**DHANALAKSHMI SRINIVASAN COLLEGE OF ENGINEERING**

**AND TECHNOLOGY**

DEPARTMENT OF MATHEMATICS

QUESTION BANK

III SEMESTER

**MA8353- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS**

V SEMESTER

Regulation – 2017

Academic Year 2018- 2019

**QUESTION BANK**

**SUBJECT : MA8353- TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS**

**SEMESTER / YEAR: III / II (CIVIL, EEE, EIE & MECH)**

|  |  |  |  |
| --- | --- | --- | --- |
| **UNIT I - PARTIAL DIFFERENTIAL EQUATIONS**  Formation of partial differential equations – Singular integrals – Solutions of standard types of first order partial differential equations – Lagrange’s linear equation – Linear partial differential equations of second and higher order with constant coefficients of both homogeneous and non-homogeneous types. | | | |
| **PART- A** | | | |
| **Q.No.** | **Question** | **Bloom’s Taxonomy Level** | **Domain** |
| 1. | Form a partial differential equation by eliminating the arbitrary constants ‘a’ and ‘b’ from *z*  *ax*2  *by*2. | BTL -6 | Creating |
| 2. | Eliminate the arbitrary function from z = ƒ(x2 − y2) and form the partial differential equation | BTL -6 | Creating |
| 3. | Construct the partial differential equation of all spheres whose centers lie on the x-axis. | BTL -3 | Applying |
| 4. | Form the partial differential equation by eliminating the arbitrary function f from z = e𝑎y ƒ(x + by). | BTL- 6 | Creating |
| 5. | Form the partial differential equation by eliminating the arbitrary constants a, b from the relation log( *az* 1)  *x*  *ay*  *b*. | BTL -6 | Creating |
| 6. | Form the PDE by eliminating the arbitrary function from  ** *z* 2  *xy*, *x*   0   *z*  | BTL -6 | Creating |
| 7. | Form the partial differential equation from  (*x*  *a*)2  ( *y*  *b*)2  *z* 2 cot 2 ** | BTL -6 | Creating |
| 8. | Form the partial differential equation by eliminating the arbitrary function  from (*x*2  *y*2 , *z*)  0 | BTL -6 | Creating |
| 9. | Form the partial differential equation by eliminating arbitrary constants a and b from (*x*  *a*)2  ( *y*  *b*)2  *z* 2  1 | BTL -6 | Creating |
| 10. | Find the complete integral of √p + √q = 1. | BTL -3 | Applying |
| 11. | Find the complete solution of *q*  2 *px* | BTL -3 | Applying |
| 12. | Find the complete integral of *p*  *q*  *pq* | BTL -3 | Applying |
| 13. | Solve *px*2  *qy*2  *z* 2 | BTL -3 | Applying |

|  |  |  |  |
| --- | --- | --- | --- |
| 14. | Solve (*D*2  7*DD*'6*D*'2 )*z*  0 | BTL -3 | Applying |
| 15. | Solve (*D*3  *D*2 *D*'8*DD*'2 12*D*'3 )*z*  0 | BTL -3 | Applying |
| 16. | 2 *z* 2 *z* *z*  Solve *x*2  *x**y*  *x*  0 | BTL -3 | Applying |
| 17. | Solve (*D*4  *D*'4 )*z*  0 | BTL -3 | Applying |
| 18. | Solve (*D*  *D*'1)(*D*  2*D*'3)*z*  0 | BTL -3 | Applying |
| 19. | Solve (*D*  *D*')3 *z*  0 | BTL -3 | Applying |
| 20. | Solve (*D* 1)(*D*  *D*'1)*z*  0 | BTL -3 | Applying |
| **PART – B** | | | |
| 1.(a) | Find the PDE of all planes which are at a constant distance ‘k’  units from the origin. | BTL -6 | Creating |
| 1. (b) | Find the singular integral of *z*  *px*  *qy*  1 *p*2  *q*2 | BTL -2 | Understanding |
| 2. (a) | Form the partial differential equation by eliminating arbitrary function  from (*x*2  *y*2  *z* 2 , *ax*  *by*  *cz*)  0 | BTL -6 | Creating |
| 2.(b) | Find the singular integral of *z*  *px*  *qy*  *p*2  *pq*  *q*2 | BTL -2 | Understanding |
| 3. (a) | Form the partial differential equation by eliminating arbitrary  functions *f* and *g* from *z*  *x f* ( *y* )  *y g*(*x*)  *x* | BTL -6 | Creating |
| 3.(b) | Solve ( *p*  *x*)2  ( *q*  *y*)2  1 2 2 | BTL -3 | Applying |
| 4. (a) | Solve *x*2 *p*  *y*2 *q*  *z*(*x*  *y*) | BTL -3 | Applying |
| 4.(b) | Form the partial differential equation by eliminating arbitrary function *f* and g from the relation z = x ƒ( x + t) + g( x + t) | BTL -6 | Creating |
| 5. (a) | Solve (*x*2  *yz*) *p*  ( *y*2  *xz*)*q*  (*z*2  *xy*) | BTL -3 | Applying |
| 5.(b) | Solve 9( *p*2 *z*  *q*2 )  4 | BTL -3 | Applying |
| 6. (a) | Find the general solution of ( 3z − 4y )p + (4x − 2z )q = 2y – 3x | BTL -2 | Understanding |
| 6.(b) | Solve ( *y*2  *z* 2 ) *p*  *xyq*  *xz*  0 | BTL -3 | Applying |
| 7. (a) | Find the complete solution of *z* 2 ( *p*2  *q*2 1)  1 | BTL -4 | Analyzing |
| 7. (b) | Find the general solution of (*D*2  2*DD*'*D*'2 )*z*  2cos *y*  *x*sin *y* | BTL -2 | Understanding |
| 8. (a) | Find the general solution of (*D*2  *D*'2 )*z*  *x*2 *y*2 | BTL -2 | Understanding |

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| 8.(b) | Find the singular integral of *z*  *px*  *qy*  *p*2  *q*2 | | BTL -2 | | Understanding | |
| 9. (a) | Solve (*D*2  3*DD*'2*D*'2 ) *z*  (2  4*x*)*ex*2 *y* | | BTL -3 | | Applying | |
| 9.(b) | Find the general solution of  (*z*2  *y*2  2*yz*) *p*  (*xy*  *zx*)*q*  (*xy*  *zx*) | | BTL -2 | | Understanding | |
| 10.(a) | Solve *x*( *y*2  *z*2 ) *p*  *y*(*z*2  *x*2 )*q*  *z*(*x*2  *y*2 ) | | BTL -3 | | Applying | |
| 10.(b) | Solve (*D*2  3*DD*'2*D*'2 )*z*  sin(*x*  5*y*) | | BTL -3 | | Applying | |
| 11.(a) | Solve the Lagrange’s equation (*x*  2*z*) *p*  (2*xz*  *y*)*q* *x*2  *y* | | BTL -3 | | Applying | |
| 11.(b) | Solve (*D*2  *DD*'2*D*'2 )*z*  2*x*  3*y*  *e*2*x*4 *y* | | BTL -3 | | Applying | |
| 12(a) | Solve (*D*2  5*DD*'6*D*'2 )*z*  *y* sin *x* | | BTL -3 | | Applying | |
| 12.(b) | Solve the partial differential equation  (*x*  2*z*) *p*  (2*z*  *y*) *q*  *y*  *x* | | BTL -3 | | Applying | |
| 13.(a) | Solve ( D2 − DD’ − 20D’ 2) z = e5s+y + s𝑖n (4x − y). | | BTL -3 | | Applying | |
| 13.(b) | Solve (*D*2  3*DD*'2*D*'2 2*D*  2*D*')*z*  sin(2*x*  *y*). | | BTL -3 | | Applying | |
| 14.(a) | Solve (*D*2  2*DD*'*D*'2 )*z*  *x*2 *y*  *ex* *y* | | BTL -3 | | Applying | |
| 14.(b) | Solve (*D*3  7*DD*'2 6*D*'3 )*z*  sin(*x*  2*y*) | | BTL -3 | | Applying | |
| **UNIT II FOURIER SERIES:**  Dirichlet’s conditions – General Fourier series – Odd and even functions – Half range Sine series – Half range Cosine series – Complex form of Fourier series – Parseval’s Identity – Harmonic analysis. | | | | | |  |
| **PART –A** | | | | | |
| **Q.No** | **Question** | **Bloom’s Taxonomy Level** | | **Domain** | |
| 1. | State the Dirichlet’s conditions for a function f(x) to be expanded as a Fourier series. | **BTL -1** | | Remembering | |
| 2. | State the sufficient condition for a function f(x) to be expressed as a Fourier Series. | **BTL -1** | | Remembering | |
| 3. | If (𝜋 − x)2 = 𝜋2 + 4 ∑*∞* 𝑐ocns 𝑖n 0 < x < 2𝜋 then deduce that  3 n=1 n2  value of ∑*∞* 1 .  n=1 n2 | **BTL -1** | | Remembering | |
| 4. | Does ƒ(x) = tanx posses a Fourier expansion? | **BTL -2** | | Understanding | |
| 5. | Determine the value of an in the Fourier series expansion of | **BTL -5** | | Evaluating | |

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
|  | ƒ(x) = x3 in (-𝜋, 𝜋). | |  | |  |
| 6. | Find the constant term in the expansion of cos2x as a Fourier  series in the interval (-𝜋, 𝜋). | | **BTL -2** | | Understanding |
| 7. | If f(x) is an odd function defined in (-l, l). What are the values of  a0 and an? | | **BTL -2** | | Understanding |
| 8. | If the function f(x) = x in the interval 0<x<2𝜋 then find the constant term of the Fourier series expansion of the function f. | | **BTL -2** | | Understanding |
| 9. | If the Fourier series of the function f(x) = x+x2 , −𝜋 < x < 𝜋.  with Period 2𝜋 is given by ƒ(x) = 𝜋2 + ∑∞ (−1)n [ 4 cos nx −  3 n=1 n2  2 sin nx] , then find the value of the infinite series 1 + 1 + 1 +…  n 12 22 32 | | **BTL -4** | | Analyzing |
| 10. | Write a0 , an in the expression x + x3 as a Fourier series in  (-𝜋, 𝜋) | | **BTL -3** | | Applying |
| 11. | Write the Complex form of Fourier Series for a function f(x) defined in −l ≤ x ≤ l. | | **BTL -3** | | Applying |
| 12. | Find the root mean square value of f(x) = x2 in (0,𝜋) | | **BTL -1** | | Remembering |
| 13. | Find the RMS value of f(x) = x(l -x) in 0≤ x ≤ l | | **BTL -3** | | Applying |
| 14. | Find the RMS value of f(x) = x2 in (0, l) | | **BTL -1** | | Remembering |
| 15. | Write down the Parseval’s formula on Fourier coefficients | | **BTL -5** | | Evaluating |
| 16. | Define the RMS value of a function f(x) over the interval (a, b) | | **BTL -6** | | Creating |
| 17. | Without finding the values of a0 , an and bnof the Fourier series, for the function f(x) = x2 in the interval (0,2𝜋) find the value of  {𝑎02 + ∑*∞* (a 2 + b 2)}  2 n=1 n n | | **BTL -4** | | Analyzing |
| 18. | Find the R.M.S value of ƒ(x) = 1 − x in 0 < x < 1. | | **BTL -1** | | Remembering |
| 19. | State Parseval’s identity for the half-range cosine expansion of f(x) in (0,1). | | **BTL -6** | | Creating |
| 20. | What is meant by Harmonic Analysis? | | **BTL -3** | | Applying |
| PART – B | | | | | |
| 1.(a) | | Find the Fourier series expansion of | | **BTL -1** | Remembering |

|  |  |  |  |
| --- | --- | --- | --- |
|  | ƒ(x) = { x ƒor 0 ≤ x ≤ 1 2 − x ƒor 1 ≤ x ≤ 2. |  |  |
| 1.(b) | Find the Fourier series of ƒ(x) = x2 𝑖n − 𝜋 < x < 𝜋.Hence  deduce the value of ∑*∞* 1  n=1 n2 | **BTL -1** | Remembering |
| 2.(a) | Obtain the Fourier series to represent the function  ƒ(x) = |x|, −𝜋 < x < 𝜋 and Deduce ∑*∞* 1 = 𝜋2 .  n=1 (2n−1)2 8 | **BTL -2** | Understanding |
| 2.(b) | Find the Fourier series of the function ƒ(x) = {0 − 𝜋 ≤ x ≤ 𝜋  s𝑖nx 0 ≤ x ≤ 𝜋  1 1 1  and Hence Evaluate 1.3 + 3.5 + 5.7 +…… | **BTL -2** | Understanding |
| 3.(a) | 1 + 2s , −𝜋 < x < 0  Expand ƒ(x) = { 𝜋 as a full range Fourier  1 − 2s , 0 < x < 𝜋  𝜋  1 1 1  series in the interval (−𝜋, 𝜋).Hence deduce that 12 + 32 + 52 +  ⋯ *∞* = 𝜋2. 8 | **BTL -1** | Remembering |
| 3.(b) | Find a Fourier series with period 3 to represent  ƒ(x) = 2x − x2 𝑖n (0,3). | **BTL -1** | Remembering |
| 4.(a) | Determine the Fourier series for the function ƒ(x) = xs𝑖nx 𝑖n  0 < x < 2𝜋. | **BTL -5** | Evaluating |
| 4.(b) | Obtain the Fourier series for the function f(x) given by ƒ(x) =  1 − x, − 𝜋 < x < 0  { 1 + x, 0 < x < 𝜋 Hence deduce that  1 + 1 + 1 + ⋯ = 𝜋2  12 32 52 8 | **BTL -3** | Applying |
| 5.(a) | Determine the Fourier series for the function ƒ(x) = |cosx|  𝑖n − 𝜋 < x < 𝜋. | **BTL -5** | Evaluating |
| 5.(b) | Expand ƒ(x) = { x, 0 < x < 1 as a series of cosines in the  2 − x , 1 < x < 2  interval (0,2). | **BTL -1** | Remembering |
| 6.(a) | Find the Fourier expansion of the following periodic function of period 4 ƒ(x) = { 2 + x, −2 ≤ x ≤ 0  2 − x, 0 ≤ x ≤ 2 | **BTL -2** | Remembering |

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  | 1 1 1 𝜋2  Hence deduce that 12 + 32 + 52 + ⋯ *∞* = 8 . | | | | | | | |  |  |
| 6.(b) | Find the half range sine series of ƒ(x) = 4x − x2 in the interval  1 1 1 1  (0,4).Hence deduce the value of the series 13 − 33 + 53 − 73 +  ⋯ *∞*. | | | | | | | | **BTL -2** | Remembering |
| 7.(a) | Find the Fourier series of ƒ(x) = |s𝑖nx| 𝑖n − 𝜋 < x < 𝜋 of periodicity 2𝜋. | | | | | | | | **BTL -4** | Analyzing |
| 7.(b) | Find the Fourier series of ƒ(x) = x + x2 in (-𝜋, 𝜋) with period  2 𝜋. Hence deduce ∑*∞* 1 𝜋2.  n=1 n2= 6 | | | | | | | | **BTL -3** | Applying |
| 8.(a) | Compute the first two harmonics of the Fourier series of f(x) from the table given | | | | | | | | **BTL -6** | Creating |
| 8.(b) | Find the complex form of the Fourier series of Cosax in (−𝜋, 𝜋)  where “a” is not an integer. | | | | | | | | **BTL -2** | Remembering |
| 9.(a) | Find the Fourier series of ƒ(x) = x2 𝑖n (−𝜋, 𝜋) and hence deduce  1 1 1 1 𝜋4  that 14 + 24 + 34 + 44 + ⋯ *∞* = 90 | | | | | | | | **BTL -2** | Remembering |
| 9.(b) |  | | | | | | | | **BTL -3** | Applying |
| t secs | 0 | T/6 | T/3 | T/2 | 2T/3 | 5T/6 | T |
| A  amps | 1.98 | 1.30 | 1.05 | 1.30 | -0.88 | -0.25 | 1.98 |
| The above table gives the variation of a periodic current over a period by harmonic analysis, Show that there is a direct current part of 0.75amps in the variable current. Also obtain the amplitude of the first harmonic. | | | | | | | |
| 10.(a) | Find the half range Fourier cosine series of ƒ(x) = (𝜋 − x)2 in the  interval (0, 𝜋). Hence Find the sum of the series 1 + 1 + 1  14 24 34  +……. | | | | | | | | **BTL -1** | Remembering |
| 10.(b) | Obtain the fourier cosine series expansion of ƒ(x) = x  𝑖n 0 < x < 4.Hence deduce the value of 1 +  1 + 1 + ⋯ *∞*.  14 34 54 | | | | | | | | **BTL -1** | Remembering |
| 11.(a) | 𝜋4 1 1  By using Cosine series sℎow tℎat 96 = 1 + 34 + 54 +  ⋯ for ƒ(x) = x 𝑖n 0 < x < 𝜋 | | | | | | | | **BTL -4** | Analyzing |

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 𝜋/3 | 2𝜋/3 | 𝜋 | 4𝜋/3 | 5𝜋/3 | 2𝜋 |
| f(x) | 1 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1 |

|  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
|  |  | | | | | | | | | | | | | | | | | | | | | | | | | |  |  | |  |
| 11.(b) | Find the Fourier cosine series up to third harmonic to represent the function given by the following data: | | | | | | | | | | | | | | | | | | | | | | | | | | **BTL -6** | Creating | |
|  | | x | | 0 | | | 1 | | | 2 | | | | 3 | | | 4 | | | 5 | |  | | | |
| y | | 4 | | | 8 | | | 15 | | | | 7 | | | 6 | | | 2 | |
| 12.(a) | Show that the complex form of Fourier series for the function  ƒ(x) = eswℎen − 𝜋 < x < 𝜋 and ƒ(x) =  ƒ(x + 2𝜋) 𝑖s ƒ(x) = c𝑖nℎ𝜋 ∑*∞* (−1)n 1+𝑖n e𝑖ns.  𝜋 n=−*∞* 1+n2 | | | | | | | | | | | | | | | | | | | | | | | | | | **BTL -1** | Remembering | |
| 12.(b) | Find the complex form of the Fourier series of  ƒ(x) = e𝑎s 𝑖n − 𝜋 < x < 𝜋. | | | | | | | | | | | | | | | | | | | | | | | | | | **BTL -4** | Analyzing | |
| 13. | Calculate the first 3 harmonics of the Fourier of f(x) from the following data | | | | | | | | | | | | | | | | | | | | | | | | | | **BTL -6** | Creating | |
| x | 0 | | 30 | 60 | 90 | | 120 | 150 | | | 180 | 210 | | | 240 | 270 | | 300 | 330 | | 360 | | |  | |
| f(x) | 1.8 | | 1.1 | 0.3 | 0.16 | | 0.5 | 1.3 | | | 2.16 | 1.25 | | | 1.3 | 1.52 | | 1.76 | 2 | | 1.8 | | |
| 14.(a) | Find the complex form of the Fourier series of  ƒ(x) = e−𝑎s 𝑖n − l < x < l. | | | | | | | | | | | | | | | | | | | | | | | | | | **BTL -4** | Analyzing | |
| 14.(b) | Find the Fourier series as far as the second harmonic to represent the function ƒ(x) With period 6 | | | | | | | | | | | | | | | | | | | | | | | | | | **BTL -6** | Creating | |
|  | x | | | 0 | | 1 | | | 2 | | | | 3 | | | 4 | | | 5 | | | |  | | |
| y | | | 9 | | 18 | | | 24 | | | | 28 | | | 26 | | | 20 | | | |
| **UNIT III -APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS**  Classification of PDE – Method of separation of variables – Fourier Series solutions of one dimensional wave equation - One dimensional equation of heat conduction – Steady state solution of two dimensional  equation of heat conduction. | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| **PART -A** | | | | | | | | | | | | | | | | | | | | | | | | | | | | | | |
| **Q.No.** | **Question** | | | | | | | | | | | | | | | | | | | | | | | | | **Bloom’s Taxonomy Level** | | | **Domain** | |
| 1. | Classify the PDE (1 − x2)Zss − 2xyZsy + (1 − y2)Zyy + xZs +  3x2yz − 2Z = 0 | | | | | | | | | | | | | | | | | | | | | | | | | BTL-4 | | | Analyzing | |
| 2. | Classify the PDE uss + usy = ƒ(x, y) | | | | | | | | | | | | | | | | | | | | | | | | | BTL-4 | | | Analyzing | |

|  |  |  |  |
| --- | --- | --- | --- |
| 3. | 𝜕u 𝜕u  Solve = 2 + u where u(x, 0) = 6e−3s by method of  𝜕s 𝜕t  separation of variables | BTL-3 | Applying |
| 4. | What are the various solutions of one dimensional wave equation | BTL-1 | Remembering |
| 5. |  2 *y*  2 *y* 2  In the wave equation  *c*2 what does C stand for?  *t* 2 *x*2 | BTL-2 | Understanding |
| 6. | What is the basic difference between the solutions of one dimensional wave equation and one dimensional heat equation  with respect to the time? | BTL-3 | Applying |
| 7. | Write down the initial conditions when a taut string of length 2lis fastened on both ends. The midpoint of the string is taken to a  height b and released from the rest in that position | BTL-1 | Remembering |
| 8. | A slightly stretched string of length *l* has its ends fastened at x = 0 and x = *l* is initially in a position given by  y(x, 0) = y0s𝑖n3 𝜋s.If it is released from rest from this position,  𝑙  write the boundary conditions | BTL-2 | Understanding |
| 9. | A tightly stretched string with end points x = 0&x = lis initially at rest in equilibrium position. If it is set vibrating giving each  point velocity𝜆x(l − x). Write the initial and boundary conditions | BTL-2 | Understanding |
| 10. | If the ends of a string of length l are fixed at both sides. The midpoint of the string is displaced transversely through a height h and the string is released from rest, state the initial and boundary  conditions | BTL-2 | Understanding |
| 11. | State the assumptions in deriving the one dimensional heat  equation | BTL-1 | Remembering |
| 12. | Write down the various possible solutions of one dimensional heat  flow equation? | BTL-1 | Remembering |
| 13. | *u*  2 *u*  In the one dimensional heat equation  *C* 2  *t* *x* 2  what is C2 ? | BTL-2 | Understanding |
| 14. | The ends A and B of a rod of length 20 cm long have their  temperature kept 300 C and 800 C until steady state prevails. Find the steady state temperature on the rod | BTL-2 | Understanding |
| 15. | An insulated rod of length 60 cm has its ends at A and B maintained at 200C and 800 C respectively. Find the steady state  solution of the rod. | BTL-2 | Understanding |
| 16. | An insulated rod of length l cm has its ends at A and B maintained at 00C and 800 C respectively. Find the steady state solution of the rod. | BTL-2 | Understanding |
| 17. | Write down the three possible solutions of Laplace equation in two dimensions | BTL-1 | Remembering |
| 18. | Write down the governing equation of two dimensional steady  state heat equation. | BTL-1 | Remembering |
| 19. | A rectangular plate with insulated surface is 10cm wide. The two  long edges and one short edge are kept at00 C, while the | BTL-2 | Understanding |

|  |  |  |  |
| --- | --- | --- | --- |
|  | temperature at short edge x =0 is given by  u = {20y , 0 ≤ y ≤ 5 Find the steady state  20(10 − y), 5 ≤ y ≤ 10  temperature at any point in the plate. |  |  |
| 20. | A plate is bounded by the lines x=0, y=0, x=l and y=l. Its faces are insulated. The edge coinciding with x-axis is kept at 1000 C. The edge coinciding with y-axis at 500 C. The other 2 edges are kept at 00 C. write the boundary conditions that are needed for solving two dimensional heat flow equation. | BTL-2 | Understanding |
| **PART-B** | | | |
| 1. | A string is stretched and fastened to two points that are distinct string *l* apart. Motion is started by displacing the string into the formy = k(lx − x2)from which it is released at time t=0. Find the  displacement of any point on the string at a distance of x from one end at time t. | BTL-2 | Understanding |
| 2. | A tightly stretched string of length 2 *l* is fastened at both ends. The Midpoint of the string is displaced by a distance b transversely and the string is released from rest in this position. Find an expression  for the transverse displacement of the string at any time during the subsequent motion. | BTL-2 | Understanding |
| 3. | A slightly stretched string of length *l* has its ends fastened at  x = 0andx = *l* is initially in a position given by  y(x, 0) = y0s𝑖n3 𝜋s. If it is released from rest from this position,  𝑙  find the displacement y at any distance xfrom one end and at any  time. | BTL-2 | Understanding |
| 4. | A tightly stretched string with fixed end points x = 0 and x = *l* is initially at rest in its equilibrium position. If it is set vibrating string giving each point a velocity 𝜆x(l − x). Find the displacement of the string at any distance x from one end at any  time t. | BTL-3 | Applying |
| 5. | A tightly stretched string with fixed end points x=0 and x=*l* is initially at rest in its equilibrium position. If it is vibrating string by  2*cx if* 0  *x*  *l*  ****  giving to each of its points a velocity *v*   *l* 2. Find  2*c*(*l*  *x*) *if l*  *x*  *l*   *l* 2  the displacement of the string at any distance x from one end at any time t. | BTL-2 | Understanding |
| 6. | A tightly stretched string of length l is initially at rest in  this equilibrium position and each of its points is given the velocity  *v* sin 3 *x* . Find the displacement y(x, t).  0 *l* | BTL-2 | Understanding |
| 7. | *u*  2 *u*  Solve  *C* 2 subject to the conditions (i) u(0,t)=0  *t* *x* 2 | BTL-3 | Applying |

|  |  |  |  |
| --- | --- | --- | --- |
|  | for all t ≤ 0 (ii) u(l, t) = 0 ƒor all t ≤ 0  *x if* 0  *x*  *l*  ****  (iii) *u*(*x*,0)  2.  *l*  *x if l*  *x*  *l*   2 |  |  |
| 8. | A rod 30 cm long has its ends A and B kept at 200 and 800 respectively until steady state conditions prevail. The temperature at each end is then suddenly reduced to 00C and kept so. Find the resulting temperature function u(x, t) taking x = 0 at A. | BTL-2 | Understanding |
| 9. | A bar 10 cm long with insulated sides has its ends A and B maintained at temperature 500C and 1000C respectively. Until steady state conditions prevails.The temperature at A  is suddenly raised to 900C and at the same time lowered to 600C at B. Find the temperature distributed in the bar at time t. | BTL-2 | Understanding |
| 10. | A square plate is bounded by the lines x = 0, y = 0, x = 20  y = 20. Its faces are insulated. The temperature along the upper horizontal edge is given by u(x, 20) = x (20 − x) when  0 < x < 20 while the other three edges are kept at 00 C. Find the steady state temperature in the plate. | BTL-2 | Understanding |
| 11. | A square metal plate is bounded by the lines x=0, x=a, y=0, y=a  .The edges x=a, y=0, y=a are kept at zero degree temperature while the temperature at the edge x=0 is ky. Find the steady state temperature distribution at in the plate. | BTL-2 | Understanding |
| 12. | A rectangular plate with insulated surface is 10 cm wide and so long compared to its width that it may be considered infinite in length without introducing appreciable error. The temperature at short edge y=0 is given by u = {20x , 0 ≤ x ≤ 5 and all  20(10 − x), 5 ≤ x ≤ 10  the other three edges are kept at 00C. Find the steady state temperature at any point in the plate. | BTL-2 | Understanding |
| 13. | An infinitely long rectangular plate with insulated surface is 10cm wide. The two long edges and one short edge are kept at 00C, while the other short edge x=0 is kept at temperature  u = {20y , 0 ≤ y ≤ 5  20(10 − y), 5 ≤ y ≤ 10. Find the steady state temperature  distribution in the plate. | BTL-2 | Understanding |
| 14. | A long rectangular plate with insulated surface is lcm . If the temperature along one short edge y=0 is u(x,0)=K( *l* x -x2) degrees, for 0 < x < *l* , while the other 2 edges x=0 and x=*l* as well as the other short edge are kept at 00C, find the steady state temperature function u(x, y). | BTL-2 | Understanding |
| **UNIT –IV FOURIER TRANSFORM**  Statement of Fourier integral theorem – Fourier transform pair – Fourier sine and cosine transforms –  Properties – Transforms of simple functions – Convolution theorem – Parseval’s identity. | | | |

|  |  |  |  |
| --- | --- | --- | --- |
| **PART –A** | | | |
| **Q.No.** | **Question** | **Bloom’s Taxonomy Level** | **Domain** |
| 1. | State Fourier integral Theorem | BTL -1 | Remembering |
| 2. | Write Fourier Transform in Pairs | BTL -1 | Remembering |
| 3. | If *F* *s* denote the Fourier Transform of *f* *x*, Prove that  *F*  *f* *ax*  1 *F*  *s* , *a*  0.     *a*  *a*  | BTL -3 | Applying |
| 4. | If the Fourier Transform of *f* *x* is  *F* *s*  *F*  *f* *x*, *then showthat F*  *f* *x*  *a*  *eias F*(*s*). | BTL -3 | Applying |
| 5. | Find the Fourier Transform of *e**a x* . | BTL -2 | Understanding |
| 6. |    *ei k x* , *if a*  *x*  *b*  Find the Fourier Transform of *f x*   .   0 , *if x*  *a* & *x*  *b* | BTL -2 | Understanding |
| 7. | State and Prove Modulation theorem on Fourier Transforms | BTL -2 | Understanding |
| 8. | Find the Fourier sine Transform of 3e−2s. | BTL -2 | Understanding |
| 9. | Define self-reciprocal with respect to Fourier Transform | BTL -1 | Remembering |
| 10. | Find the infinite Fourier sine Transform of 1 .  *x* | BTL -2 | Understanding |
| 11. | Find the Fourier sine Transform of *f* (*x*)  *e* *x* 2 . | BTL -2 | Understanding |
| 12. | Give an example of a function which is self- reciprocal under  Fourier Sine & Cosine Transform | BTL -3 | Applying |
| 13. | Write down the Fourier cosine Transform pair of formulae | BTL -1 | Remembering |
| 14. | If F(s) is the Fourier Transform of *f* *x*. Show that the Fourier  Transform of *eia x f* (*x*)*is F*(*s*  *a*). | BTL -3 | Applying |
| 15. | Show that the Fourier Transform of the derivatives of a function   *d n*   *n*  *F*  *n f* (*x*)   *is F* (*s*).   *dx*  | BTL -3 | Applying |
| 16. | If *F* *s*  *F*  *f* *x*, *then find F* *xn f* *x*. | BTL -2 | Understanding |
| 17. | Find the Fourier cosine Transform of *e* 2 *x* . | BTL -2 | Understanding |
| 18. | Let *F* *s* be the Fourier cosine Transform of *f* *x*. Prove that  *c*  *F*  *f* (*x*) cos *ax*  1 *F* *s*  *a* *F* *s*  *a*.  *c* 2 *c c* | BTL -3 | Applying |
| 19. | State Convolution theorem in Fourier Transform | BTL -1 | Remembering |

|  |  |  |  |
| --- | --- | --- | --- |
| 20. | State Parseval’s Identity on Fourier Transform | BTL -1 | Remembering |
| PART-B | | | |
| 1.(a) | 1 , *x*  *a*  Find the Fourier Transform of *f* *x*   0 , *x*  *a*  0    sin *t*   and hence evaluate  *dt*. Also using Parseval’s  0  *t*     sin 2 *t*  **  Identity Prove that  *t* 2 *dt*  2  0   | BTL -2 | Understanding |
| 1. (b) | Find the Fourier Cosine Transform of the function  *a x* *b x*  *f* *x* *e*  *e* , *x*  0  *x* | BTL -2 | Understanding |
| 2. (a) | Find the Fourier Transform of the function  1  *x* , *if x*  1   sin *t* 2 **  f(x) =  Hence deduce that (i)   *dt*    0 , *if x*  1 0  *t*  2    sin *t* 4 **  (*ii*)  *t*  *dt*  3 .  0   | BTL -2 | Understanding |
| 2.(b) | * *x* 2   Show that the function *e* 2 is self-reciprocal under the Fourier  Transform by finding the Fourier Transform of *e* *a*2 *x*2 , *a*  0 | BTL -3 | Understanding |
| 3. | Show that the Fourier Transform of *f* *x*   *a*  *x* , *if x*  *a* is   0 , *if x*  *a*  0    2 1  cos *as*    sin *t* 2 **  **  *s* 2 . Hence deduce that (*i*)  *t*  *dt*  2 ,    0      sin *t* 4 **  (*ii*)  *t*  *dt*  3 .  0   | BTL -3 | Applying |
| 4. (a) | 1  *x* 2 , *if x*  1  Find the Fourier Transform of *f* (*x*)   Hence   0 , *if x* 1    sin *s*  *s* cos *s*  *s* 3**  Show that  *s*3 cos 2 *ds*  16  0   | BTL -1 | Remembering |
| 4.(b) |  *x* , 0  *x*  1  Find the Fourier Sine Transform of *f* *x*    *x* , 1  *x*  2  2  0 , *x*  2   | BTL -2 | Understanding |
| 5. |  *a* 2  *x* 2 , *x*  *a*  Show that the Fourier transform of *f* (*x*)   is   0 , *x*  *a*  0  2  sin *as*  *as* cos *as*   2 **  *s*3 .    | BTL -3 | Applying |

|  |  |  |  |
| --- | --- | --- | --- |
|  |   sin *t*  *t* cos *t*  **  Hence deduce that (*i*) *t* 3 *dt*  4 ,  0      sin *t*  *t* cos *t* 2 **  (*ii*) *t* 3  *dt*  15 .  0   |  |  |
| 6. (a) | Find the Fourier Transform of *f* *x* 1 .  *x* | BTL -2 | Understanding |
| 6.(b) | Using Parseval’s Identity evaluate the following integrals.    2  *i* *dx* , *ii*  *x dx where a*  0.  *a* 2  *x* 2  2   2  2 2  0 0 *a x* | BTL -5 | Evaluating |
| 7. (a) | Find the Fourier sine transform of *e**a x* (a>0). Hence find  *a x*   *F* *xe* *a x*  and *F*  *e*  hence deduce the value of sin *sx*  *s s*  *x*   *s dx*    0 | BTL -3 | Applying |
| 7. (b) |  *dx*  Evaluate  *x*2  *a* 2 *x*2  *b*2  , using Fourier Transform  0 | BTL -5 | Evaluating |
| 8. (a) | *e**a s*  Find the function whose Fourier Sine Transform is , *a*  0  *s* | BTL -1 | Remembering |
| 8.(b) | State and Prove Convolution Theorem on Fourier Transform | BTL -3 | Applying |
| 9. (a) | Find the Fourier Transform of *e* *x* and hence find the Fourier Transform of *f* *x* *e* *x* cos 2*x*. | BTL -2 | Understanding |
| 9.(b) | Find the Fourier Cosine transform of e– 4x. Deduce that   cos 2*x *   *x* sin 2*x *    *dx*  *e* 8 *and*  *dx*  *e* 8 .  *x* 2  16 8 *x* 2  16 2  0 0 | BTL -2 | Understanding |
| 10.(a) | Find the Fourier Transform of *e* *a x* and hence deduce that   cos *xt *     (*i*) *dt*  *e a x* (*ii*) *F xe**a x* = *i* 2 2*as* , hereF  *a* 2  *t* 2 2*a *  2  2 2  0 *a s*  stands for Fourier Transform. | BTL -4 | Analyzing |
| 10.( b) | Using Fourier Sine Transform prove that   *x*2 *dx *   (*x*2  *a* 2 )(*x*2  *b*2 )  2(*a*  *b*)  0 | BTL -3 | Applying |
| 11.(a) | Find the Fourier cosine & sine Transform of *e* *x* . Hence evaluate    2  *i* 1 *dx and* *ii*  *x dx*.  *x*2  12  *x*2  12  0 0 | BTL -2 | Understanding |
| 11.(b) | Find *F* *xe* *a x*  *and F* *xe* *a x* . *s c* | BTL -2 | Understanding |
| 12.(a) | Find the Fourier Sine Transform of the function  *f* *x* sin *x* , 0  *x*  *a*     0 , *x*  *a* | BTL -2 | Understanding |

|  |  |  |  |
| --- | --- | --- | --- |
| 12.(b) | Find the Fourier Cosine Transform of *f* *x* *e* *a* 2 *x*2 and hence find  2   * *x*   the Fourier Cosine Transform of *e* 2 and the Fourier Sine   * *x*2   Transform of *xe* 2 . | BTL -2 | Understanding |
| 13.(a) | s  Find the Fourier sine transform of s2+1 and Fourier  1  cosine transform ofs2+ .  1 | BTL -2 | Understanding |
| 13.(b) |  *dx*  Using Parseval’s Identity evaluate  *x*2  25*x*2  9.  0 | BTL -5 | Evaluating |
| 14.(a) | Verify the Convolution Theorem for Fourier Transform if  *f* *x*  *g*(*x*)  *e* *x*2 | BTL -5 | Evaluating |
| 14.(b) | Prove that  *F* *x f* (*x*)  *d* *F* *f* *x* *and F* *x f* (*x*)  *d* *F* *f* *x*  *c ds s s ds c* | BTL -3 | Applying |
| **UNIT -V Z - TRANSFORMS AND DIFFERENCE EQUATIONS**  Z- Transforms - Elementary properties – Inverse Z - transform (using partial fraction and residues) – Initial and Final value theorem – Convolution theorem - Formation of difference equations – Solution of difference equations using Z - transform. | | | |
| **PART –A** | | | |
| **Q.No.** | **Question** | **Bloom’s Taxonomy**  **Level** | **Domain** |
| 1. | Define Z – Transform of the sequence *f* *n*. | BTL -1 | Remembering |
| 2. | Find *Z* 3*n*  2 and *Z*  os 2 *n*   c 2     | BTL -2 | Understanding |
| 3. | Find  *an*   *Z*  *n*!     | BTL -2 | Understanding |
| 4. | Find *Z*  1       *n*! | BTL -2 | Understanding |
| 5. | Find *Z* 1    *n* *n*  1    | BTL -2 | Remembering |
| 6. | Define the unit step sequence .write its z-transform | BTL -1 | Remembering |
| 7. | State initial value theorem and final value theorem | BTL -1 | Remembering |
| 8. | Find *Z* *n*2 . | BTL -2 | Understanding |
| 9. | z  Find inverse Z transform of(z−1)(z−2) | BTL -2 | Understanding |
| 10. | 1   Find *Z* *n* 1    ! | BTL -2 | Understanding |
| 11. | Find *Z* *et* sin 2*t*. | BTL -2 | Understanding |

|  |  |  |  |
| --- | --- | --- | --- |
| 12. | Prove that *Z* *a n f* (*n*) *f* ( *z* )  *a* | BTL -5 | Evaluating |
| 13. | Prove that *Z* *a n*  *z*  *z*  *a* | BTL -5 | Evaluating |
| 14. | Findz−1[ z ]  (z−1)2 | BTL -2 | Analyzing |
| 15. | 1  Find Z transform of n | BTL -2 | Understanding |
| 16. | Findz−1[ z ]  (z+1)2 | BTL -2 | Understanding |
| 17. | Solve *yn*+1 + 2*yn* =0 given that *y*(0)=2 | BTL -3 | Applying |
| 18. | State Convolution theorem in Z – Transforms | BTL -1 | Remembering |
| 19. | Form the difference equation by eliminating arbitrary constants from yn = 𝐴2n+1 | BTL -6 | Creating |
| 20. | Prove that Z[ƒ(n + 1)] = z𝐹(z) − zƒ(0). | BTL -6 | Creating |
| **PART-B** | | | |
| 1.(a) | Find the z transform of *f* *n*   2*n*  3  *n* 1*n*  2 | BTL -2 | Understanding |
| 1. (b) | Find the *Z* 1 10*z*       *z* 2  3*z*  2  | BTL -2 | Understanding |
| 2. (a) | Find the z-transform of (n+1)2 and sin(3n+5) | BTL -2 | Understanding |
| 2.(b) | Find the inverse Z – Transform of *z* *z* 2  *z*  2  *z*  1*z*  12 . | BTL -2 | Understanding |
| 3. (a) | Find *i**Z* *r n* cos *n*  , *ii* *Z* *r n* sin *n*  *iii*)*Z*(*e**at* cos *bt*) | BTL-2 | Understanding |
| 3.(b) | Using convolution theorem find inverse Z transform of   *z* 2   *z*  *a**z*  *b*    | BTL -3 | Applying |
| 4. (a) | Find the *Z* 1 *z*   *z* 1*z*  2    | BTL -2 | Understanding |
| 4.(b) | Using convolution theorem find the inverse Z – Transform of  12*z*2  3*z* 14*z*  1 | BTL -3 | Analyzing |
| 5. (a) | 3  Find inverse Z -Transform of  *z*   by the method of  *z* 12 *z*  2  Partial fraction | BTL -2 | Understanding |
| 5.(b) | Using convolution theorem find inverse Z transform of  8z2  (2z−1)(4z+1). | BTL -3 | Applying |
| 6. (a) | −1 4z3  Using residue find Z [(2z−1)2(z−1 ].  ) | BTL -3 | Applying |
| 6.(b) | Using convolution theorem find   *z* 2   *Z* 1    *z*  4*z*  5 | BTL -3 | Applying |

|  |  |  |  |
| --- | --- | --- | --- |
|  |  |  |  |
| 7.(a) | Using Z transform Solve *yn*  2  3*y n* 1  2 *yn*  0  given that y(0)=0,y(1)=1 | BTL -3 | Analyzing |
| 7.(b) | ( ) 2z2+3z+12  If ƒ z = , Find the values of u2and u3 by residue  (z−1)4  method. | BTL -2 | Understanding |
| 8.(a) | Using Z transform solve yn+2 + 4yn+1 + 3yn = 3n  given that y(0)=0,y(1)=1 | BTL -3 | Applying |
| 8.(b) | Using the inversion method (Residue theorem )find the inverse Z   *z* 2   transform of U(z)= *z*  2*z*  4    | BTL -3 | Applying |
| 9.(a) | Using Residue method find *Z* 1  *z*       *z* 2  2*z*  2  | BTL -3 | Applying |
| 9.(b) |   *z* 2   1  Using convolution theorem evaluate *Z* *z*  1*z*  3    | BTL -3 | Applying |
| 10.(a) | Z−1 [ z2 ]  Using convolution theorem evaluate (z−1)(z−1)  2 4 | BTL -3 | Applying |
| 10.(b) | Using Z transform solve yn+2 − 3yn+1 − 10yn = 0 with y(0)=0,y(1)=1 | BTL -3 | Applying |
| 11.(a) | Find the Z transform {an} and {nan} | BTL -3 | Applying |
| 11.(b) | Using Z transform solve yn+2 − 7yn+1 + 12yn = 2n  given that y(0)=0,y(1)=0 | BTL -3 | Applying |
| 12.(a) | Form the difference equationy(k + 3) − 3y(k + 1) + 2y(k) = 0 w𝑖tℎ y(0) = 4, y(1) = 0and y(2) = 8 | BTL -6 | Creating |
| 12.(b) | State and prove final value theorem and their inverse transformation | BTL -3 | Applying |
| 13.(a) | Find the Z transform of {1} and{cosn 𝜋}  n 2 | BTL -2 | Understanding |
| 13.(b) | Solve the difference equation *yn*  3  3*y n* 1  2*yn*  0  given that y(0)=4 ,y(1)=0,y(2)=8 | BTL -3 | Applying |
| 14.(a) | Solve the equation using Z – Transform *yn*  2  5 *y n* 1  6 *yn*  36  given that y(0) = y(1) = 0 | BTL -3 | Applying |
| 14.(b) | Find *Z* 1 *z*  by convolution theorem.      *z* 2  7*z*  10  | BTL -2 | Understanding |