DHANALAKSHMI SRINIVASAN COLLEGE OF ENGINEERING AND TECHNOLOGY
ECR, MAMALLAPURAM, CHENNAI-603 104

QUESTION BANK(REG 2017)

SUBJECT:TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS
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UNIT 1

PARTIAL DIFFERENTIAL EQUATIONS

Part A

(1)Form a partial differential equation by eliminating arbitrary constants a and b from $z=(x+a)^2+(y+b)^2$

(2) Solve: $(D^2-2DD'+D^2)z=0$

(3)Form a partial differential equation by eliminating the arbitrary constants a and b from the equation $(x-a)^2+(y-b)^2=z^2\cot^2\alpha$.

(4)Find the complete solution of the partial differential equation $p^2+q^2-4pq=0$.

(5)Find the PDE of all planes having equal intercepts on the x and y axis.

(6)Find the solution of $px^2+qy^2=z^2$.

(7)Find the singular integral of the partial differential equation $z=px+qy+p^2-q^2$.

(8)Solve: $p^2+q^2=m^2$.

(9)Form a partial differential equation by eliminating the arbitrary constants a and b from $z=ax^n+by^n$.

(10)Solve:
$$(D^3+D^2D'+DD'^2+D'^3)z=0.$$

(11)Form a partial differential equation by eliminating the arbitrary constants a and b from $z=(x^2+a^2)(y^2+b^2)$.

(12)Solve:
$$(D^2-DD'+D'-1)z=0$$

Part B

(1)(i) Form a partial differential equation by eliminating arbitrary functions from $z=xf(2x+y)+g(2x+y)$

(ii) Solve: $p^2y(1+x^2)=qx^2$

(2)(i) Solve: $x(z^2-y^2)p+y(x^2-z^2)q=z(y^2-x^2)$

(ii) Solve: $\frac{\partial^3 z}{\partial x^3}-2\frac{\partial^3 z}{\partial x^2\partial y}=e^{x+2y}+4\sin(x+y)$

(3)(i) Solve: $\left(x^2-y^2-z^2\right)p+2xyq-2xz=0$
(ii) Solve: $\left(D^2-DD'-2D'^2\right)z=2x+3y+e^{3x+4y}$

(4)(i) Solve: $z^2\left(p^2+q^2\right)=x^2+y^2$
(ii) Solve: $\left(D^2+3DD'-4D'^2\right)z=\sin y$

(5)(i) Solve: $\left(x^2+y^2+yz\right)p+\left(x^2+y^2-xz\right)q=z(x+y)$
(ii) Solve: $\left(D^2-DD'-20D'^2\right)z=e^{5x+y}+\sin(4x-y)$

(6)(i) Solve: $z=p^2+q^2$
(ii) Solve: $\left(D^2+DD'-6D'^2\right)z=x^2y+e^{3x+y}$

(7)(i) Solve: $\left(y^2+z^2\right)p-xyq+xz=0$
(ii) Solve: $\left(D^2-6DD'+5D'^2\right)z=xy+e^x\sinh y$

(8)(i) Solve: $p\left(1-q^2\right)=q(1-z)$
(ii) Solve: $\left(D^2-4DD'+4D'^2\right)z=xy+e^{2x+y}$

(9)(i) Solve: $\left(x^2-yz\right)p+\left(y^2-zx\right)q=z^2-xy$
(ii) Solve: $x(y-z)p+y(z-x)q=z(x-y)$

(10)(i) Solve: $p^2+x^2y^2q^2=x^2z^2$
(ii)Solve: $\left(D^2-D'^2-3D+3D'\right)z=xy$

(11)(i) Form the partial differential equation by eliminating $f\ and\ \phi$ from
$z=f(y)+\phi(x+y+z)$
(ii) Solve: $\left(D'^2-2DD'\right)z=x^3y+e^{2x}$

(12)(i) Find the complete integral of $p+q=x+y$
(ii) Solve: $y^2p-xyq=x(z-2y)$

(13)(i) Solve: $(3z-4y)p+(4x-2z)q=2y-3x$
(ii) Solve: $\left(D^2-2DD'-D'^2+3D+3D'+2\right)z=\left(e^{3x}+2e^{-2y}\right)^2$
(iii) Solve: $\left(D^2+4DD'-5D'^2\right)z=\sin(x-2y)+e^{2x-y}$

(14)(i) Solve $z^2=1+p^2+q^2$
(ii) Solve: $(y-z)p-(2x+y)q=2x+z$

(15)(i) Form a partial differential equation by eliminating arbitrary functions f and g in $z = x^2 f(y) + y^2 g(x)$

(ii) Solve: $(D^2 - DD' - 20D'^2)z = \sin(4x - y) + e^{5x+y}$

(16)(i) Form a partial differential equation by eliminating arbitrary functions f and g in $z = f(x^3 + 2y) + g(x^3 - 2y)$

(ii) Solve: $(y - xy)p + (yz - x)q = (x + y)(x - y)$

(17)(i) Find the singular solution of $z = px + qy + \sqrt{p^2 + q^2 + 1}$

(ii) Solve: $(D^2 - DD' - 30D'^2)z = xy + e^{6x+y}$

(18) (i) Solve: $(D^3 - 7DD'^2 - 6D'^3)z = \sin(x + 2y) + e^{2x+y}$

(ii) Find the singular integral of a partial differential equation $z = px + qy + p^2 - q^2$

(19)(i) Solve: $(4D^2 - 4DD' + D'^2)z = 16\log(x + 2y)$

(ii) Form a partial differential equation by eliminating arbitrary functions 'f' from $f(z - xy, x^2 + y^2) = 0$

(20)(i) Solve: $(D^2 + D'^2)z = \sin 2x \sin 3y + 2\sin^2(x + y)$

(ii) Solve: $p^2 + x^2y^2q^2 = x^2z^2$

UNIT 2

FOURIER SERIES

PART – A

1. Determine the value of a_n in the Fourier series expansion of $f(x) = x^3 \ in - \pi < x < \pi.$
2. Find the root mean square value of $f(x) = x^2$ in the interval $(0, \pi)$.
3. 3. Find the coefficient b_5 of $\cos 5x$ in the Fourier cosine series of the function

   $f(x) = \sin 5x$ in the interval $(0, 2\pi)$
4. 4. If $f(x) = \begin{cases} \cos x, & if \ 0 < x < \pi \\ 50, & if \ \pi < x \le 2\pi \end{cases}$ and $f(x) = f(x + 2\pi)$ for all x, find the sum of the

   Fourier series of $f(x)$ at $x = \pi$.
5. 5. Find b_n in the expansion of x^2 as a Fourier series in $(-\pi, \pi)$.
6. 6. If $f(x)$ is an odd function defined in (-*l* , *l*) what are the values of a_0
7. 7. Find the Fourier constants b_n for $x \sin x$ in $(-\pi, \pi)$.

8. 8. State Parseval's identity for the half-range cosine expansion of $f(x)$ in (0 , 1).

9. 9. Find the constant term in the Fourier series expansion of $f(x) = x$ in $(-\pi, \pi)$.

10. 10. State Dirichlet's conditions for Fourier series.

PART B

1. (i) Express $f(x) = x\sin x$ as a Fourier series in $0 \le x \le 2\pi$.

(ii) Show that for 0 < x <*l*, $x = \frac{2l}{p}\left(\sin\frac{\pi x}{l} - \frac{1}{2}\sin\frac{2\pi x}{l} + \frac{1}{3}\sin\frac{3\pi x}{l}\cdots\right)$. Using root mean square value of x, deduce the value of $\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots$

2. (i) Find the Fourier series of periodicity 3 for $f(x) = 2x - x^2$ in 0 < x < 3.

(ii) Find the Fourier series expansion of period 2π for the function $y = f(x)$ which is defined in $(0, 2\pi)$ by means of the table of values given below. Find the series upto the third harmonic.

x	0	$\frac{\pi}{3}$	$\frac{2\pi}{3}$	π	$\frac{4\pi}{3}$	$\frac{5\pi}{3}$	2π
$f(x)$	1.0	1.4	1.9	1.7	1.5	1.2	1.0

3.(i) Find the Fourier series of periodicity 2π for $f(x) = x^2$ for 0 < x < 2π.

(ii) Show that for 0 < x <*l*, $x = \frac{l}{2} - \frac{4l}{\pi^2}\left(\cos\frac{\pi x}{l} + \frac{1}{3^2}\cos\frac{3\pi x}{l} + \cdots\right)$. Deduce that

$\frac{1}{1^4} + \frac{1}{3^4} + \frac{1}{5^4} + \ldots = \frac{\pi^4}{96}$.

4. (i) Find the Fourier series for $f(x) = \begin{cases} l - x, & 0 < x \le l \\ 0, & l \le x \le 2l \end{cases}$. Hence deduce the sum to infinity of the series $\sum_{n=0}^{\infty} \frac{1}{(2n+1)^2}$.

(ii) Find the complex form of Fourier series of $f(x) = e^{ax} (-\pi < x < \pi)$ in the form

$$e^{ax} = \frac{\sinh a\pi}{\pi}\sum_{-\infty}^{\infty}(-1)^n \frac{a+in}{a^2+n^2} e^{inx}$$ and hence prove that $$\frac{\pi}{a\sinh a\pi} = \sum_{-\infty}^{\infty}\frac{(-1)^n}{n^2+a^2}.$$

5. Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$.

6. Find the Fourier series for $f(x) = |\cos x|$ in the interval $(-\pi, \pi)$.

7. (i) Expanding $x(\pi - x)$ as a sine series in $(0, \pi)$ show that $1 - \frac{1}{3^3} + \frac{1}{5^3} + \ldots = \frac{\pi^3}{32}$.

(ii) Find the Fourier series as far as the second harmonic to represent the function given in the following data.

x	0	1	2	3	4	5
$f(x)$	9	18	24	28	26	20

8. Obtain the Fourier series for $f(x)$ of period $2l$ and defined as follows

$$f(x) = \begin{cases} L + x & in\ (-L, 0) \\ L - x & in\ (0, L) \end{cases}$$

Hence deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \ldots = \frac{\pi^2}{8}$.

9. Obtain the half range cosine series for $f(x) = x$ in $(0, \pi)$.

10. (i) Find the Fourier series of $f(x) = \begin{cases} 1 & in\ (0, \pi) \\ 2 & in\ (\pi, 2\pi) \end{cases}$

(ii) Obtain the sine series for the function

$$f(x) = \begin{cases} x & in\ 0 \le x \le \frac{l}{2} \\ l - x & in\ \frac{l}{2} \le x \le l \end{cases}$$

11. (i) Find the Fourier series for the function

$$f(x) = \begin{cases} 0 & in\ (-1, 0) \\ 1 & in\ (0, 1) \end{cases}$$ and $f(x+2) = f(x)$ for all x.

(ii) Determine the Fourier series for the function

$$f(x) = \begin{cases} \pi x, & 0 \le x \le 1 \\ \pi(2 - x), & 1 \le x \le 2 \end{cases}$$

12. Obtain the Fourier series for $f(x) = 1 + x + x^2$ in $(-\pi, \pi)$. Deduce that

$$\frac{1}{1^2} + \frac{1}{2^2} + \frac{1}{3^2} + \ldots = \frac{\pi^2}{6}.$$

13. Obtain the constant term and the first harmonic in the Fourier series expansion for $f(x)$ where $f(x)$ is given in the following table.

x	0	1	2	3	4	5	6	7	8	9	10	11
$f(x)$	18.0	18.7	17.6	15.0	11.6	8.3	6.0	5.3	6.4	9.0	12.4	15.7

14. (i) Express $f(x) = x \sin x$ as a Fourier series in $(-\pi, \pi)$.

(ii) Obtain the half range cosine series for $f(x) = (x-2)^2$ in the interval $0 < x < 2$.

15. Find the half range sine series of $f(x) = x \cos x$ in $(0, \pi)$.

16. (i) Find the Fourier series expansion of $f(x) = e^{-x}$ *in* $(-\pi, \pi)$

(ii) Find the half-range sine series of $f(x)$ = sin ax in (0 , *l*).

17. Expand $f(x) = x - x^2$ as a Fourier series in -1 < x < 1 and using this series find the r.m.s. value of $f(x)$ in the interval.

18. The following table gives the variations of a periodic function over a period T.

x	0	$\frac{T}{6}$	$\frac{T}{3}$	$\frac{T}{2}$	$\frac{2T}{3}$	$\frac{5T}{6}$	T
$f(x)$	1.98	1.3	1.05	1.3	-0.88	-0.25	1.98

Show that $f(x) = 0.75 + 0.37 \cos\theta + 1.004 \sin\theta$, where $\theta = \frac{2\pi x}{T}$

19. Find the Fourier series upto the third harmonic for the function y = $f(x)$ defined in $(0, \pi)$ from the table

x	0	$\frac{\pi}{6}$	$\frac{2\pi}{6}$	$\frac{3\pi}{6}$	$\frac{4\pi}{6}$	$\frac{5\pi}{6}$	π
$f(x)$	2.34	2.2	1.6	0.83	0.51	0.88	1.19

20. (i) Find the half-range (i) cosine series and (ii) sine series for $f(x) = x^2$ in $(0, \pi)$

(ii) Find the complex form of the Fourier series of $f(x)$ = cos ax in $(-\pi, \pi)$.

UNIT 3
APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS
PART A

1. Classify the partial differential equation $3u_{xx} + 4u_{xy} + 3u_y - 2u_x = 0.$
2. The ends A and B of a rod of length 10 cm long have their temperature kept at 20^0 C and 70^0 C. Find the steady state temperature distribution on the rod.
3. Solve the equation $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0,$ given that $u(x,0) = 4e^{-x}$ by the method of separation of variables.
4. Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is $f(x)$ and the initial velocity imparted at each point x is $g(x)$.
5. What is the basic difference between the solution of one dimensional wave equation and one dimensional heat equation.
6. In steady state conditions derive the solution of one dimensional heat flow equation.
7. What are the possible solutions of one dimensional wave equation.
8. In the wave equation $\frac{\partial^2 y}{\partial t^2} = \alpha^2 \frac{\partial^2 y}{\partial x^2}$ what does c^2 stand for?
9. State Fourier law of conduction.
10. What is the constant a^2 in the wave equation $u_{tt} = a^2 u_{xx}$.

PART B

(1) A tightly stretched string with fixed end points x=0 and $x = l$ is initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity $\lambda x(l - x)$., then show that

$$y(x,t) = \frac{8\lambda l^3}{\pi^4 a} \sum_{n=1,3,5}^{\infty} \frac{1}{n^4} \sin\frac{n\pi x}{l} . \sin\frac{n\pi at}{l}$$

(2) A rectangular plate is bounded by lines x=0, y=0, x=a, y=b. It's surface are insulated. The temperature along x=0 and y=o are kept at 0^0 C and others at 100^0 C. Find the steady state temperature at any point of the plate.

(3)A metal bar 10 cm. long, with insulated sides has its ends A and B kept at 20^0 C and 40^0 C respectively until steady state conditions prevail. The temperature at A is then suddenly raised to 50^0 C and at the same instant that at B is lowered to 10^0 C. Find the subsequent temperature at any point of the bar at any time.

(4) A tightly stretched string of length l has its ends fastened at x=0, $x = l$. The mid-point of the string is mean taken to height 'b' and then released from rest in that position.
Find the lateral displacement of a point of the string at time't' from the instant of release.

(5)If a square plate is bounded by the lines $x = \pm a$ *and* $y = \pm a$ and three of its edges are kept at temperature 0^0 C, while the temperature along the edge y=a is kept at $u = x + a, -a \le x \le a,$ Find the steady state temperature in the plate.

(6)A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve

(i) $y = k\sin^3\left(\frac{\pi x}{l}\right)$ and

(ii) $y = kx(l - x)$

and then releasing it from this position at time t=0. Find the displacement of the point of the string at a distance x from one end at time t.

(7) A long rectangular plate with insulated surface is 1 cm wide. If the temperature along one short edge(y=0) is $u(x,0) = k(2lx - x^2)$ degrees, for $0 < x < l$, while the two long edges x=0 and x=l as well as the other short edge are kept at temperature 0^0 C, Find the steady state temperature function $u(x, y)$.

(8)Find the temperature distribution in a homogeneous bar of length π which is insulated laterally, if the ends are kept at zero temperature and if, initially, the temperature is k at the centre of the bar and falls uniformly to zero at its ends.

(9) solve the one dimensional wave equation $\frac{\partial^2 y}{\partial t^2} = a^2 \frac{\partial^2 y}{\partial x^2} \quad in -l \le x \le l, t \ge 0,$

given that $y(-l,t) = 0, y(l,t) = 0, \frac{\partial y}{\partial t}(x,0) = 0 \ and \ y(x,0) = \frac{b}{l}(l - |x|).$

(10) A rectangular plate with insulated surface is 20 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge x=0 is given by

$$u = \begin{cases} 10y, & for \ \ 0 \le y \le 10 \\ 10(20 - y), & for \ 10 \le y \le 20 \end{cases}$$

and the two long edges as well as the other short edge are kept at 0^0 C. Find the steady state temperature distribution in the plate.

(11) Solve the one dimensional heat flow equation

$$\frac{\partial u}{\partial t} = \alpha^2 \frac{\partial^2 u}{\partial x^2}$$

Satisfying the following boundary conditions.

(i) $\frac{\partial u}{\partial x}(0,t) = 0, \quad for\ all\ t \ge 0$

(ii) $\frac{\partial u}{\partial x}(\pi,t) = 0, \quad for\ all\ t \ge 0$

(iii) $u(x,0) = \cos^2 x, \quad for \quad 0 < x < \pi$

(12)A rectangular plane with insulated surface is a cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the two long edges x=0 and x=a and the short edge at infinity are kept at temperature 0^0 C, while the other short edge y=0 is kept at temperature

(i) $u_0 \sin^3 \frac{\pi x}{a}$ *and* (ii) T (constant). Find the steady state temperature at any point (x, y) of the plate.

(13) A tightly stretched strings with fixed end points x=0 and x=50 is initially at rest in its equilibrium position. If it is said to vibrate by giving each point a velocity

(i) $v = v_0 \sin^3 \frac{\pi x}{50}$ *and*

(ii) $v = v_0 \sin\frac{\pi x}{50}\cos\frac{2\pi x}{50}$,

Find the displacement of any point of the string at any subsequent time.

(14) An infinitely long metal plate in the form of an area is enclosed between the lines $y = 0$ *and* $y = \pi$ for positive values of x. The temperature is zero along the edges $y = 0$, $y = \pi$ and the edge at infinity. If the edge x=0 is kept at temperature ky, Find the steady state temperature distribution in the plate.

(15) A taut string of length $2l$, fastened at both ends, is disturbed from its position of equilibrium by imparting to each of its points an initial velocity of magnitude $k(2lx - x^2)$. Find the displacement function $y(x,t)$

UNIT-4

FOURIER TRANSFORMS

PART A

1. State the Fourier integral theorem.
2. State the convolution theorem of the Fourier transform.
3. Write the Fourier transform pair.
4. Find the Fourier sine transform of $f(x) = e^{-ax}$ (a > 0).
5. If the Fourier transform of $f(x)$ is F(s) then prove that . $F[f(x-a)] = e^{isa}F(s)$
6. State the Fourier transforms of the derivatives of a function.
7. Find the Fourier sine transform of $f(x) = e^{-x}$.
8. Prove that $F_C[f(ax)] = \frac{1}{a} F_C\left(\frac{s}{a}\right)$, $a > 0$
9. If F(s) is the Fourier transform of $f(x)$ then prove that $F[x.f(x)] = (-i)\frac{dF(s)}{ds}$
10. Find the Fourier sine transform of $f(x) = e^{ax}$

PART B

1. Find the Fourier Transform of

$$f(x)=\begin{cases}1-x^2 & in\ |x|\le 1\\ 0 & in\ |x|>1\end{cases}$$

Hence prove that $\int_0^\infty \frac{\sin s - s\cos s}{s^3}\cos\frac{s}{2}\,ds=\frac{3\pi}{16}$.

2. Find the Fourier cosine transform of $\frac{e^{-ax}}{x}$.

3. Find the Fourier Transform of $f(x)$ if

$$f(x)=\begin{cases}1-|x|, & |x|<1\\ 0, & |x|>1\end{cases}$$

Hence deduce that $\int_0^\infty \left(\frac{\sin t}{t}\right)^4 dt=\frac{\pi}{3}$

4. Evaluate $\int_0^\infty \frac{dx}{(x^2+a^2)(x^2+b^2)}$ using transforms

5. Find the Fourier transform of $e^{-a|x|}$ and hence deduce that

(i) $\int_0^\infty \frac{\cos xt}{a^2+t^2}dt=\frac{\pi}{2a}e^{-a|x|}$

(ii) $F\left[xe^{-a|x|}\right]=i\sqrt{\frac{2}{\pi}}\frac{2as}{(s^2+a^2)^2}$

6. Show that the Fourier transform of $f(x)=\begin{cases}a^2-x^2 & |x|<a\\ 0 & |x|>a>0\end{cases}$ is

$2\sqrt{\frac{2}{\pi}}\left(\frac{\sin as - as\cos as}{s^3}\right)$. Hence deduce that $\int_0^\infty \frac{\sin t - t\cos t}{t^3}dt=\frac{\pi}{4}$. Using Parseval's identity

show that $\int_0^\infty \left(\frac{\sin t - t\cos t}{t^3}\right)^2 dt=\frac{\pi}{15}$.

7. . Find the Fourier transform of $f(x)$ if

$$f(x)=\begin{cases}1, & |x|<1\\ 0, & otherwise\end{cases}$$

Hence deduce that $\int_0^\infty \frac{\sin x}{x} dx = \int_0^\infty \frac{\sin^2 x}{x^2} dx = \frac{\pi}{2}$

8. Derive the parseval's identity for Fourier transforms.

9. Find the Fourier sine transform of

$$f(x) = \begin{cases} \sin x, & 0 < x \le a \\ 0, & a \le x < \infty \end{cases}$$

10. Find the Fourier transform of e^{-4x} and hence deduce that

(i) $\int_0^\infty \frac{\cos 2x}{x^2+16} dx = \frac{\pi}{8} e^{-8}$

(ii) $\int_0^\infty \frac{x \sin 2x}{x^2+16} dx = \frac{\pi}{2} e^{-8}$

11. State and prove convolution theorem for Fourier transforms.

12. Using Parseval's identity calculate

(i) $\int_0^\infty \frac{dx}{(a^2+x^2)^2}$, (ii) $\int_0^\infty \frac{x^2}{(a^2+x^2)^2} dx$ if a > 0.

13. Find the Fourier cosine transform of $e^{-a^2x^2}$

14. (i) Find the Fourier cosine transform of e^{-x^2}
(ii) Find the Fourier sine transform of

$$f(x) = \begin{cases} x, & 0 < x < 1 \\ 2 - x, & 1 < x < 2 \\ 0 & x > 2 \end{cases}$$

15. Find Fourier sine and cosine transform of e^{-x} and hence find the Fourier sine transform of $\frac{x}{x^2+1}$ and Fourier cosine transform of $\frac{1}{x^2+1}$.

UNIT 5
PART A
Z-Transforms

(1)Form the difference equation from $y_n = a + b3^n$
(2)Express $Z\{f(n+1)\}$ in terms of $\bar{f}(z)$
(3)Find the value of $Z\{f(n)\}$ when $f(n) = na^n$

(4)Define bilateral Z-transform.
(5)Find the z-transform of $(n+1)(n+2)$
(6)Find $Z\{e^{-iat}\}$ using z-transform.
(7)Define unilateral Z-transform
(8)Find $Z\left\{\frac{a^n}{n!}\right\}$ using z-transform.
(9) State and prove initial value theorem in z-transform.
(10)Find the z-transform of n.

Part B

(1) (a)Prove that $Z\left(\frac{1}{n}\right) = \log\left(\frac{z}{z-1}\right), \quad n \neq 0$.

Find $Z\left(\sin\frac{n\pi}{2}\right) \quad and \quad Z\left(\cos\frac{n\pi}{2}\right)$

(b)Find $Z^{-1}\left(\frac{8z^2}{(2z-1)(4z+1)}\right)$ using Convolution theorem.

(2) (a) Prove that $Z(a^n) = \left(\frac{z}{z-a}\right)$

(b) Solve: $y_{n+2} + 6y_{n+1} + 9y_n = 2^n \quad given\, y_0 = y_1 = 0,$ using z- transform.

(3)(a) Find the z-transform of $f(n) = \left(\frac{2n+3}{(n+1)(n+2)}\right)$

(b)Using Convolution theorem find $Z^{-1}\left(\frac{z^2}{(z+2)^2}\right)$

(4)(a) Solve the difference equation

$$y(n+3) - 3y(n+1) + 2y(n) = 0 \quad given\, y(0) = 4, y(1) = 0 \; and \; y(2) = 8.$$

(b) Find $Z^{-1}\left\{\frac{z^2}{(z+2)(z^2+4)}\right\}$, by the method of partial fractions.

(5)(a) Find $Z^{-1}\left\{\frac{z^3}{(z-1)^2(z-2)}\right\}$, by the method of partial fractions.

(b) Solve the difference equation

$$y(k+2) - 4y(k+1) + 4y(k) = 0 \quad given\, y(0) = 1, y(1) = 0$$

(6)(a) Prove that $Z\left(\frac{1}{n+1}\right) = z\log\left(\frac{z}{z-1}\right)$,

(b) State and prove the second shifting theorem in z-transform.

(7)(a) Using Convolution theorems evaluate inverse z-transform of $\left(\frac{z^2}{(z-1)(z-3)}\right)$.

(b) Solve the difference equation

$$y(n)+3y(n-1)-4y(n-2)=0,\ n\geq 2,\quad given\ y(0)=3,\ y(1)=-2$$

(8)(a) Find the inverse Z-transform of $\frac{1+2z^{-1}}{1-z^{-1}}$, by the long division method.

(b) Find Z-transforms of

(i) $f(n)=\frac{1}{n(n-1)}$, and

(ii) $f(n)=\frac{2n+3}{(n+1)(n+2)}$

(9) (a) Solve: $y_{n+2}-7y_{n+1}+12y_n=2^n \quad given\ y_0=y_1=0,$ using z- transform.

(b) Find $Z^{-1}\left\{\frac{z}{(z-1)(z-2)^2}\right\}$

(10)(a) Find the bilateral Z-transforms of

(i) $a^n\delta(n-k)$,

(ii) $-\alpha^n U(-n-1)$,

(iii) $-n\alpha^n U(-n-1)$,

(b) Solve the equation $x_{n+2}-5x_{n+1}+6x_n=36,$ given that $x_0=x_1=0.$

(11)(a) Use convolution theorem to find the sum of the first n natural numbers.

(b) Find the inverse Z-transform of $\frac{1}{1+4z^{-2}}$, by the long division method.

(12) (i) Use initial value theorem to find $f(0)$, when

$$\bar{f}(z)=\frac{ze^{aT}\left(ze^{aT}-\cos bT\right)}{z^2e^{2aT}-2ze^{aT}\cos bT+1}$$

(ii) Use final value theorem to find $f(\infty)$, when

$$\bar{f}(z) = \frac{Tze^{aT}}{\left(ze^{aT} - 1\right)^2}$$

(13) (a) Find $Z^{-1}\left\{\frac{2z^2 + 4z}{(z-2)^3}\right\}$, by using Residue theorem.

(b) Find the Z-transforms of

(ii) $\sin^3\left(\frac{n\pi}{6}\right)$, and (iii) $\cos\left(\frac{n\pi}{2} + \frac{\pi}{4}\right)$