Sample** | **Size** | **Sample mean** | **Sum of squares of deviations from the mean** |
| **I** | **10** | **15** | **90** |
| **II** | **12** | **14** | **108** |

Test whether the samples come from the same normal population at 5% level of Significance.

1. **In 200 tosses of a coin, 115 heads and 85 tails were observed. Test the hypothesis that the coin is fair using a level of significance of 0.05. (Nov/Dec 2020) (APR/MAY 2021)**
2. **The following data gives the number of aircraft accidents that occurred during the various days of a week. Find whether the accidents are uniformly distributed over the week.**

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| **Days:** | **Sun** | **Mon** | **Tue** | **Wed** | **Thu** | **Fri** | **Sat** |
| **No. of accidents:** | **14** | **16** | **8** | **12** | **11** | **9** | **14** |

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| **Health** | **Social Status** | | **Total** |
| **Poor** | **Rich** |
| **Below normal** | **130** | **20** | **150** |
| **Normal** | **102** | **108** | **210** |
| **Above normal** | **24** | **96** | **120** |
| **Total** | **256** | **224** | **480** |

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| **18.** | **Using the data given in the following table to test at 1% level of significance whether a person’s ability in Mathematics is independent of his/her interest in Statistics.**  **(Nov/Dec 2017)** | | | | | |
|  | **Interest in**  **Statistics** | **Ability in Mathematics** | | |  |
| **Low** | **Average** | **High** |
| **Low** | **63** | **42** | **15** |
| **Average** | **58** | **61** | **31** |
| **High** | **14** | **47** | **29** |
| **19.** | **In an investigation into the health and nutrition of two groups of children of different social status, the following results are got and their economic conditions. What conclusion can you draw from the following data?**  **Discuss the relation between the Health and their Social Status.** | | | | | |
| **20.** | **Fit a binomial distribution for the following data and also test the goodness of fit.**  ***x* : 0 1 2 3 4 5 6 Total**  ***f*(*x*) : 5 18 28 12 7 6 4 80** | | | | | |

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| **UNIT II - DESIGN OF EXPERIMENTS** | |
| **PART A** | |
| **1.** | **What do you understand by “Design of an experiments”?** |
|  | The sequence of steps taken to ensure a scientific analysis leading to valid inferences about the  hypothesis is called design of experiment. |
| **2.** | **What are the three essential steps to plan Design of experiment?** |
|  | To plan an experiment, the following three are essential   * A Statement of the objective - Statement should clearly mention the hypothesis to be tested * A description of the experiment - Description should include the type of experimental material, size of the experiment and the number of replications. * The outline of the method of analysis - The outline of the method consists of analysis of   variance |
| **3.** | **Define a treatment and a yield in an experimental design. (APR/MAY 2021)** |
|  | Treatments: Various objects of comparison in a comparative experiment is called treatments. Yield: we change the experimental factors and measure the response outcome which in this case is the yield of the desired product. |
| **4.** | **What are the basic principles of design of experiments? (APR/MAY 2017, 2018)** |
|  | Basic Principle of Design of Experiments are  (i) Randomization (ii) Replication and (iii) Local Control |
| **5.** | **Define Experimental error. (NOV /DEC 2016)** |
|  | Factors beyond the control of the experiment are known as experimental error. |

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| **S. V** | **D.F** | **S. S** | **M.S** | **F cal** |
| **Treatments** | **4** | **A** | **6.86** | **2.37** |
| **Error** | **B** | **C** | **16.26** | **-** |
| **Total** | **24** | **D** | **-** | **-** |

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| **6.** | **Define Analysis of Variance. (NOV /DEC 2016)** | | | |
|  | Analysis of Variance is a statistical method used to test the differences between more than two  means. | | | |
| **7.** | **What are uses of ANOVA (APR/MAY 2017)** | | | |
|  | * To test the homogeneity of several means. * It is now frequently used in testing the linearity of the fitted regression line or in the significance of the correlation ratio. | | | |
| **8.** | **What are the assumptions of analysis of variance? (APR/MAY 2014)** | | | |
|  | * The samples are drawn from normal populations. * The samples are independently drawn from these populations. * All the populations have same variances. | | | |
| **9.** | **Define completely randomized design. (APR/MAY 2017)** | | | |
|  | In completely randomized design the treatments are given to the experimental units by a procedure of random allocation. It is used when the units are homogeneous. | | | |
| **10.** | **What are the advantages of Completely Randomized Block Design?** | | | |
|  | The advantages of Completely Randomized Experimental Design as follows:  (a) Easy to lay out. (b) Allow flexibility. (c) Simple Statistical Analysis.  (d) Lots of information due to missing data is smaller than with any other design. | | | |
| **11.** | **Find A, B, C, D from the ANOVA table** | | | |
|  | A = D.F(treatment)  M.S(treatment) **=** 4  6.86 = 27.44, B = Total D.F – Treatments = 24 – 4 = 20, C = D.F(error) M.S(error) =20  16.26 = 325.20, D = A + C = 27.44 + 325.20 =  352.64 | | | |
| **12.** | **State the null and alternative hypotheses for a completely randomized design.** | | | |
|  | Null Hypotheses  (i.e. The treatment means are equal.)  Alternative hypotheses  not all  equal. | | | |
| **13.** | **Explain the situation in which RBD is considered an improvement over CRD.** | | | |
|  |  | **RBD** | **CRD** |  |
| It is flexible and so any number of treatments  and any number of replications may be used | There is complete flexibility as the number  of replications is not fixed |  |
| The analysis of design is simple as it results in a two-way classification analysis of variance | The analysis of design is simple as it results in a one-way classification analysis  of variance |  |
| **14.** | **What do you mean by two-way classification in ANOVA?** | | | |
|  | When data are classified according to two factors one classification is taken column wise and the other row wise. Such a classification is called two-way classification | | | |
| **15.** | **Write the ANOVA table for randomized block design.** | | | |

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|  | Source of  Variation | S.S | | D.F | | | M.S.S | | | F cal |  |
| Column  Treatments | SSC | | c – 1 | | | *MSC*  *SSC*  *c* 1 | | | *F*  *MSC*  *C MSE* |
| Row  Treatments | SSR | | r – 1 | | | *MSR*  *SSR*  *r* 1 | | | *F*  *MSR*  *R MSE* |
| Error | SSE | | (c – 1) (r – 1) | | | *MSE*   *SSE*   *r* 1*c* 1 | | | - |
| Total | SST | | rc-1 | | |  | | |  |
| **16.** | **Write the advantages of Latin Square Design.** | | | | | | | | | | | |
|  | * Latin square design controls variation in two directions of the experimental materials as rows and columns resulting in the reduction of experimental error. * The analysis of remains relatively simple even with missing data. | | | | | | | | | | | |
| **17.** | **Compare RBD, LSD, CRD.** | | | | | | | | | | | |
|  | **CRD RBD LSD**  To influence one To influence two factors To influence more than two factors factor  No restriction No restriction on treatment The number of replications of each further treatments & replications treatment is equal to the number of  treatments | | | | | | | | | | | |
| **18.** | **Construct 5**  **5 Latin Square Design.** | | | | | | | | | | | |
|  | Answer: Each Treatments appears only once in each row and each column  (5  5 LSD) | | | | | | | | | | | |
|  | | | A | B | C | D | | E |  | | |
| B | C | D | E | | A |
| C | D | E | A | | B |
| D | E | A | B | | C |
| E | A | B | C | | D |
| **19.** | **Why a 2**  **2 Latin Square is not possible? Explain. (NOV/DEC 2015) (MAY/JUNE 2016)** | | | | | | | | | | | |
|  | Consider a *n*  *n* Latin square design,  The degrees of freedom for SSE  (*n*2 1) (*n* 1) (*n* 1)  (*n* 1)   (*n*2 1)  *n* 1 *n* 1 *n* 1   *n*2 1 3*n*  3   *n*2  3*n*  2   (*n* 1)(*n*  2) | | | | | | | | | | | |

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|  | For *n* = 2, d.f. of SSE = 0 and hence MSE is not defined. Comparisons are not possible.  Hence a 2  2 Latin Square Design is not possible. | | | | | | | | | | | | | | | | |
| **20.** | **What is contrasts and orthogonal contrast in a**  **‐factorial design?** | | | | | | | | | | | | | | | | |
|  | A contrast is a linear combination of 2 or more factor level means with coefficients that sum to zero.  Two contrasts are orthogonal if the sum of the products of corresponding coefficients (i.e. coefficients for the same means) adds to zero. | | | | | | | | | | | | | | | | |
| **PART B** | | | | | | | | | | | | | | | | | |
| **1.** | **As head of the department of a consumer’s research organization you have the responsibility of testing and comparing life times of 4 brands of electric bulbs. suppose you test the life time of 3 electric bulbs each of 4 brands, the data is given below, each entry representing the life time of an electric bulb, measured in hundreds of hours.**  **(NOV/DEC 2017)**  **A B C D**  **20 25 24 23**  **19 23 20 20**  **21 21 22 20** | | | | | | | | | | | | | | | | |
| **2.** | **A farmer wishes to test the effects of four different fertilizers A, B, C, D on the yield of wheat. In order to eliminate sources of error due to variability in soil fertility he uses the fertilizers in a Latin square arrangement as indicated below where the number indicate yields in Kilograms per unit area. Perform an analysis of variance to determine if there is a significant difference between the fertilizers at 0.01 level of significance.**  **(APR/MAY 2019)** | | | | | | | | | | | | | | | | |
|  | | | **A 18** | | | **C 21** | | **D 25** | | | **B 11** | | |  | | |
| **D 22** | | | **B 12** | | **A 15** | | | **C 19** | | |
| **B 15** | | | **A 20** | | **C 23** | | | **D 24** | | |
| **C 22** | | | **D 21** | | **B 10** | | | **A 17** | | |
| **3.** | **Table below shows the yields for hectare of four different plant crops grown on plots treated with three different types of fertilizers. Using a suitable design of experiment, test at the 0.05 level of significance whether there is a significant difference in yield per hectare**  **due to fertilizers and there is a significant difference in yield per hectare due to crops.** | | | | | | | | | | | | | | | | |
|  |  | | | **Types of Crops** | | | | | | | | | | | |  |
| **Fertilizers** | | |  | | **I** | | **II** | | **III** | | | **IV** | | |
| **A** | | **4.5** | | **6.4** | | **7.2** | | | **6.7** | | |
| **B** | | **8.8** | | **7.8** | | **9.6** | | | **7.0** | | |
| **C** | | **5.9** | | **6.8** | | **5.7** | | | **5.2** | | |
| **4.** | **Analyze the following RBD and find the conclusion.** | | | | | | | | | | | | | | | | |
|  | | **Treatments** | | | **T1** | | **T2** | | **T3** | | | **T4** | | |  | |
| **Blocks** | **B1** | | **12** | | **14** | | **20** | | | **22** | | |
| **B2** | | **17** | | **27** | | **19** | | | **15** | | |
| **B3** | | **15** | | **14** | | **17** | | | **12** | | |
| **B4** | | **18** | | **16** | | **22** | | | **12** | | |
| **B5** | | **19** | | **15** | | **20** | | | **14** | | |

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|  | | **DAYS** | | | | |
| **MON** | **TUE** | **WED** | **THU** | **FRI** |
| **ROUTES** | **1** | **22** | **26** | **25** | **25** | **31** |
| **2** | **25** | **27** | **28** | **26** | **29** |
| **3** | **26** | **29** | **33** | **30** | **33** |
| **4** | **26** | **28** | **27** | **30** | **30** |

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|  | | **Chemists** | | | |
| **A** | **B** | **C** | **D** |
| **COAL** | **I** | **8** | **5** | **5** | **7** |
| **II** | **7** | **6** | **4** | **4** |
| **III** | **3** | **6** | **5** | **4** |

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| **5.** | **The accompanying data results from an experiment comparing the degree of soiling for fabric co-polymerized with the three different mixtures of methacrylic acid. Analysis is the given classification. (APR /MAY 2017)**  **Mixture 1 0.56 1.12 0.90 1.07 0.94**  **Mixture 2 0.72 0.69 0.87 0.78 0.91**  **Mixture 3 0.62 1.08 1.07 0.99 0.93** | | | | | | | | | | | | | |
| **6.** | **Table below shows the seeds of 4 different types of corns planted in 3 blocks. Test at 0.05 level of significance whether the yields in kilograms per unit area vary significantly with**  **different types of corns. (APR /MAY 2019)** | | | | | | | | | | | | | |
|  |  | | **Types of Corns** | | | | | | | | | |  |
| **Blocks** | |  | | **I** | | **II** | | **III** | | **IV** | |
| **A** | | **4.5** | | **6.4** | | **7.2** | | **6.7** | |
| **B** | | **8.8** | | **7.8** | | **9.6** | | **7.0** | |
| **C** | | **5.9** | | **6.8** | | **5.7** | | **5.2** | |
| **7.** | **The following data represent a certain person to work from Monday to Friday by 4 different routes.**  **Test at 5% level of significance whether the difference among the means obtained for the different routes are significant and also whether the differences among the means obtained from the different days of the week are significant. (NEV/ DEC 2017)** | | | | | | | | | | | | | |
| **8.** | **A set of data involving “four tropical feed stuffs A, B, C, D” tried on 20 chicks is given below. All the twenty chicks are treated alike in all respects except the feeding treatments**  **and each feeding treatment is given to 5 chicks. Analyze the data. (APR/MAY 2017)** | | | | | | | | | | | | | |
|  | | **A** | | **55** | | **49** | | **42** | **21** | **52** | |  | |
| **B** | | **61** | | **112** | | **30** | **89** | **63** | |
| **C** | | **42** | | **97** | | **81** | **95** | **92** | |
| **D** | | **169** | | **137** | | **169** | **85** | **154** | |
| **9.** | **Three varieties of coal were analyzed by 4 chemists and the ash content is given below. Perform an ANOVA Table (MAY/JUNE 2016)** | | | | | | | | | | | | | |

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| **Salesmen** | | **A** | **B** | **C** | **D** |
| **Season** | **Summer** | **45** | **40** | **28** | **37** |
| **Winter** | **43** | **41** | **45** | **38** |
| **Monsoon** | **39** | **39** | **43** | **41** |

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| **10.** | **A company appoints 4 salesmen A, B, C and D and observes their sales in 3 seasons, summer, winter and monsoon. The figures are given in the following table:**  **(NOV/DEC 2016)**  **Carry out an analysis of variances.** | | | | | |
| **11.** | **The following is the Latin Square of a design when 4 varieties of seed are being tested. Set up the analysis of variance table and state your conclusion. You can carry out the suitable change of origin and scale.**  **(A) 110 (B) 100 (C)130 (D) 120**  **(C) 120 (D) 130 (A) 110 (B) 110**  **(D) 120 (C) 100 (B) 110 (A) 120**  **(B) 100 (A) 140 (D) 100 (C) 120** | | | | | |
| **12.** | **In a Latin square experiment given below are the yield in quintals per acre on the paddy crop carried out for testing the effect of five fertilizers A, B, C, D, E. Analyze the data for variations. (NOV / DEC 2017)**  **B 25 A 18 E 27 D 30 C 27**  **A 19 D 31 C 29 E 26 B 23**  **C 28 B 22 D 33 A 18 E 27**  **E 28 C 26 A 20 B 25 D 33**  **D 32 E 25 B 23 C 28 A 20** | | | | | |
| **13.** | **A variable trial was conducted on wheat with 4 varieties in a Latin Square Design. The plan of the experiment and the per plot yield are given below:**  **(APR/MAY 2017) (NOV/DEC 2016)** | | | | | |
|  | **C 25** | **B 23** | **A 20** | **D 20** |  |
| **A 19** | **D 19** | **C 21** | **B 18** |
| **B 19** | **A 14** | **D 17** | **C 20** |
| **D 17** | **C 20** | **B 21** | **A 15** |

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| **14.** | **A sales of four salesmen in 3 seasons are tabulated here.**  **Salesmen**  **Seasons A B C D Summer 36 36 21 35**  **Winter 28 29 31 32**  **Monsoon 26 28 29 29 Carry out an analysis of variance.** | | | | | |
| **15.** | **Given the following observation for the 2 factors A & B at two levels, Compute (i) the**  **main effect (ii) make an analysis of variance.** | | | | | |
|  | **Treatment**  **Combination** | **Replication**  **I** | **Replication**  **II** | **Replication**  **III** |  |
| **(1)** | **10** | **14** | **9** |
| **A** | **21** | **19** | **23** |
| **B** | **17** | **15** | **16** |
| **AB** | **20** | **24** | **25** |

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|  | **UNIT III**  **SOLUTION OF EQUATIONS AND EIGEN VALUE PROBLEMS** |
|  | **PART A** |
| **1** | **State fixed point theorem.** |
|  | Let *f*  *x*  0 be the given equation whose actual root is *a*. The equation *f*  *x*  0 be  written as *x*  *g* *x*. Let *I* be the interval containing the root *a* . If *g* ' *x*  1 for all *x* in *I* ,  then the sequence of successive approximations *x*0 , *x*1, *x*2 ... , will converges to *a*, if the initial starting value *x*0 is chosen in *I* . |
| **2** | **Write down the order of convergence and condition for convergence of fixed-point iteration method *x***  ***g***  ***x***  |
|  | The order of convergence is one and condition for convergence is *g* ' *x*  1 , for *x*  *I* where  *I* is the interval containing the root of *x*  *g*  *x*. |
| **3** | **Locate the negative root for the equation *x*3**  **2*x***  **5**  **0, approximately.** |
|  | Let *f*  *x*  *x*3  2*x*  5 ;  When *x*  1 *f* 1  13  2 1  5  1 2  5  6 (Positive) When *x*  2  *f* 2  23  2 2  5  8  4  5  1 (Positive)  When *x*  3  *f* 3  33  2 3  5  27  6  5  16 (Negative) The negative root in magnitude lies in the interval (–3, –2). |
| **4** | **Locate the positive root for the equation *xex***  **cos *x*** |
|  | Let *f*  *x*  *xex*  cos *x*  When *x*  0  *f* 0  0*e*0  cos0  0 1  1 (Negative)  When *x*  1 *f* 1  1*e*1  cos1= 2.718  0.999  1.719 (Positive) The positive root in magnitude lies in the interval (0, 1). |
| **5** | **Using iteration method, find the root between 0 and 1 of *ex***  **3*x***  **0 correct to two decimal places.** |



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|  | Given *f* *x*  *ex*  3*x*  *f*  *x*  0  *ex*  3*x*  0  3*x*  *ex*  *x*  1 *ex*  *g*(*x*) , *g* ' *x*  1 *ex*  3 3  Here *g* ' *x*  1 for all x in the interval (0,1). Hence, the iteration converges. Let *x*0  0.6 . We obtain the following results.  *x*  1 *ex*0  *x*  1 *e*0.6  *x*  0.60737  10 3 1 3 1  *x*  1 *ex*1  *x*  1 *e*0.60737  *x*  0.61187  11 3 2 3 2  *x*  1 *ex*2  *x*  1 *e*0.61187  *x*  0.61452  12 3 3 3 3  The last two iteration values are equal. We take the required root as *x*  0.61 |
| **6** | **Write down the Newton-Raphson method formula.** |
|  | *x*  *x*  *f*  *xn*  *n*  0,1,...  *n*1 *n f* ' *x*   *n* |
| **7** | **State the order and criterion of convergence of Newton-Raphson method for**  ***f***  ***x***   **0** . **[Apr/May 2021]** |
|  | The order of convergence of Newton-Raphson method is 2  The criterion of convergence of Newton-Raphson Method is *f*  *x* *f* '' *x*  *f* ' *x* 2 |
| **8** | **If the Newton-Raphson formula for *a* is *x***  **1**  ***x***  ***a***  **then find the value**  ***n*****1 2**  ***n x***    ***n***   **of 5** |
|  | Given *x*  *a*, then by Newton-Raphson method *x*  1  *x*  *a*  here *a*  5  *n*1 2  *n x*    *n*   *x*  5  *x*2  5  *x*2  5  0  Let *f* *x*  *x*2  5  *f* ' *x*  2*x*  When *x*  2  *f* 2  4  5  1(negative) When *x*  3  *f* 3  9  5  4 (positive) Therefore *x* lies between 2 and 3  Choose *x*0  2.25  *x*  1  *x*  5   1 2.25  5   1 2.25  2.2222  4.4722  2.2361  1 2  0 *x*  2  2.25  2 2   0  |



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|  | *x*  1  *x*  5   1 2.2361 5   1 2.2361 2.2360  4.4721  2.2361  2 2  1 *x*  2  2.2361 2 2   1   Here *x*1  *x*2  2.2361   5  2.2361 | | | | |
| **9** | **What are the merits of Newton’s method of iteration?** | | | | |
|  | 1. Newton’s method is successfully used to improve the results obtained by other methods. 2. It is applicable to the solution of equations involving algebraic functions as well as transcendental functions. | | | | |
| **10** | **Using Newton’s method, find the root between 0 and 1 of *x*3**  **6*x***  **4 correct to 4 decimal places.** | | | | |
|  | Given *f*  *x*  *x*3  6*x*  4  *f* ' *x*  3*x*2  6  *x*  *x*  *f* (*xn* ) *x*3  6*x*  4  We know that *n*1 *n*   *xn*  *n n*  *f* (*xn* ) 3*x*2  6  *n*   3*x*3  6*x*  *x*3  6*x*  4  2*x*3  4  *n n n n n*  3*x*2  6 3*x*2  6  *n n*  Take *x*0  1,  2*x*3  4 2 13  4 2  *x*1  0    0.6666 3*x*2  6 312  6 3  0  2*x*3  4 20.66663  4  *x*2  1   0.7301 3*x*2  6 30.66662  6  1  2*x*3  4 20.73013  4  *x*3  2   0.7320 3*x*2  6 30.73012  6  2  2*x*3  4 2 0.73203  4  *x*4  3   0.7320 3*x*2  6 30.73202  6  3  The required root as *x*  0.7320 | | | | |
| **11** | **For solving a linear system of equations, compare Gauss Elimination method and Gauss Jordan method. [APR/MAY 2019]** | | | | |
|  |  | **S.No.** | **Gauss-Elimination method** | **Gauss – Jordan method** |  |
| 1. | Coefficient matrix is transformed into upper triangular matrix. | Coefficient matrix is transformed into diagonal matrix. |
| 2. | Solution is obtain by back substitution method. | Solution is obtain by directly |
| **12** | **Solve by Gauss Elimination method 2*x***  ***y***  **1, *x***  **3 *y***  **0** | | | | |
|  | Given 2*x*  *y*  1, *x*  3*y*  0 | | | | |

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|  | The given system is equivalent to 2 1  *x*   1  1 3  *y* 0         *AX*  *B*   *A*, *B*  2 1 1     1 -3 0  Now, we will make the matrix *A* as a upper triangular matrix  2 -1 1  *R*  2*R*  ~ 0 5 1 *R*2 1 2      5*y*  1 and 2*x*  *y*  1   *y*  1 and 2 *x*  1  1  5 5  2*x*  1 1  6  5 5  Hence *x*  3 , *y*  1  5 5 |
| **13** | **Solve the system of equations *x***  ***y***  **2** and **2*x***  **3 *y***  **5 by Gauss elimination method.** |
|  | The given system is equivalent to 1 1  *x*  2  2 3  *y* 5         *AX*  *B*   *A*, *B*  1 1 2  2 3 5       Now, we will make the matrix *A* as a upper triangular matrix  1 1 2   2*R*  *R*  ~ 0 -1 1 *R*2 1 2       *y*  1 and *x*  *y*  2   *y*  1 and *x* 1  2  *x*  2 1  1  Hence *x*  1, *y*  1. |
| **14** | **Solve by Gauss-Jordan method for the following system 5*x***  **4 *y***  **15, 3*x***  **7 *y***  **12** |

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|  | The given system is equivalent to 5 4  *x*   15  3 7  *y* 12         *A X*  *B*  5 4 15 [ *A*, *B*]  3 7 12     Now, we will make the matrix *A* as a diagonal matrix  5 4 15  5*R*  3*R*  ~ 0 23 15 *R*2 2 1     115 0 285  23*R*  4*R*  ~  0 23 15  *R*1 1 2     115*x*  285  *x*  57  2.4783  23  23*y*  15  *y*  15  0.6522  23 |
| **15** | **Which of the iteration method for solving linear system of equation converges faster? Why?** |
|  | Gauss – Seidel method is faster than other iterative methods. In Gauss – Seidel method the latest values of unknowns at each stage of iteration are used in proceeding to the next stage of iteration. Hence the convergence in Gauss – Seidel method is more rapid than Gauss – Jacobi method. |
| **16** | **State the condition for convergence of Jacobi’s iteration method for solving a system of simultaneous algebraic equations.** |
|  | The process of iteration by Gauss-Jacobi method will converge if in each equation of the system, the absolute value of the largest coefficient is greater than the sum of the absolute values of the remaining coefficients.  The coefficient of matrix should be diagonally dominate.  *a*1*x*  *b*1 *y*  *c*1*z*  *d*1 *a*2 *x*  *b*2 *y*  *c*2 *z*  *d*2 *a*3 *x*  *b*3 *y*  *c*3 *z*  *d*3  The convergence condition is  *a*1  *b*1  *c*1  *b*2  *a*2  *c*2 *c*3  *b*3  *a*3 |
| **17** | **Find the first iteration values of *x*, *y*, *z* satisfying 28*x***  **4 *y***  ***z***  **32,**  ***x***  **2 *y***  **10*z***  **24 and 2*x***  **17 *y***  **4*z***  **35 by Gauss – Seidel method.** |
|  | Interchanging the equation,  28*x*  4 *y*  *z*  32, 2*x* 17 *y*  4*z*  35 *x*  2 *y* 10*z*  24  Now it is diagonally dominant,  *x*  1 32  4 *y*  *z* , *y*  1 35  2*x*  4*z* , *z*  1 24  *x*  2 *y*   28 17 10  Put *y*  0, *z*  0  First iteration  *x*  1 32  4 *y*  *z*   1 32  4 0  0  1 32  1.142  28 28 28 |



|  |  |
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|  | *y*  1 35  2*x*  4*z*   1 35  2 1.142  4 0  1 35  2.284  1 32.716  1.924 17 17 17 17  *z*  1 24  *x*  2 *y*   1 24 1.142  21.924  1 19.01  1.901  10 10 10 |
| **18** | **Let A have eigenvalues 2, 5, 0, -7, and -2. Does A have a dominant eigenvalue? If so, which is dominant?** |
|  | Since 7  5  2  2  0  A has a dominant eigenvalue is -7 |
| **19** | **Find the dominant eigenvalue of the matrix**  **1 2**  **by power method.**   **3 4**     |
|  | Let *X*  1 be initial eigenvector  0 1     *A X*  1 2 1  3  7 0.43  7 *X*  0 3 4 1 7  1  1           *A X*  1 2 0.43  2.43  5.29 0.46  5.29 *X*  1 3 4  1  5.29  1  2           *AX*  1 2 0.46  2.46  5.38 0.46  5.38*X*  2 3 4  1  5.38  1  3           The dominant eigenvalue is 5.38 and the corresponding eigenvector is 0.46   1     |
| **20** | **Find all the Eigenvalues and Eigenvectors of *A***  **2 2**  **by Jacobi method.**  **2** **1**     **[APR/MAY 2021]** |
|  | Let *A*  2 2   2 1     The largest off-diagonal element is *a*12  *a*21  2 and *a*11  2, *a*22  1. Here *a*11  *a*22  Let *S*  cos sin  be the rotaion matrix.  1 sin cos      Since *a*  *a* tan 2  2*a*12  2 2  4  2  tan1  4   0.927    0.463    11 22 *a*  *a* 2  1 3  3   11 22    sin  0.446  1 , cos  0.894  2  5 5   2 1    5 5   *S*1   1 2       5 5   Then the transformation is *B*  *ST AS*  1 1 1   2 1   2 1    5 5  2 2   5 5  3 0   *B*1   1 2  2 1  1 2   0 2            5 5   5 5  |

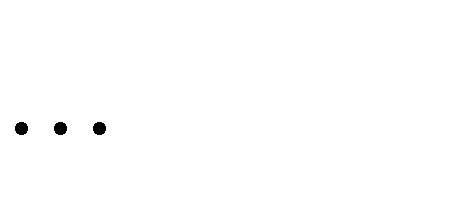
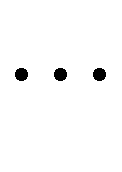


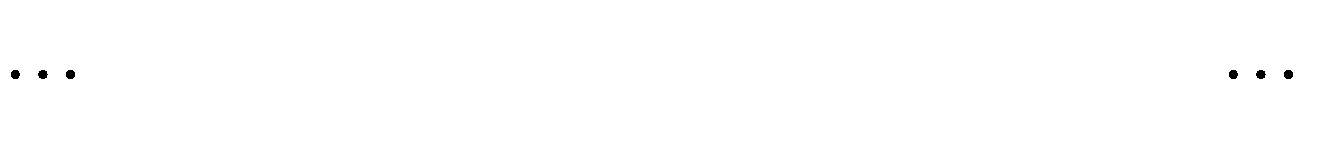
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|  | Thus *B*1 is diagonal matrix. So the eigenvalues of *A* are 3, 2 and the corresponding eigenvectors are the columns of *S*1   2   1    5   5  2 1    ,   *or*   ,     1   2  1  2    5   5  |
|  | **PART B** |
| **1(i)** | **Find a positive root of *f***  ***x***   **3*x***  **1**  **sin *x* , using fixed point iteration.**  **[APR/MAY 2021]** |
| **(ii)** | **Find a positive root of the equation cos *x***  **3*x***  **1**  **0 by the method of fixed-point iteration.** |
| **2(i)** | **Find the real root of the equation *x*3**  ***x*2**  **1**  **0 using fixed point iteration.** |
| **(ii)** | **Find the positive root of 3*x***  **log10 *x***  **6 using fixed point iteration method.** |
| **3(i)** | **Find Newton’s iterative formula to find the reciprocal of a given number *N* and hence**  **find the value of** 1  19 |
| **3(ii)** | **Using Newton-Raphson methods find the real root of 3*x***  **sin *x***  ***ex***  **0 by choosing initial approximation *x*0**  **0.5** |
| **4(i)** | **Compute the real root of *x* log10 *x***  **1.2 correct to three decimal places using Newton- Raphson method.** |
| **(ii)** | **Find the approximate root of *xe x***  **3 by Newton-Raphson method correct to three decimal places.** |
| **5(i)** | **Solve the following linear system of equations by gauss elimination method**  **2*x***  **3 *y***  ***z***  **1, 5*x***  ***y***  ***z***  **9, 3*x***  **2 *y***  **4*z***  **11 [APR/MAY 2021]** |
| **(ii)** | **Solve the following linear system of equations by gauss elimination method**  ***x***  **2 *y***  **5*z***  **9, 3*x***  ***y***  **2*z***  **5, 2*x***  **3 *y***  ***z***  **3** |
| **6(i)** | **Apply Gauss Jordan method to find the solution of the following system**  **3*x***  ***y***  **2*z***  **12, *x***  **2 *y***  **3*z***  **11, 2*x***  **2 *y***  ***z***  **2** |
| **(ii)** | **Using Gauss Jordan method solve the given system of equations 2*x***  ***y***  **3*z***  **8,**   ***x***  **2 *y***  ***z***  **4, 3 *x***  ***y***  **4*z***  **0** |
| **7(i)** | **Solve the following system of equations by Gauss-Jacobi method 27 *x***  **16 *y***  ***z***  **85,**  ***x***  ***y***  **54*z***  **110, 6*x***  **15 *y***  **2*z***  **72** |
| **(ii)** | **Solve the following system of equations by Gauss-Jacobi method 8*x***  ***y***  ***z***  **18, 2*x***  **5 *y* – 2*z***  **3, *x***  ***y* – 3*z***  **6** |
| **8(i)** | **Use Gauss - Seidel iterative method to obtain the solution of the equations**  **30*x***  **2 *y***  **3*z***  **75, 2*x***  **2 *y***  **18*z***  **30, *x***  **17 *y***  **2*z***  **48 (APR/MAY**  **2021)** |
| **(ii)** | **Find the solution of the system of following equations by Gauss - Seidel method upto three iterations *x***  **2 *y***  **5*z***  **12, 5*x***  **2 *y***  ***z***  **6, 2*x***  **6 *y***  **3*z***  **5** |
| **9(i)** | **Find the dominant eigenvalue and the corresponding eigenvector by power**   **1 3** **1**  **method for the matrix *A***   **3 2 4**  **[APR/MAY 2021]**     **1 4 10** |



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| **(ii)** | **5 0 1**  **Find the dominant eigenvalue and eigenvector *A***  **0** **2 0** **, using power method.**     **1 0 5** |
| **10(i)** | **1 0 0**   **Find the eigenvalues and eigenvectors of the matrix *A***  **0 3** **1** **, by using Jacobi**     **0** **1 3**   **method.** |
| **(ii)** |  **1 2 2**      **Find the Eigen values and Eigen vectors of the matrix *A***   **2 3 2**  **, by using**      **2 2 1**   **Jacobi method.** |

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|  | **UNIT IV - INTERPOLATION, NUMERICAL DIFFERENTIATION AND**  **NUMERICAL INTEGRATION** | | | | | | | | | | | | | | |
|  | **PART – A** | | | | | | | | | | | | | | |
| **1.** | **What is meant by Interpolation and Extrapolation?** | | | | | | | | | | | | | | |
|  | Interpolation is the process of computing intermediate values of a function from a given set of tabular values of the function (i.e.) inside the interval  *x*0 , *xn*  **.** Extrapolation is the process of finding the values outside the interval  *x*0, *xn*  . | | | | | | | | | | | | | | |
| **2.** | **Write down Inverse Lagrange’s Interpolation formula.** | | | | | | | | | | | | | | |
|  | Inverse Lagrange’s Interpolation formula is  ***x***  ( ***y***  ***y1*** )( ***y***  ***y2*** )....( ***y***  ***yn*** ) ***x***  ( ***y***  ***yo*** )( ***y***  ***y2*** ) ( ***y***  ***yn*** )  ( ***y***  ***y*** )( ***y***  ***y*** )....( ***y***  ***y*** ) ***o*** ( ***y***  ***y*** )( ***y***  ***y*** )....( ***y***  ***y***  ***o 1 o 2 o n 1 o 1 2 1 n*** | | | | | | | | ) | ***x1*** | |  .....  | | ( ***y***  ***y0*** )( ***y***  ***y1*** )....( ***y***  ***yn******1*** ) ***x***  ( ***y***  ***y*** )( ***y***  ***y*** )....( ***y***  ***y*** ) ***n n o n 1 n n******1*** | |
| **3.** | **Obtain Lagrangian interpolation polynomial from the data.** | | | | | | | | | | | | | | |
|  |  | | **-1** | | **1** | | | | | **2** | | | |  |
|  | | **7** | | **5** | | | | | **15** | | | |
|  | *f* (*x*)  (*x*  *x*1 )(*x*  *x*2 ) *y*  (*x*  *xo* )(*x*  *x*2 ) *y*  (*x*  *x*0 )(*x*  *x*1 ) *y*  (*x*  *x* )(*x*  *x* ) *o* (*x*  *x* )(*x*  *x* ) 1 (*x*  *x* )(*x*  *x* ) 2  *o* 1 *o* 2 1 *o* 1 2 2 *o* 2 1  *f* (*x*)  (*x* 1)(*x*  2) (7)  (*x* 1)(*x*  2) (5)  (*x* 1)(*x* 1)  (15) (11)(11) (11)(1 2) (2 1)(2 1)  *f* (*x*)  7 (*x*2  3*x*  2)  5 (*x*2  *x*  2)  15 (*x*2 1)  1 11*x*2  3*x*  7 6 2 3 3 | | | | | | | | | | | | | | |
| **4.** | **Construct the difference table for the data given** | | | | | | | |  |  | |  | | **(APR/MAY 2021)** | |
|  | |  | | **4** | | **6** | **8** | **10** | | | |  | | |
|  | | **1** | | **3** | **8** | **16** | | | |

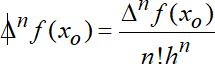




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| --- | --- | --- | --- |
| X | y = f(x) | ∆y | ∆2y |
| 0 | -1 |  |  |
|  |  | 2 |  |
| 1 | 1 |  | 1 |
|  |  | 3 |  |
| 2 | 4 |  |  |

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|  |  | | | | | | | |
|  | x | f(x) |  ***f* ( *x*)** | **2 *f* ( *x*)** | | **3 *f* ( *x*)** |  |
| 4 | 1 |  | |  |  |
|  |  | 3 - 1 = 2 | |  |  |
|  |  |  | | 5 - 2 = 3 |  |
| 6 | 3 |  | |  |  |
|  |  |  | |  | 3 - 3 = 0 |
|  |  | 8 - 3 = 5 | |  |  |
|  |  |  | | 8 - 5 = 3 |  |
| 8 | 8 |  | |  |  |
|  |  | 16 - 8 = 8 | |  |  |
| 10 | 16 |  | |  |  |
| **5.** | **State Newton’s Backward formula.** | | | | | | | |
|  | y(x)  yn  vyn  v(v 1)  2 yn  v(v 1)(v  2) 3 yn  ..... v(v 1) (v  (n 1)) n yn ,  2! 3! n!  Where v  x  x n  h | | | | | | | |
| **6.** | **Find the polynomial which takes the following values given f (0) = - 1, f (1) = 1 and**  **f (2) = 4 using Newton’s interpolating formula. (NOV/DEC 2016)** | | | | | | | |
|  | There are 3 data points given. Hence, the interpolating polynomial of degree will be 2.  *y*( *x*)  *y*  *u* *y*  *u*(*u*  1)  2 *y*  *u*(*u*  1)(*u*  2) 3 *y*   *u*(*u*  1)...(*u*  (*n*  1))  *n y*   *o o* 2! *o* 3! *o n*! *o*  where *u*  *x*  *x*0  *h*  *x*0  0 *h*  1  *u*  *x*  *x*(*x* 1) *x*2  *x* 1    *y*(*x*)  1  *x*(2)  (1)  1  2*x*   *x*2  3*x*  2  2! 2 2 | | | | | | | |
| **7.** | **Write down Newton’s divided difference formula.** | | | | | | | |
|  | Newton’s divided difference formula is  *f* (*x*)  *f* (*x*0 )  (*x*  *x*0 ) *f* (*x*0 , *x*1)  (*x*  *x*0 )(*x*  *x* ) *f* (*x*0 , *x* , *x*2 )   1 1   (*x*  *x*0 )(*x*  *x*1) (*x*  *xn*1 ) *f* (*x*0 , *x* , *x*2 , *xn* )  1 | | | | | | | |



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| **8.** | **State any two properties of Divided difference. (NOV/DEC 2016)** | | | | | | | | | | | | |
|  | 1. The divided differences are symmetrical in all their arguments. i.e. the value of any divided difference is independent of the order of the arguments. 2. The nth order divided differences of a polynomial of degree ‘n’ are constants. | | | | | | | | | | | | |
| **9.** | **When we use Newton’s forward method and Newton’s backward method?** | | | | | | | | | | | | |
|  | Newton’s forward formula is used to interpolate value of *y* nearer to the beginning value of the table. Newton’s backward formula is used to interpolate value of *y* nearer to the end of set  of tabular values. This may also be used to extrapolate closure to right of *yn* . | | | | | | | | | | | | |
| **10.** | **Form the divided difference table for 1, 3, 6, 11 and f(x) = x3**  **x**  **2 (APRIL/MAY 2021)** | | | | | | | | | | | | |
|  |  | | | *x:* | | 1 | | 3 | 6 | 11 | |  | |
| *f(x) =* **x3**  **x**  **2** | | 4 | | 32 | 224 | 1344 | |
|  | | | | | | | | | | | | |
|  | X | f(x) | |  | | 2 | | | | 3 | |  |
| 1  3  6  11 | 4  32  224  1344 | | 32  4  14 | |  | | | |  | |
| 3 1  224  32  64  6  3  1344  224  224  11 6 | | 64 14  10  6 1  224  64  20  11 3 | | | | 20 10  1  111 | |  |
| **11.** | **What is the relationship between the divided differences and forward differences?** | | | | | | | | | | | | |
|  | If the arguments are equally spaced, then  where h is the interval of difference. | | | | | | | | | | | | |
| **12.** | **Define Numerical Differentiation.** | | | | | | | | | | | | |
|  | *dy d* 2 *y*  Numerical differentiation is the process of computing the values of *dx* , *dx*2 …. for some  particular values of x from the given data (xi,yi) where y is not known explicitly. | | | | | | | | | | | | |
| **13.** | **Define Numerical Integration** | | | | | | | | | | | | |
|  | *b*  The process of computing the values of definite integrals  *y dx* from a set of (n + 1) paired  *a*  values (xi,yi), i = 0,1,2,…,n, where xo = a, xn = b of the function y = f(x) is called Numerical integration. | | | | | | | | | | | | |

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| **14.** | ***dy d* 2 *y***  **State Newton’s formula to find the derivatives *dx* , *dx* 2 at x = x0 using forward**  **differences.** | | | |
|  | *y*'  *f* '(*x*)  *dy*  1 *y*  1 2 *y*  1 3 *y*    *dx h*  *o* 2 *o* 3 *o* .  *d* 2 *y* 1  2 3 11 4   *y*''  *f* ''(*x*)    *y*   *y*   *y* .......  *dx*2 *h*2  *o o* 12 *o*  | | | |
| **15.** | ***dy d* 2 *y***  **Write down Newton’s formula to find the derivatives , using backward**  ***dx dx* 2**  **differences. (APR / MAY 2016)** | | | |
|  | *dy* 1  (2*v*  1) (3*v*2  6*v*  2)   *y* '  *f* '(*x*)  *dx*  *h* *yn*  2!  *yn*  3!  *yn*    2 3    *d* 2 *y* 1   6*v*2 18*v* 11   *x*  *x y* ''  *f* ''(*x*)   2 *y*  (*v* 1)3 *y*  4 *y*  ... *where v*   *n*  *dx*2 *h*2  *n n*  12  *n*  *h*   | | | |
| **16.** | **Write the formula for trapezoidal and Simpson’s 1/3rd rules. (APR/MAY 2019)** | | | |
|  | Trapezoidal rule: *xn h*  *I*   *f* (*x*)*dx*  2 [( *y*0  *yn* )  2( *y*1  *y*2  ...  *yn*1)]  *x*0  Simpson’s 1/3rd rule:  *xn h*  *I*   *f* (*x*)*dx*  3 [( *y*0  *yn* )  2( *y*2  *y*4 ...  *yn*2 )  4( *y*1  *y*3  ... *yn*1)]  *x*0 | | | |
| **17.** | **What is the order of error in Trapezoidal and Simpson’s 1/3rd rule?**  **(APR/MAY 2016, NOV/DEC 2017)** | | | |
|  | Trapezoidal rule: Error  (b  a) h 2 y' ' (  ) , a  x  b; Order of the error  h 2  12  Simpson’s rule: Error  (b  a) h 4 yiv (  ) , a  x  b ; Order of the error  h 4  180 | | | |
| **18.** | **Compare Trapezoidal rule and Simpson’s one-third rule.** | | | |
|  |  | **Trapezoidal rule** | **Simpson’s one-third rule** |  |
| It has no specific restriction on number of segments.  i.e. n = any number | It requires even number of segments and odd number of points  i.e. n = even |
| Degree of interpolating polynomial y(x) is one. | Degree of interpolating polynomial y(x) is two. |
| It is approximated by trapeziums. | It is approximated by set of parabolas. |
| **19.** | **Given the following data, find 4 by using Simpson’s 1 rule with h = 1**   **exdx**  **0 3** | | | |

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|  | *x* : 0 1 2 3 4  *y*  *ex* : 1 2.72 7.39 20.09 54.60  4 1   *exdx*  1 54.60  2 7.39  4 2.72  20.09  53.8733  0 3 | | | | | | | | | | | | | | | | | | | | | |
| **20.** | **Write the Trapezoidal rule to evaluate**  *f* (*x*, *y*)*dxdy* | | | | | | | | | | | | | | | | | | | | | |
|  | Trapezoidal rule for Double Integration is   sumof valuesof f (x, y) at the remaining nodes  I  hk sumof valuesof f x, yat the 4 corners  2 on theboundary  4     4sum of valuesof f (x, y) at the interior nodes  | | | | | | | | | | | | | | | | | | | | | |
|  | **PART – B** | | | | | | | | | | | | | | | | | | | | | |
| **1.** | **(i) Using Lagrange’s inverse interpolation formula, find the value of *x* when *y* = 13.5**  **from the given data** | | | | | | | | | | | | | | | | | | | | | |
|  | | **x** | **93.0** | | **96.2** | | **100.0** | | **104.2** | | | **108.7** | | |  | | | | | | |
| **y** | **11.38** | | **12.80** | | **14.70** | | **17.07** | | | **19.91** | | |
|  | **(ii) Using Lagrange’s interpolation calculate the profit in the year 2000 from the following data:** | | | | | | | | | | | | | | | | | | | | | |
|  | | **Year** | | | | | | **1997** | | | **1999** | | | **2001** | | | **2002** | |  | | |
| **Profit(in Lakhs)** | | | | | | **43** | | | **65** | | | **159** | | | **248** | |
|  | **(iii) Find y** **10** **given y** **5**  **12*,* y** **6**  **13*,* y** **9**  **14 and y** **11**  **16 by Lagrange’s**  **formula. [APRIL/MAY 2021]** | | | | | | | | | | | | | | | | | | | | | |
|  | **(iv) From the following table find the value of tan 45** **15'**  ***x*** **: 45 46 47 48 49 50**  **tan *x*** **: 1 1.03553 1.07237 1.11061 1.15037 1.19175** | | | | | | | | | | | | | | | | | | | | | |
| **2.** | **(i) Using Newton’s forward interpolation formula, find the cubic polynomial which takes the following values. Hence find *f*** **3** **. [APRIL/MAY 2019]** | | | | | | | | | | | | | | | | | | | | | |
|  | | | | ***x*** | | | **0** | | **2** | | **4** | | **6** | | |  | | | | | |
| ***f***  ***x*** | | | **-14** | | **6** | | **18** | | **118** | | |
|  | **(ii) The following table gives the population of a town during the last six censuses.**  **Estimate, using Newton’s interpolation formula, the increase in the population during the period 1946 to 1948.** | | | | | | | | | | | | | | | | | | | | | |
|  | **Year** | | | | | | **1911** | | | **1921** | | | **1931** | | | **1941** | | **1951** | | **1961** |  |
| **Population (in thousands)** | | | | | | **12** | | | **13** | | | **20** | | | **27** | | **39** | | **52** |
|  | **(iii) Find the number of students who scored marks not more than 45, from the following data [APRIL/MAY 2021]**  ***x* 30-40 40-50 50-60 60-70 70-80**  ***y* 35 48 70 40 22** | | | | | | | | | | | | | | | | | | | | | |
| **3.** | **(i) Using Newton’s divided difference formula, find the polynomial and hence find *y*** **3**  **from the data. [APRIL/MAY 2019]** | | | | | | | | | | | | | | | | | | | | | |
|  | | ***x*** | **-2** | **0** | | **1** | **4** | |  | | | | | | | | | | | | |
| ***y*** | **0** | **-2** | | **0** | **90** | |

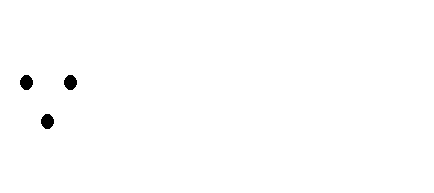
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|  | **(ii) If f** **1**  **1*,* f** **2**  **5*,* f** **7**  **5 and f** **8**  **4*,* find a polynomial that satisfies this data**  **using Newton’s divided difference formula. hence, find f** **6** **[APRIL/MAY 2021]** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| **4.** | **(i) Given the data below, find y'(6) and the maximum value of y. [NOV/DEC 2021]** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | ***x*** | **0** | | | **2** | | | **3** | | | **4** | | | **7** | | | | **9** | | |  | | | | | | | | | |
| ***y*** | **4** | | | **26** | | | **58** | | | **112** | | | **466** | | | | **922** | | |
|  | **(ii) Find the first, second and third derivatives of the function f(x) at x = 1.5** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | ***x*** | | **1.5** | | | **2.0** | | | | **2.5** | | | **3.0** | | | | | | **3.5** | | | **4.0** | | | |  | | | | |
| ***f(x)*** | | **3.375** | | | **7.0** | | | | **13.625** | | | **24.0** | | | | | | **38.875** | | | **59.0** | | | |
| **5.** | ***dy d*** 2 ***y***  **(i) Given the following values of x and y, find and at x = 1.05**  ***dx dx***2 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | ***x*** | | **1** | | | **1.05** | | | **1.10** | | | **1.15** | | | | | **1.20** | | | | **1.25** | | | **1.30** | | | |  | | |
| ***f(x)*** | | **1** | | | **1.025** | | | **1.049** | | | **1.072** | | | | | **1.095** | | | | **1.118** | | | **1.140** | | | |
|  | **(ii) A jet fighter’s position on an aircraft carrier’s runway was timed during landing.**  **t 1.0 1.1 1.2 1.3 1.4 1.5 1.6**  **y 7.989 8.403 8.781 9.129 9.451 9.750 10.031**  **Where y is the distance from one end of the carrier. Estimate velocity and acceleration at**  **t = 1.1** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| **6.** | **1**  **(i) Find the first two derivatives of**  ***x*** **3 at x=50 and x= 56, for the following table.** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | ***x*** | | | **50** | | | **51** | | | | **52** | | | | **53** | | | | | **54** | | | **55** | | | | **56** | | |  |
| **1**  **y =**  ***x*** **3** | | | **3.6840** | | | **3.7084** | | | | **3.7325** | | | | **3.7563** | | | | | **3.7798** | | | **3.8030** | | | | **3.8259** | | |
|  | **(ii) The table given below reveals the velocity v of a body during the time‘t’. Find its acceleration at t = 1.1 [APR/MAY 2019]** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | | ***t*** | | | | **1.0** | | | | | **1.1** | | | | | **1.2** | | | | | **1.3** | | | | **1.4** | | | |  | |
| ***v*** | | | | **43.1** | | | | | **47.7** | | | | | **52.1** | | | | | **56.4** | | | | **60.8** | | | |
| **7.** | **3**  **(i) Evaluate I**   **x4dx correct to three decimals dividing the range of integration into 8**  **3**  **equal parts using Trapezoidal and Simpson's** 1 **rule**  3 | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | **1 2**  **(ii) Find the value of log 21/3 from**  ***x dx* using Simpson’s 1 rule with *h***  **0.25**  **1**  ***x*3 3**  **0** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| **8** |   **(i) By dividing the range into 10 equal parts, evaluate**  **sin *x dx* by using Trapezoidal**  **0**  **rule. Verify your results by actual integration.** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
|  | **(ii) A curve passes through the points** **1**, **2**,**1**.**5**, **2**.**4**, **2**.**0**, **2**.**7**, **2**.**5**, **2**.**8**, **3**, **3**  **3**.**5**, **2**.**6** **and** **4**.**0**, **2**.**1**. **Obtain the area bounded by the curve, the X axis and x=1, x=4.** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |



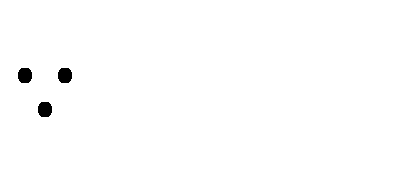
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|  | **Also find the volume of solid of revolution got by revolving this area about the X axis.** |
| **9.** | **2 4 dxdy**  **(i) Evaluate**   **x**  **y** **2 by using Trapezoidal and Simpson’s rule with h = k = 0.5.**  **1 3**  **[NOV/DEC 2021]** |
|  | 2 2  **(ii) Evaluate numerically**  xy dxdy **by using Trapezoidal rule taking h = k = 0.25**  x  y  1 1 |
| **10** | **1.4 2.4 *dx dy***  **(i) Evaluate**   ***xy* by using Trapezoidal rule, verify your results by actual**  **1 2**  **integration.** |
|  | **1 1**    **2 2 sin( *xy*)**  **(ii) Evaluate**   **1**  ***xy dx dy* by using Simpson’s rule with h=k=0.25**  **0 0** |

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|  | **UNIT V - INITIAL VALUE PROBLEMS FOR ORDINARY DIFFERENTIAL EQUATIONS** | | | | |
|  | **PART A** | | | | |
| **1.** | **Compare single-step method and multi-step methods.** | | | | |
|  |  | S.No | Single-step method | Multi-step method |  |
| 1 | It requires only one numerical value in order to compute the next  value | It requires not only the numerical value  but also at least four of the past values ,  , and . |  |
| 2 | Taylor series, Euler’s and R-K methods are single step methods | Milne’s, Adam’s methods are multi step methods |  |
| **2.** | **What are the different methods of solving an ordinary differential equation?**  **(APR /MAY 2021)** | | | | |
|  | The following methods are used for solving an ordinary differential equation: Taylor’s Series method, Euler’s and Modified Euler’s Algorithm method, Runge – Kutta (R.K) Method, Milne’s Predictor & Corrector Method and Adam’s Bash-Forth Predictor & Corrector method. | | | | |
| **3.** | **Are the multistep methods self-starting? Give reason.** | | | | |
|  | * Multistep methods are not self-starting, since kth – step method requires the k previous values. * The k-values that are required for starting the application of the method are obtained by using single step methods. | | | | |
| **4.** | **Write down the Taylor series formula for solving first order ODE.** | | | | |
|  | If  with  then  *y*  *y*  *h y*  *h*2 *y*   *h*3 *y* ..., where, *h*   *x*  *x*  is the step size.  *n*1 *n* 1 ! *n* 2 ! *n* 3 ! *n n*1 *n* | | | | |
| **5.** | **Write the truncation error in Taylor’s series method?** | | | | |
|  | (*x*  *x* )*n*  Error  *o yn* ( ). It is of order hn .  *n*! | | | | |
| **6.** | **Write the merits and demerits of the Taylor’s series method.** | | | | |
|  | **Merit:** Taylor’s series method is powerful single step method if successive derivatives can be obtained easily. | | | | |



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| *y*  *x*2  *y* | *y*0  *x*02  *y*0  0  0  0 |
| *y*   2*x*  *y* | *y*0  2*x*0  *y*0  2(0)  0  0 |
| *y*   2  *y*  | *y*0  2  *y*0  2 |
| *yiv*   *y* | *y iv*   *y*0 2  0 |

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|  | **Demerit:** (i) This method suffers from the time consumed in calculating the higher derivatives.  (ii) Accuracy of the answer depends on number of terms which is taken in Taylor’s  formula. |
| **7.** | **Find y (0.1) by Taylor’s series method if *dy***  ***x*2**  ***y* with *y*** **0**  **0**  ***dx*** |
|  | Given *dy*  *x*2  *y* (*i*.*e*) *y*  *x*2  *y* and *y* 0  0 (*i*.*e*) *y*  0, *x*  0 & *h*  0.1  *dx* 0 0  Taylor series formula is, *y*  *y*  *h y*  *h*2 *y*   *h*3 *y*  ...  *n*1 *n* 1! *n* 2! *n* 3! *n*  *Put n*  0, *y*  *y*  *h y*  *h*2 *y*   *h*3 *y*  *h*4 *yiv*  ...  1 0 1! 0 2! 0 3! 0 4!  Here, *x*0  0, *y*0  0  0.1 0.12 0.13 0.14   *y*1  0  1! 0  2! 0  3! 2  4! 2  ...  *y* 0.1  0  0  0  0.00033  0.000008333  0.000325 |
| **8.** | **What are the limitations of Euler’s method?** |
|  | 1. The attainable accuracy is limited by length of step h. 2. The method is slow and has limited accuracy. |
| **9.** | **Write the error of Euler’s method?** |
|  | *h*2 ' ' 2  Error *at* (*x*  *x*1 )  2! *y* (*x*1 , *y*1 ). It is of order h . |
| **10.** | **Find y(0.1) by Euler’s method, if *dy***  ***x*2**  ***y*2 , *y*(0)**  **0.1**  ***dx*** |
|  | Given *dy*  *x*2  *y*2 (*i*.*e*) *f* (*x*, *y*)  *x*2  *y*2 *and y*(0)  0.1 (*i*.*e*) *x*  0, *y*  0.1  *dx* 0 0  we need to find *y*(0.1)  *y*1 *at x*1  0.1  *h*  0.1  Euler’s method: *yn*1  *yn*  *h f* (*xn* , *yn* )  Put *y*1  *y*0  *h f* (*x*0 , *y*0 ) , *where h*  *x*1  *x*0  0.1 0  0.1.  *y*1  0.1 0.1 *f* (0,0.1)  0.1 0.102  0.12   *f* (*x*, *y*)  *x*2  *y*2   *y*1 0.1  0.1 0.10.01  0.1 0.001  0.101 |
| **11.** | **Write the formula for Modified Euler’s method.** |
|  | If with and *h*  *xn*1  *xn*  *y*(*x* )  *y*  *y*  *h f*  *x*  *h* , *y*  *h f* (*x* , *y* )   *n*1 *n*1 *n*  *n* 2 *n* 2 *n n*     |
| **12.** | **Why Modified Euler’s method is superior to Euler method? (APR/ MAY 2021)** |
|  | * Modified Euler method provides a more accurate approximation than Euler’s method. * It decreases the error than Euler’s Method. |



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| **13.** | **Use Modified Euler’s method to y(0.1) given dy**  **x2**  **y2 , y(0)**  **1 (APR/MAY 2021)**  **dx** | | | | | | |
|  | Given *dy*  *x*2  *y*2 (*i*.*e*) *f* (*x*, *y*)  *x*2  *y*2 *and y*(0)  1 (*i*.*e*) *x*  0, *y*  1  *dx* 0 0  Tofind *y*(0.1)(*i*.*e*) *y*1 *at x*1  0.1for *h*  0.1  Modified Euler’s formula**:** *y*(*x* )  *y*  *y*  *h f*  *x*  *h* , *y*  *h f* (*x* , *y* )   *n*1 *n*1 *n*  *n* 2 *n* 2 *n n*      Put *n*  0  *y*(0.1)  *y*  *y*  *h f*  *x*  *h* , *y*  *h f* (*x* , *y* )   1 0  0 2 0 2 0 0      *y*(0.1)  1 0.1 *f*  0  0.1,1 0.1 *f* (0,1)   1 0.1 *f* 0.05,1.05(02 12 )  *f* (*x*, *y*)  *x*2  *y*2    2 2       1 0.1 *f* 0.05,1.05)  1 0.1(0.052 1.052)  *y*(0.1)  1 0.1(0.25 1.1025)  1 0.1(0.1105)  1.1105 | | | | | | |
| **14.** | **Write down 4th order Runge-Kutta formula to solve first order differential equation.** | | | | | | |
|  | If with Then *y*  *y*  *y* where *y*  1 *k*  2*k*  *k*   *k*  ,  1 0 6 1 2 3 4  *k*  *h f*  *x y*  , *k*  *h f*  *x*  *h* , *y*  *k*1  , *k*  *h f*  *x*  *h* , *y*  *k*2  , *k*  *h f* *x*  *h*, *y*  *k*   1 0, 0 2  0 2 0 2  3  0 2 0 2  4 0, 0 3      | | | | | | |
| **15.** | **Compare Taylor’s Method and Runge-Kutta method.** | | | | | | |
|  | 1. The use of R-K method gives quick convergence to the solutions of the differential equations than Taylor’s series method. 2. The labour involved in R-K method is comparatively lesser .   (ii) No need to find the higher order derivatives in R.K Methods but Taylors series requires higher order derivatives | | | | | | |
| **16.** | **Write down Adam’s Bashforth predictor and corrector formulae to solve *dy***  ***dx***  **[APRIL/MAY 2021]** | | | | | | |
|  | The predictor formula is *y*  *y*  *h* [55 *y* '  59 *y* '  37 *y* '  9 *y* ' ]  *i* 1, *p i* 24 *i i* 1 *i*  2 *i* 3  The corrector formula is *y*  *y*  *h* [9 *y* ' 19 *y* '  5 *y* '  *y* ' ]  *i* 1, *c i* 24 *i* 1 *i i* 2 *i* 3 | | | | | | |
| **17.** | **Write down the Milne’s predictor and corrector formulae.** | | | | | | |
|  | The predictor formula is *y*  *y*  4*h* (2 *y* '  *y* '  2 *y* ' )  *n*1,*P n*3 3 *n*2 *n*1 *n*  The corrector formula is *y*  *y*  *h* ( *y* '  4 *y* '  *y* ' )  *n*1,*C n*1 3 *n*1 *n n*1 | | | | | | |
| **18.** | **Compare Adam’s Bashforth method with RungeKutta method.** | | | | | | |
|  |  | S.No | Adam’s Bashforth Method | Runge-Kutta Method | | |  |
| 1 | Multi step method | Single step method | | |
| 2 | Need four prior values of yi’s | Need only the last prior value | | |
| **19.** | **Write down the error in Adam’s predictor and corrector formulas.** | | | | | | |
|  | Order of error is h5 ; Error in predictor : 251 *h*5 *y*( *v* ) ( ) ; Error in corrector :  720 | | | | 19  120 | *h*5 *y*( *v* ) ( ) |  |
| **20.** | **Write down the error in Milne’s predictor and corrector formulas.** | | | | | | |
|  | Order of error is h5 ; Error in predictor : 14 *h*5 *y*( *v* ) ( ) ; Error in corrector :  45 | | | | 1  90 | *h*5 *y*( *v* ) ( ) |  |





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|  | **PART B** |
| **1.** | **(i) Find the value of y(0.2) and y(0.4) from , by Taylor’s series method.** |
|  | **(ii) Using Taylor’s series method up to fourth order find at and 1.2 by solving the equation with up to four decimals.**  **(APR/MAY 2021)** |
| **2.** | ***dy***   ***y*2**  **(i) Solve , *y*(0)**  **1 by Euler’s method by choosing h = 0.1, find y(0.1) and**  ***dx* 1**  ***x***  **y(0.2)** |
|  | **(ii) Compute y(4.1) and y(4.2) by using Euler method given that**  **5*xy*'**  ***y*2**  **2**  **0, *y*(4)**  **1 .** |
| **3.** | **(i) Solve *dy***  ***y***  **2*x* , given *y*(0)**  **1 and find values of y(0.1) and y(0.2) using**  ***dx y***  **Modified Euler’s method, correct to four decimal places. (APR/ MAY 2021)** |
|  | **(ii) Solve dy**  **log (x**  **y), y(0)**  **2 by Euler’s Modified method and find the values of y**  **dx 10**  **(0.2), y (0.4), and y (0.6), taking h=0.2** |
| **4.** | ***dy***  **(i) Compute given**  ***y***  ***xy*2**  **0, *y*(0)**  **1 by taking using Runge –**  ***dx***  **Kutta method of fourth order, correct to four decimal accuracy. (APR/MAY 2021)** |
|  | ***dy y*2**  ***x*2**  **(ii) Using Runge – Kutta method of fourth order, solve**  **2 2 , *y*(0)**  **1 at x = 0.2**  ***dx y***  ***x***  **& 0.4** |
| **5.** | **(i) Use Runge‐Kutta method of the fourth order to find**  **and**  **given that**  **by taking (up to four decimal places).**  **(APR/MAY 2021)** |
|  | **(ii) Compute y(0.1) and y(0.2) by Runge – kutta method of fourth order for the**  **differential equation *dy***  ***xy***  ***y*2 with y(0)=1.**  ***dx*** |
| **6.** | **(i) Given 5 *x dy***  ***y*2**  **2**  **0 , *y*(4)**  **1, *y*(4.1)**  **1.0049, *y*(4.2)**  **1.0097, *y*(4.3)**  **1.0143 .**  ***dx***  **Compute y(4.4) by Milne’s Predictor-Corrector Method.** |
|  | **(ii) Given**  **and**   **,**  **. Evaluate**  **by Milne’s predictor‐corrector method up to four decimal places. (APR/MAY 2021)** |
| **7** | ***dy***  **(i) Given**  ***x*2** **1**  ***y*****, *y*(1)**  **1, *y*(1.1)**  **1.233, *y*(1.2)**  **1.548, *y*(1.3)**  **1.979 .**  ***dx***  **Evaluate y(1.4) by Adam’s – Bashforth method.** |
|  | **(ii) Given that , y(1.1)**  **Find**  **using Adam’s Bashforth up to four decimals. (APR/ MAY 2021)** |
| **8.** | ***dy***  **(i) Using Taylor’s series method, solve**  ***xy***  ***y*2 , *y*(0)**  **1 at x = 0.1, 0.2, 0.3.**  ***dx***  **Continue the solution at x=0.4 by Milne’s Predictor – Corrector method.** |



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|  | **(APR/ MAY 2021)** |
|  | **(ii) Given *y***  ***x*( *x*2**  ***y*2 )*e*** ***x* , y(0) = 1 find y at x=0.1, 0.2 and 0.3 Taylor series method and compute y(0.4) by Milne’s method.** |
| **9** | **(i) Find y (0.1), y (0.2), y (0.3) from** y '  x2  y **; y (0) = 1 using Taylor’s series method and hence obtain y (0.4) using Adams-Bashford method.** |
|  | ***dy x***  ***y***  **(ii) Compute the first three steps of the initial value problems**  **, y(0)=1.0 by**  ***dx* 2**  **Taylor’s series method and next step by Milne’s method with step length h=0.1** |
| **10.** | **a) Consider the initial value problem *dy***  ***y***  ***x*2**  **1, *y*(0)**  **0.5**  ***dx***   1. **Using the Modified Euler method, find y(0.2) ;** 2. **Using R.K. Method of order 4, find y(0.4) and y(0.6) ;**   **(ii) Using Adam- Bashforth predictor corrector method, find y(0.8)** |
|  | 1. **Given and , Find**    1. **by Euler method**    2. **and**  **by modified Euler method**    3. ***y* (0.4) by Milne’s method.** |