**DHANALAKSHMI SRINIVASAN COLLEGE OF ENGINEERING AND TECHNOLOGY**

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**QUESTION BANK(REG 2017)**

**SUBJECT:TRANSFORMS AND PARTIAL DIFFERENTIAL EQUATIONS**

**SUBCODE: MA8353**

 **SEM/YEAR: III/II**

UNIT 1

 **PARTIAL DIFFERENTIAL EQUATIONS**

Part A

(1)Form a partial differential equation by eliminating arbitrary constants a and b from

 

(2) Solve: 

(3)Form a partial differential equation by eliminating the arbitrary constants a and b from the equation.

(4)Find the complete solution of the partial differential equation 

(5)Find the PDE of all planes having equal intercepts on the x and y axis.

(6)Find the solution of.

(7)Find the singular integral of the partial differential equation 

(8)Solve: 

(9)Form a partial differential equation by eliminating the arbitrary constants a and b from

 

(10)Solve:

 

(11)Form a partial differential equation by eliminating the arbitrary constants a and b from

 

(12)Solve:

 

Part B

 (1)(i) Form a partial differential equation by eliminating arbitrary functions from 

(ii) Solve: 

(2)(i) Solve: 

(ii) Solve: 

(3)(i) Solve: 

(ii) Solve: 

(4)(i) Solve: 

(ii) Solve: 

(5)(i) Solve: 

(ii) Solve: 

(6)(i) Solve: 

(ii) Solve: 

(7)(i) Solve: 

(ii) Solve: 

(8)(i) Solve: 

(ii) Solve: 

(9)(i) Solve: 

(ii) Solve: 

(10)(i) Solve: 

(ii)Solve: 

(11)(i) Form the partial differential equation by eliminating  from 

(ii) Solve: 

(12)(i) Find the complete integral of 

(ii) Solve: 

(13)(i) Solve: 

(ii) Solve: 

(iii) Solve: 

(14)(i) Solve 

(ii) Solve: 

(15)(i) Form a partial differential equation by eliminating arbitrary functions f and g in 

(ii) Solve: 

(16)(i) Form a partial differential equation by eliminating arbitrary functions f and g in 

(ii) Solve: 

(17)(i) Find the singular solution of 

(ii) Solve: 

(18) (i) Solve: 

(ii) Find the singular integral of a partial differential equation 

(19)(i) Solve: 

(ii) Form a partial differential equation by eliminating arbitrary functions ‘f’ from



(20)(i) Solve: 

(ii) Solve: 

**UNIT 2**

**FOURIER SERIES**

**PART – A**

1. Determine the value of  in the Fourier series expansion of 
2. Find the root mean square value of  in the interval .
3. 3. Find the coefficient  of  in the Fourier cosine series of the function  in the interval 
4. 4. If  and  for all x, find the sum of the Fourier series of  at .
5. 5. Find  in the expansion of  as a Fourier series in .
6. 6. If  is an odd function defined in (-*l , l*) what are the values of 
7. 7. Find the Fourier constants  for  in .
8. 8. State Parseval’s identity for the half-range cosine expansion of  in (0 , 1).
9. 9. Find the constant term in the Fourier series expansion of  in .
10. 10. State Dirichlet’s conditions for Fourier series.

**PART B**

1. (i) Express  as a Fourier series in 

 (ii) Show that for 0 < x <*l*,  . Using root mean square value of x, deduce the value of 

2. (i) Find the Fourier series of periodicity 3 for  in 0 < x < 3.

 (ii) Find the Fourier series expansion of period 2 for the function  which is defined in  by means of the table of values given below. Find the series upto the third harmonic.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 |  |  |  |  |  |  |
|  | 1.0 | 1.4 | 1.9 | 1.7 | 1.5 | 1.2 | 1.0 |

3.(i) Find the Fourier series of periodicity 2 for  for 0 < x < 2.

 (ii) Show that for 0 < x <*l*,  . Deduce that 

4. (i) Find the Fourier series for . Hence deduce the sum to infinity of the series 

 (ii) Find the complex form of Fourier series of  in the form

 and hence prove that 

5. Obtain the half range cosine series for  in 

6. Find the Fourier series for  in the interval .

7. (i) Expanding  as a sine series in  show that 

 (ii) Find the Fourier series as far as the second harmonic to represent the function given in the following data.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 |
|  | 9 | 18 | 24 | 28 | 26 | 20 |

8. Obtain the Fourier series for  of period 2*l* and defined as follows

 

Hence deduce that 

9. Obtain the half range cosine series for  in 

10. (i) Find the Fourier series of 

 (ii) Obtain the sine series for the function

 

11. (i) Find the Fourier series for the function

  and  for all x.

 (ii) Determine the Fourier series for the function

 

12. Obtain the Fourier series for  in . Deduce that 

13. Obtain the constant term and the first harmonic in the Fourier series expansion for  where  is given in the following table.

|  |  |  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|  | 18.0 | 18.7 | 17.6 | 15.0 | 11.6 | 8.3 | 6.0 | 5.3 | 6.4 | 9.0 | 12.4 | 15.7 |

14. (i) Express  as a Fourier series in 

 (ii) Obtain the half range cosine series for  in the interval 0 < x < 2.

15. Find the half range sine series of  in 

16. (i) Find the Fourier series expansion of  = 

 (ii) Find the half-range sine series of  = sin ax in (0 , *l*).

17. Expand  = - as a Fourier series in -1 < x < 1 and using this series find the r.m.s. value of  in the interval.

18. The following table gives the variations of a periodic function over a period T.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
|  x |  |  |  |  |  |  |  |
|  | 1.98 | 1.3 | 1.05 | 1.3 | -0.88 | -0.25 | 1.98 |

 Show that  = 0.75 + 0.37 +1.004 , where 

19. Find the Fourier series upto the third harmonic for the function y =  defined in (0 , ) from the table

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| x | 0 |  |  |  |  |  |  |
|  | 2.34 | 2.2 | 1.6 | 0.83 | 0.51 | 0.88 | 1.19 |

20. (i) Find the half-range (i) cosine series and (ii) sine series for  =  in (0 , )

 (ii) Find the complex form of the Fourier series of  = cos ax in (- , ).

**UNIT 3**

**APPLICATIONS OF PARTIAL DIFFERENTIAL EQUATIONS**

**PART A**

1. Classify the partial differential equation 

2. The ends A and B of a rod of length 10 cm long have their temperature kept at  C and C. Find the steady state temperature distribution on the rod.

3. Solve the equation  given that  by the method of separation of variables.

4. Write the one dimensional wave equation with initial and boundary conditions in which the initial position of the string is  and the initial velocity imparted at each point x is .

5. What is the basic difference between the solution of one dimensional wave equation and one dimensional heat equation.

6. In steady state conditions derive the solution of one dimensional heat flow equation.

7. What are the possible solutions of one dimensional wave equation.

8. In the wave equation  what does  stand for?

9. State Fourier law of conduction.

10. What is the constant  in the wave equation 

**PART B**

(1) A tightly stretched string with fixed end points x=0 andis initially at rest in its equilibrium position. If it is set vibrating giving each point a velocity , then show that

 ****

(2) A rectangular plate is bounded by lines x=0, y=0, x=a, y=b. It’s surface are insulated. The temperature along x=0 and y=o are kept at C and others at C. Find the steady state temperature at any point of the plate.

(3)A metal bar 10 cm. long, with insulated sides has its ends A and B kept at C and C respectively until steady state conditions prevail. The temperature at A is then suddenly raised toC and at the same instant that at B is lowered to C. Find the subsequent temperature at any point of the bar at any time.

(4) A tightly stretched string of length  has its ends fastened at x=0, . The mid-point of the string is mean taken to height ‘b’ and then released from rest in that position.

Find the lateral displacement of a point of the string at time‘t’ from the instant of release.

(5)If a square plate is bounded by the lines  and three of its edges are kept at temperature C, while the temperature along the edge y=a is kept at  Find the steady state temperature in the plate.

(6)A uniform string is stretched and fastened to two points l apart. Motion is started by displacing the string into the form of the curve

 (i)  and

 (ii) 

and then releasing it from this position at time t=0. Find the displacement of the point of the string at a distance x from one end at time t.

(7) A long rectangular plate with insulated surface is 1 cm wide. If the temperature along one short edge(y=0) isdegrees, for, while the two long edges x=0 and x=l as well as the other short edge are kept at temperature C, Find the steady state temperature function.

(8)Find the temperature distribution in a homogeneous bar of length  which is insulated laterally, if the ends are kept at zero temperature and if, initially, the temperature is k at the centre of the bar and falls uniformly to zero at its ends.

(9) solve the one dimensional wave equation 

given that 

(10) A rectangular plate with insulated surface is 20 cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the temperature of the short edge x=0 is given by

 

and the two long edges as well as the other short edge are kept at C. Find the steady state temperature distribution in the plate.

(11) Solve the one dimensional heat flow equation

 

Satisfying the following boundary conditions.

 (i) 

 (ii) 

 (iii)

(12)A rectangular plane with insulated surface is a cm wide and so long compared to its width that it may be considered infinite in length without introducing an appreciable error. If the two long edges x=0 and x=a and the short edge at infinity are kept at temperatureC, while the other short edge y=0 is kept at temperature

(i) (ii) T (constant). Find the steady state temperature at any point

(x, y) of the plate.

(13) A tightly stretched strings with fixed end points x=0 and x=50 is initially at rest in its equilibrium position. If it is said to vibrate by giving each point a velocity

(i) 

(ii) 

Find the displacement of any point of the string at any subsequent time.

(14) An infinitely long metal plate in the form of an area is enclosed between the lines

 for positive values of x. The temperature is zero along the edges

 and the edge at infinity. If the edge x=0 is kept at temperature ky,

 Find the steady state temperature distribution in the plate.

(15)A taut string of length, fastened at both ends, is disturbed from its position of equilibrium by imparting to each of its points an initial velocity of magnitude Find the displacement function

**UNIT-4**

**FOURIER TRANSFORMS**

**PART A**

1. State the Fourier integral theorem.

2.State the convolution theorem of the Fourier transform.

3. Write the Fourier transform pair.

4. Find the Fourier sine transform of  (a > 0).

5. If the Fourier transform of  is F(s) then prove that . 

6. State the Fourier transforms of the derivatives of a function.

7. Find the Fourier sine transform of .

8. Prove that 

9. If F(s) is the Fourier transform of  then prove that 

10. Find the Fourier sine transform of 

**PART B**

1. Find the Fourier Transform of

 

 Hence prove that 

2. Find the Fourier cosine transform of 

3. Find the Fourier Transform of  if

 

Hence deduce that 

4. Evaluate  using transforms

5. Find the Fourier transform of  and hence deduce that

(i) 

(ii) 

6. Show that the Fourier transform of  is . Hence deduce that  Using Parseval’s identity show that 

7. . Find the Fourier transform of  if

 

Hence deduce that 

1. Derive the parseval’s identity for Fourier transforms.

9. Find the Fourier sine transform of

 

10. Find the Fourier transform of  and hence deduce that

(i) 

(ii) 

11. State and prove convolution theorem for Fourier transforms.

12. Using Parseval’s identity calculate

(i)  (ii)  if a > 0.

13. Find the Fourier cosine transform of 

14. (i) Find the Fourier cosine transform of 

 (ii) Find the Fourier sine transform of

 

15. Find Fourier sine and cosine transform of  and hence find the Fourier sine transform of  and Fourier cosine transform of .

 **UNIT 5**

 **PART A**

 **Z-Transforms**

 (1)Form the difference equation from ****

(2)Express in terms of 

(3)Find the value of  when ****

(4)Define bilateral Z-transform.

(5)Find the z-transform of 

(6)Find  using z-transform.

(7)Define unilateral Z-transform

(8)Find  using z-transform.

(9)State and prove initial value theorem in z-transform.

(10)Find the z-transform of n.

Part B

(1) (a)Prove that**.**

 Find ****

(b)Find **** using Convolution theorem.

(2) (a) Prove that ****

 (b) Solve: **** using

 z- transform.

(3)(a) Find the z-transform of ****

 (b)Using Convolution theorem find ****

(4)(a) Solve the difference equation

 ****

(b) Find **** by the method of partial fractions.

(5)(a) Find **** by the method of partial fractions.

 (b) Solve the difference equation

  ****

(6)(a) Prove that****

 (b) State and prove the second shifting theorem in z-transform.

(7)(a) Using Convolution theorems evaluate inverse z-transform of**.**

 (b) Solve the difference equation

 ****

(8)(a) Find the inverse Z-transform of , by the long division method.

 (b) Find Z-transforms of

(i), and

(ii) 

 (9) (a) Solve: **** using

 z- transform.

 (b) Find ****

(10)(a) Find the bilateral Z-transforms of

(i) 

(ii) 

(iii) 

(b) Solve the equation given that 

(11)(a)Use convolution theorem to find the sum of the first n natural numbers.

 (b) Find the inverse Z-transform of, by the long division method.

(12)(i) Use initial value theorem to find, when

 ****

(ii) Use final value theorem to find, when

 ****

(13) (a) Find , by using Residue theorem.

 (b) Find the Z-transforms of

 (ii), and (iii)