DHANALAKSHMI SRINIVASAN COLLEGE OF ENGINEERING AND TECHNOLOGY
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DEPARTMENT OF INFORMATION TECHNOLOGY

QUESTION BANK(REG 2017)

SUBJECT:DISCRETE MATHEMATICS
SUBCODE: MA8351
SEM/YEAR: III/II

PART A

UNIT I(2 Marks)

LOGIC AND PROOFS

1. Using truth table show that the proposition $P \vee \neg (P \wedge Q)$ is a tautology.
2. Find the truth table for the statement $P \rightarrow Q$.
3.Does $P \rightarrow (P \Rightarrow Q)$ form a tautology.
4.Explain the two types of quantifiers through examples.
5. Show that $(P \rightarrow (Q \rightarrow R)) \rightarrow ((P \rightarrow Q) \rightarrow (P \rightarrow R))$ is a tautology.
6. When do you say that two compound propositions are equivalent.
7.State the truth table of " If tigers have wings then the earth travels round the sun" .
8. S.T . $\neg P \rightarrow (P \rightarrow Q)$ is a tautology.
9.S.T $(x)(P(x) \rightarrow Q(x)) \wedge (x)(Q(x) \rightarrow R(x)) \Rightarrow (x)(P(x) \rightarrow R(x))$
10.Define functionally complete set and give example for it.

PART B

UNIT I(8 Marks)

LOGIC AND PROOFS

1. Obtain the PDNF and PCNF of the statement $p \vee (\neg p \rightarrow (q \vee (\neg q \rightarrow r)))$
2. Prove that $\forall x(P(x) \rightarrow Q(x)),\ \forall x(R(x) \rightarrow \neg Q(x)) \Rightarrow \forall x(R(x) \rightarrow \neg P(x))$
3.Show that $(P \rightarrow Q) \wedge (R \rightarrow S), (Q \wedge M) \wedge (S \rightarrow N), \neg(M \wedge N)$ and $(P \rightarrow R) \Rightarrow \neg P$
4.Prove that $\sqrt{2}$ is irrational by giving a proof using contradiction.
5.Use indirect method to prove that that $\forall x(P(x) \vee Q(x)) \Rightarrow (\forall x)P(x) \vee (\exists x)Q(x)$
6.Prove that the premises $a \rightarrow (b \rightarrow c), d \rightarrow (b \wedge \neg P)$ and $(a \wedge d)$ are inconsistent.
7. Without using truth table prove that $\neg P \rightarrow (q \rightarrow r) \equiv q \rightarrow (p \vee r)$
8 . Without constructing the truth table obtain the product of sums canonical form of the formula. $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$. Hence find the sum of products canonical form.
9. S.T (i) $\neg(P \leftrightarrow Q) \Leftrightarrow (P \vee Q) \wedge \neg(P \wedge Q)$
(ii) $\neg(P \leftrightarrow Q) \Leftrightarrow (P \wedge \neg Q) \vee (\neg P \wedge Q)$

10.Explain Nested quantifiers and give example for it.

PART A

UNIT2 (2 Marks)

COMBINATORICS

1.What is the solution of the recurrence relation $a_n = a_{n-1} + 2a_{n-2}$?
2.State Pigeonhole principle.
3. In how many ways can all the letters in MATHEMATICAL is arranged.

4. Twelve students want to place order of different ice-creams in ice-cream parlour, which has the six type of ice-creams. Find the number of orders that the twelve students can place.
5.Find the recurrence relation satisfying the equation=$A(3)^n+B(-4)^n$
6.Use mathematical induction to show that $n!\geq 2^{n-1}$
7..How many integers between 1 to 100 that are divisible by3 but not by 7.
8.Define permutation and give example for it.

9.Define combination and give example for it.

10. Show that if 7 colours are used to paint 50 bicycles, atleast 8 bicycles will be the same colour.

PART B

UNIT2 (8Marks)

COMBINATORICS

1.Using Mathematical induction show that $\sum_{k=1}^{n} k^2 = \frac{n(n+1)(2n+1)}{6}$
2.Using generating function solve $y_{n+2}-5y_{n+1}+6y_n=0, n\geq 0$ with $y_0=1$ and $y_1=1$
3.Find the number of distinct permutation that can be formed from all the letters of each word(i)RADAR (ii)UNUSUAL
4.Prove by mathematical induction that for all $n\geq 1$, n^3+2n is a multiple of 3.

5.How many positive integers n can be formed using the digits 3,4,4,5,5,6,7 if n has to exceed 5000000?
6.Find the number of integers between 1 and 250 both inclusive that are divisible by any of the integers 2,3,5,7.

7.Using generating function, solve the difference equation $y_{n+2}-y_{n+1}-6y_n=0$, $y_1=1$, $y_0=2$

8. Solve the recurrence relation, S (n)= S (n-1)+2(n-1), with S (0) = 3, S (1) = 1 by finding its generating functions

9.Solve the recurence relation of the Fibonacci sequence of numbers

$$f_n = f_{n-1} + f_{n-2} \; ; \; n > 2$$

10.Any positive integer $n\geq 2$ is either a prime or a product of primes. To prove this we use the principle of strong mathematical induction.

PART A
UNIT 3(2 marks)
GRAPHS

1.Define graph & Complete Graph with example.
2.When a simple graph G bipartite and give an example.
3.State Hand Shaking theorem.
4. Define Strongly connected graph.
5. Define Pseudo graph
6. Define isomorphism of two graphs.

7. Give an example of an Euler graph.
8.State the necessary and sufficient conditions for the existence of an Eulerian path in a connected graph.
9..State any two properties of a graph.
10.Define adjacency matrix of a graph
11.Draw the complete graph K_5.
12.Give an example of a graph which is Eulerian but not Hamiltonian
13.Give an example of non-Eulerian graph which is Hamiltonian.

PART B

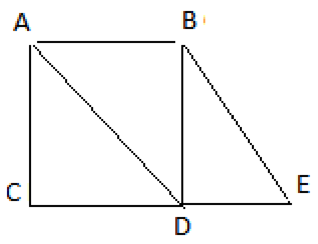
UNIT 3(8 marks)

GRAPHS

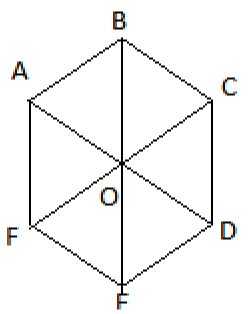
1.State and prove Handshaking Theorem.
2.Prove that number of vertices of odd degree in a graph is always even.
3.The adjacency matrices of two pairs of graph as given below. Examine the isomorphism of G and H finding a permutation matrix.

$$A_G = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad A_H = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 0 \\ 1 & 0 & 0 \end{bmatrix}$$

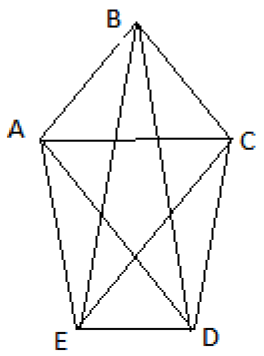
4. Draw the graph with 5 vertices A,B,C,D,E such that deg.(A)= 3; deg(B) = odd; deg.(C) = 2; & D&E are adjacent..
5. Find an Euler path or Euler circuit if it exists in each of three graph below. If not explain why?



G1

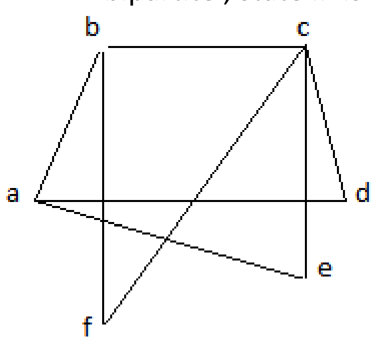


G2

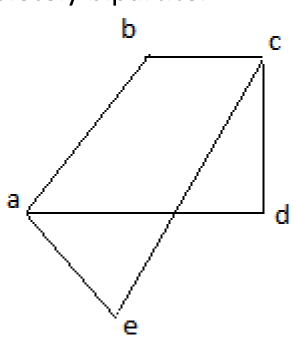


G3

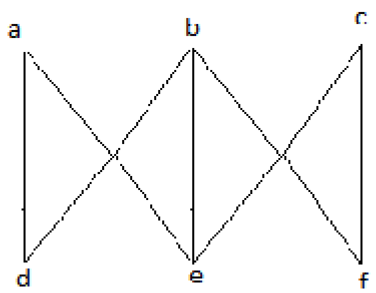
6. Determine which of the following graph are bipartite & which are not. If a graph is bipartite , state if is completely bipartite.



G1



G2



G3

7. If all the vertices of an undirected graph are each of degree k, show that the number of edges of the graph is a multiple of k

8. Draw the complete graph K_5 with vertices A,B,C,D,E. Draw all complete subgraph of K_5 with 4 vertices.

9.Prove that a simple graph with n vertices and k components can have at most (n-k)(n-k+1)/2 edges.

10.Let G be a simple undirected graph with n vertices. Let u and v be two non adjacent vertices in G such that deg(u) + deg(v) ≥ n in G. Show that G is Hamiltonian if and only if G + uv is Hamiltonian.

PART A
UNIT 4(2 marks)
ALGEBRAIC STRUCTURES

1.Define Semi Group
2.Define Monoid
3.Prove that identity element in a group is unique.
4.State any two properties of a group.
5.Show that the inverse of an element in a group (G, *) is unique
6.Prove that identity element of a subgroup is same as that of the group.
7.Prove that every subgroup of an abelian group is a normal.
8.State Lagrange's theorem.
9.Define Ring
10.Define field in an algebraic system
11.Deefine the homomorphism of two groups.
12.When is group(G,*) called abelian?

PART B
UNIT 4(8 marks)
ALGEBRAIC STRUCTURES

1. If(G *) is an abelian group , S.T $(a*b)^2 = a^2 * b^2$
2. State and prove Lagrange's Theorem
3. If * is a binary operation on the set R of real numbers defined a * b = a+b-2ab
   - Find < R , * > is semi group.
   - Find identity element if it is exists.
   - Which element has inverse & what are they.
4. If (G,*) is finite cyclic group generated by a element a Є G & order n, then $a^n = e$,
   So that G = { a, a^2 ,a^3 ,.....a^n =e} also n is the least +ve integer forw with a^n =e.

5.Show that (Z,+,X) is an integral domain where Z is the set of all integers.

6.If (Z,+) and (E,+) where Z is the set of all integers and E is the set of all even integers, show that the two semi groups (Z,+) and (E,+) are isomorphic.

7.Show that every finite semi group has an idempotent element.

8.If any group (G,*),show that $(a*b)^{-1}=b^{-1}*a^{-1}$

9.State and prove the fundamental theorem of homomorphism.

10. Prove that the group homomorphism preserves identity and inverse element.

11. Prove that intersection of any two normal subgroups of a group (G, *) is a normal subgroup of a group (G, *).

12. Discuss Ring and Fields with suitable examples.

UNIT 5(2marks)PART A
LATTICES AND BOOLEAN ALGEBRA

1.In a Lattice □***L***, ≤ □ ,prove that $a \wedge (a \vee b) = a$ for all ***a*** , ***b*** □ ***L***

2.Show that in a lattice if a≤b≤c, then a+b=b*c and (a*b)+(b*c)=(a+b)*(b+c)

3.Show that in a distributive lattice, if complement of an element exists then it must *be* unique.

4.Define Boolean Algebra.

5.Define sub-lattice.

6.When is a lattice called complete?

7.Show that in a Boolean algebra ab´+a´b=0 if and only if a=b.

8.Define Lattice.

9.Draw the Hasse diagram for $\{(a,b), a \text{ divides } b\}$ on $\{1,2,3,4,6,8,12\}$.

10.Let a, b, c ∈ B , S.T (i) a.0 =0 (ii) a+1 = 1.

11.Draw the Hasee diagram of (X,≤), where X={2,4,5,10,12,20,25} and the relation ≤ be such that x≤y if x divides y.

UNIT 5(8marks)PART B
LATTICES AND BOOLEAN ALGEBRA

1.Show that every finite partial ordered set has a maximal and minimal element.

2.Show that (N,≤) is a partially ordered set where N is set of all positive integers and ≤ is defined by m≤n iff(n-m) is a non-negative integer.

3.In a Boolean Algebra, prove that (a ∧b)’ = a’∨b’.

4.In a Lattice (L,≤), prove that x∨(y∧z) ≤ (x∨y) ∧(x∨z)

5.In S_{42} is the set of all divisors of 42 and D is the relation “divisor of” on S_{42}, prove that {S_{42},D} is a complemented Lattice.

6. Show that the direct product of any two distributive lattices is a distributive lattice.

7.Let ***B*** be a finite Boolean algebra and let ***A*** be the set of all atoms of ***B***.Then prove that the Boolean algebra ***B*** is isomorphic to the Boolean algebra ***P* (*A*)** ,here ***P* (*A*)** is the power set of ***A***

8. Draw the Hasse diagram representing the partial ordering {(A,B):A subset of B} on the power set P(S) where S={a,b,c}. Find the maximal, minimal, greatest and least elements of the poset.

9.Simplify the Boolean Expression

a´.b´.c+a.b´.c+a´.b´.c´ using Boolean algebra identities.

10.In a Boolean , prove that a(a+b)=a, for all a,b € B.

11. (L,≤) be a lattice for any a,b∈L. a≤b if a∧b=a and a∨b=b.

12.Show that in a complemented distributive lattice a≤b⟷a∧ b =0,

a ∨b= b ≤ a

13.If (L, ∧, ∨) is a complemented distributive lattice, then prove the Demorgan’s laws.

14.In any Boolean algebra show that a. b + b. a = (a+b)(a + b)