



SUBJECT:GRAPH THEORY AND APPLICATIONS
SUBCODE: CS6702
SEM/YEAR: VII/IV

UNIT I INTRODUCTION

PART – A

1. Define Graph.
2. Define Simple graph.
3. Write few problems solved by the applications of graph theory.
4. Define incidence, adjacent and degree.
5. What are finite and infinite graphs?
6. Define Isolated and pendent vertex.
7. Define null graph.
8. Define Multigraph
9. Define complete graph
10. Define Regular graph
11. Define Cycles
12. Define Isomorphism.
13. What is Subgraph?
14. Define Walk, Path and Circuit.
15. Define connected graph. What is Connectedness?
16. Define Euler graph.
17. Define Hamiltonian circuits and paths
18. Define Tree
19. List out few Properties of trees.
20. What is Distance in a tree?
21. Define eccentricity and center.
22. Define distance metric.
23. What are the Radius and Diameter in a tree.
24. Define Rooted tree
25. Define Rooted binary tree

PART – B

1. Explain various applications of graph.
 2. Define the following kn , cn , kn,n , dn , trail, walk, path, circuit with an example.
 3. Show that a connected graph G is an Euler graph iff all vertices are even degree.
 4. Prove that a simple graph with n vertices and k components can have at most $(n-k)(n+k-1)/2$ edges.
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5. Are they isomorphic?
 6. Prove that in a complete graph with n vertices there are $(n-1)/2$ edges-disjoint Hamiltonian circuits, if n is odd number ≥ 3 .
 7. Prove that, there is one and only one path between every pair of vertices in a tree T .
 8. Prove the given statement, “A tree with n vertices has $n-1$ edges”.
 9. Prove that, any connected graph with n vertices has $n-1$ edges is a tree.
 10. Show that a graph is a tree if and only if it is minimally connected.
 11. Prove that, a graph G with n vertices has $n-1$ edges and no circuits are connected.

UNIT II TREES, CONNECTIVITY & PLANARITY

PART – A

1. Define Spanning trees.
2. Define Branch and chord.
3. Define complement of tree.
4. Define Rank and Nullity.
5. How Fundamental circuits created?
6. Define Spanning trees in a weighted graph.
7. Define degree-constrained shortest spanning tree.
8. Define cut sets and give example.
9. Write the Properties of cut set
10. Define Fundamental circuits
11. Define Fundamental cut sets
12. Define edge Connectivity.
13. Define vertex Connectivity.
14. Define separable and non-separable graph.
15. Define articulation point.
16. What is Network flows.
17. Define max-flow and min-cut theorem (equation).
18. Define component (or block) of graph.
19. Define 1-Isomorphism.
20. Define 2-Isomorphism.
21. Briefly explain Combinational and geometric graphs.
22. Distinguish between Planar and non-planar graphs.
23. Define embedding graph.
24. Define region in graph.
25. Why the graph is embedding on sphere.

PART – B

1. Find the shortest spanning tree for the following graph.

	v_1	v_2	v_3	v_4	v_5	v_6
v_1	–	10	16	11	10	17
v_2	10	–	9.5	∞	∞	19.5
v_3	16	9.5	–	7	∞	12
v_4	11	∞	7	–	8	7
v_5	10	∞	∞	8	–	9
v_6	17	19.5	12	7	9	–

2. Explain 1 - isomorphism and 2 - isomorphism of graphs with your own example.
3. Prove that a connected graph G with n vertices and e edges has $e-n+2$ regions.
4. Write all possible spanning tree for K_5 .
5. Prove that every cut-set in a connected graph G must contain at least one branch of every spanning tree of G .
6. Prove that the every circuit which has even number of edges in common with any cut-set.
7. Show that the ring sum of any two cut-sets in a graph is either a third cut set or an edge disjoint union of cut sets.
8. Explain network flow problem in detail.
9. If G_1 and G_2 are two 1-isomorphic graphs, the rank of G_1 equals the rank of G_2 and the nullity of G_1 equals the nullity of G_2 , prove this.
10. Prove that any two graphs are 2-isomorphic if and only if they have circuit correspondence.

UNIT III MATRICES, COLOURING AND DIRECTED GRAPH

PART – A

1. What is proper coloring?
2. Define Chromatic number
3. Write the properties of chromatic numbers (observations).
4. Define Chromatic partitioning
5. Define independent set and maximal independent set.
6. Define uniquely colorable graph.
7. Define dominating set.
8. Define Chromatic polynomial.
9. Define Matching (Assignment).
10. What is Covering?
11. Define minimal cover.
12. What is dimer covering?
13. Define four color problem / conjecture.
14. State five color theorem
15. Write about vertex coloring and region coloring.
16. What is meant by regularization of a planar graph?
17. Define Directed graphs .
18. Define isomorphic digraph.
19. List out some types of directed graphs.
20. Define Simple Digraphs.
21. Define Asymmetric Digraphs (Anti-symmetric).
22. What is meant by Symmetric Digraphs?
23. Define Simple Symmetric Digraphs.
24. Define Simple Asymmetric Digraphs.
25. Give example for Complete Digraphs.
26. Define Complete Symmetric Digraphs.
27. Define Complete Asymmetric Digraphs (tournament).
28. Define Balance digraph (a pseudo symmetric digraph or an isograph).
29. Define binary relations.
30. What is Directed path?
31. Write the types of connected digraphs.
32. Define Euler graphs.

PART – B

1. Prove that any simple planar graph can be embedded in a plane such that every edge is drawn as a straight line.
2. Show that a connected planar graph with n vertices and e edges has $e-n+2$ regions.
3. Define chromatic polynomial. Find the chromatic polynomial for the following graph.



4. Explain matching and bipartite graph in detail.
5. Write the observations of minimal covering of a graph.
6. Prove that the vertices of every planar graph can be properly colored with five colors.
7. Explain matching in detail.
8. Prove that a covering g of graph G is minimal iff g contains no path of length three or more.
9. Illustrate four-color problem.
10. Explain Euler digraphs in detail.

UNIT IV PERMUTATIONS & COMBINATIONS

PART – A

1. Define Fundamental principles of counting
2. Define rule of sum.
3. Define rule of Product
4. Define Permutations
5. Define combinations
6. State Binomial theorem
7. Define combinations with repetition
8. Define Catalan numbers
9. Write the Principle of inclusion and exclusion formula.
10. Define Derangements
11. What is meant by Arrangements with forbidden (banned) positions.

PART – B

1. Explain the Fundamental principles of counting.
2. Find the number of ways of ways of arranging the word APPASAMIAP and out of it how many arrangements have all A's together.
3. Discuss the rules of sum and product with example.
4. Determine the number of (staircase) paths in the xy -plane from $(2, 1)$ to $(7, 4)$, where each path is made up of individual steps going 1 unit to the right (R) or one unit upward (U). iv.
Find the coefficient of a^5b^2 in the expansion of $(2a - 3b)^7$.
5. State and prove binomial theorem.
6. How many times the print statement executed in this program segment?

```
for i := 1 to 20 do
  for j := 1 to i do
    for k := 1 to j do
      print (i * j + k)
```
7. Discuss the Principle of inclusion and exclusion.
8. How many integers between 1 and 300 (inc.) are not divisible by at least one of 5, 6, 8?
9. How 32 bit processors address the content? How many address are possible?
10. Explain the Arrangements with forbidden positions.

UNIT V GENERATING FUNCTIONS

PART – A

1. Define Generating function.
2. What is Partitions of integer?
3. Define Exponential generating function
4. Define Maclaurin series expansion of e^x and e^{-x} .
5. Define Summation operator
6. What is Recurrence relation?
7. Write Fibonacci numbers and relation
8. Define First order linear recurrence relation
9. Define Second order recurrence relation
10. Briefly explain Non-homogeneous recurrence relation.

PART – B

1. Explain Generating functions
2. Find the convolution of the sequences $1, 1, 1, 1, \dots$ and $1, -1, 1, -1, 1, -1$.
3. Find the number of non negative & positive integer solutions of for $x_1+x_2+x_3+x_4=25$.
4. Find the coefficient of x^5 in $(1-2x)^7$.
5. The number of virus affected files in a system is 1000 and increases 250% every 2 hours.
6. Explain Partitions of integers
7. Use a recurrence relation to find the number of viruses after one day.
8. Explain First order homogeneous recurrence relations.
9. Solve the recurrence relation $a_{n+2}-4a_{n+1}+3a_n=-200$ with $a_0=3000$ and $a_1=3300$.
10. Solve the Fibonacci relation $F_n = F_{n-1}+F_{n-2}$.
11. Find the recurrence relation from the sequence $0, 2, 6, 12, 20, 30, 42, \dots$.
12. Determine $(1+\sqrt{3}i)^{10}$.
13. Discuss Method of generating functions.

