PART A (2MARKS) AND PART B (16 MARKS) QUESTIONS WITH ANSWERS

AE 6007 FATIGUE AND FRACTURE

SEMESTER VIII, DEPARTMENT OF AERONAUTICAL ENGINEERING

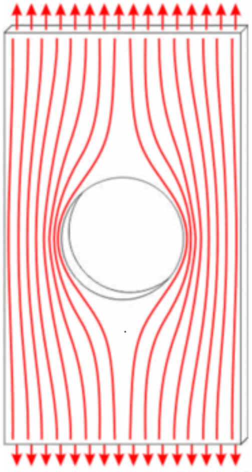
2 Marks questions

Unit 1

Fatigue of structures

1. What is meant by stress concentration? Explain how its value can be reduced.

A stress concentration is a location in an object where stress is concentrated. An object is strongest when force is evenly distributed over its area, so a reduction in area, e.g. caused by a crack, results in a localized increase in stress. Fatigue cracks always start at stress raisers, so removing such defects increases the fatigue strength.



Internal force lines are denser near the hole

2. How does one determine the endurance limit stress?

(i) Endurance limit stress related to hardness:

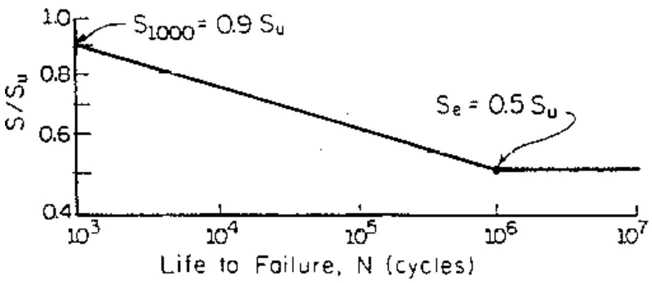
S_e(Mpa) $\approx$ (0.25x6.897)xBHN for BHN $\leq$ 400, if BHN > 400, $S_e \approx$ (100x6.897) Mpa, where BHN is the Brinell hardness number.

(ii) Endurance limit stress related to ultimate strength:

$S_e \approx 0.5 \times S_u$ for $S_u \leq$ (200x6.897) Mpa

If $S_u >$ (200x6.897)Mpa, $S_e \approx$(100x6.897) Mpa

The alternating stress level corresponding to a life of 1000 cycles. S_{1000}, be estimated as 0.9 times the ultimate strength. The line connecting this and the endurance limit is the estimate used for the S-N design line point as shown in figure.

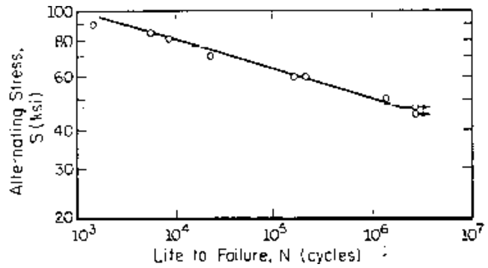


Generalized S–N curve for wrought steels on log–log plot.

3. What is an SN curve? Explain the significant points in the curve.

S-N diagram, which is a plot of alternating stress, S, versus cycles to failure, N.

Actual S-N line representing the mean of the data. Certain materials, primarily body-centered cubic (BCC) steels, have an endurance of fatigue limit, S_e, which is a stress level below which the material has an “infinite” life. For engineering purposes, this “infinite” life is usually considered to be 1 million cycles.



S-N curve for 1045 steel.

4. What are the effects of notches and cutouts in the loaded structures?

The existence of irregularities or discontinuities, such as holes, grooves or notches and cutouts in a part increases the magnitude of stresses significantly in the immediate vicinity of the discontinuity. Fatigue failure mostly originates from such places. Hence its effect must be accounted and normally a fatigue stress concentration factor K_f, is applied when designing against fatigue, even if the materials behaviour is ductile.

Fatigue stress concentration factor,

$$K_f = \frac{S_e^{(unnotched)}}{S_e^{(notched)}}$$

5. Explain the terms:

   (a) Endurance limit
   (b) Endurance Strength

(a) <u>Endurance limit</u>

The endurance limit, also known as fatigue limit, is a <u>stress</u> level below which a material has an "infinite" life. Infinite life is commonly considered to be 1 million cycles.

S_e(Mpa) ≈ (0.25x6.897)xBHN for BHN ≤ 400, if BHN > 400, S_e ≈ (100x6.897) Mpa, where BHN is the Brinell hardness number.

(b) <u>Endurance Strength</u>

Endurance limit related to Ultimate Strngth

$S_e \approx 0.5$ x S_u for $S_u \leq$ (200x6.897) Mpa

If $S_u >$ (200x6.897)Mpa, $S_e \approx$(100x6.897) Mpa

6. What is fatigue according to ASTM standards?

The process of progressive localized permanent structural changes occurring ina material subject to conditions that produce fluctuating stresses and strains at some point or points and that may culminate in cracks of complete fracture after a sufficient number of fluctuations.

7. What are the various variables that affect the S-N curve?

   Variables

   Stress amplitude or Alternating stress
   Fatigue strength
   Yield tensile stress
   Ultimate tensile stress
   Life to failure (N)

   Factors

   Material Composition
   Heat treatment
   Operating Temperature
   Grain size and grain direction
   Welding
   Surface condition

8. Explain the term: Notch sensitivity.

   Notch sensitivity ‘q’ is defined by the equation,

$$q = \frac{K_f - 1}{K_t - 1}$$

$$q = \frac{\text{Actual intensification of stresses over nominal stress}}{\text{Theoretical intensification of stress over nominal stresses}}$$

The values of q are between zero and unity. It is evident that if q=0 then $K_f = 1$ and the material has no sensitivity to notches at all. On the other hand q=1, then $K_f = K_t$, and the material has full notch sensitivity. In analysis or design work, find K_t first, from geometry of the part. Then select or specify the material, find q and solve for K_f from the equation

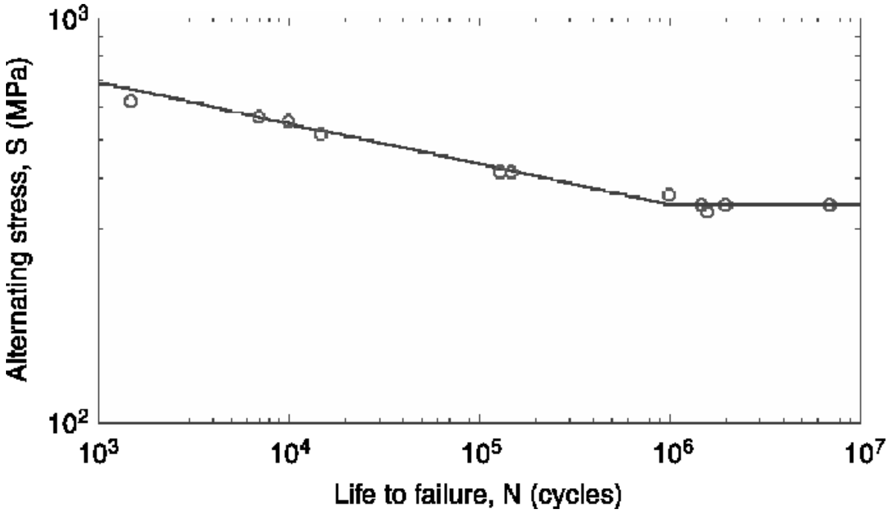
$$K_f = 1 + q(K_t - 1)$$

9. Explain the methods of reducing stress concentrations.

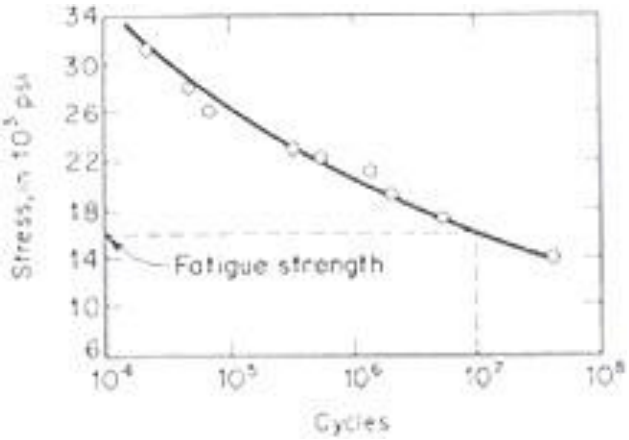
   Stress concentrations can arise at sharp corners and abrupt changes in section. Fillets should therefore be provided at re-entrant corners and cut-outs, such as windows and access panels should be reinforced. Rivets should not be used in areas of high stress and stiffness should be bonded to plates rather than attached by rivets. In machined panels the material thickness should be increased around bolt holes, while holes in primary bolted joints should be reamed to improve surface finish. Surface scratches and machine marks are sources of fatigue crack initiation.

10. Draw typical S-N curves for mild steel and aluminium and explain the differences.

At higher stress, of course, the component has a short fatigue life. For mild steel, it is found that below the endurance limit σ_e material does not fail. However distinct endurance limit is not observed for Aluminium (non ferrous materials).



S-N curve for mild steel



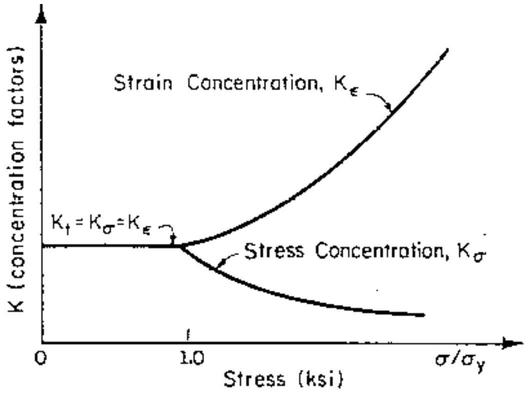
S-N curve for Aluminium

11. What is limiting range of stress?

The greatest range of stress (mean stress zero) that a metal can withstand for an indefinite number of cycles without failure. If exceeded, the metal fractures after a certain no. of cycles which decreases as the range of stress increases. Also called endurance range; half this range is the fatigue limit or endurance limit.

12.What is Neuber's rule?

$K_t = \sqrt{K_\sigma K_\varepsilon}$ states that the theoretical stress concentration the geometric mean of the stress and strain concentration or the square root of the product of K_σ and K_ε. This seems intuitively reasonable since after yielding occurs, K_σ decreases while K_ε increases as shown in Fig. although this method was proven only for one notch geometry, it is assumed that this relationship holds true for most notch geometries. This versions of equation termed Neuber's rule, are often used in the local strain approach to relate nominal stresses and strains to local values.



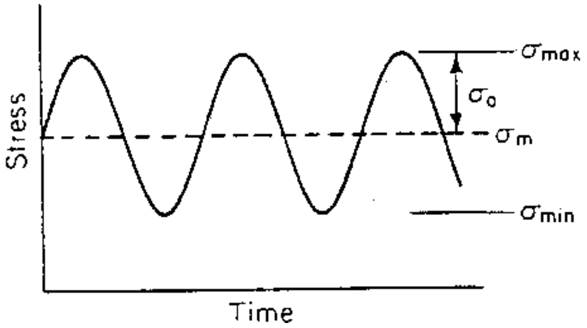
Effect of yielding on K_σ and K_ε.

13.Define: Stress amplitude

A test parameter of a dynamic fatigue test. One half the algebraic difference between the maximum and minimum stress in one cycle.

Stress amplitude or Alternating stress

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$



Terminology for alternating stress.

14.Explain the term: Mean stress

Mean stress,

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

15.Define the term: Stress range

Stress range is the difference between maximum stress and minimum stress.

Stress range, $\Delta_\sigma = \sigma_{max} - \sigma_{min}$

16. Define the following term: Stress ratio

Stress ratio, is the minimum stress and maximum stress.

$$R = \frac{\sigma_{min}}{\sigma_{max}}$$

17. Explain the following term: Amplitude ratio

Amplitude ratio is the ratio between amplitude stress and mean stress

Amplitude stress,

$$A = \frac{\sigma_a}{\sigma_m}$$

The R and A values corresponding to several n loading situation are:

Fully reversed: R=-1, A=∞

Zero to max: R=0, A=1

Zero to min: R=∞, A= -1

Where R=Stress ratio

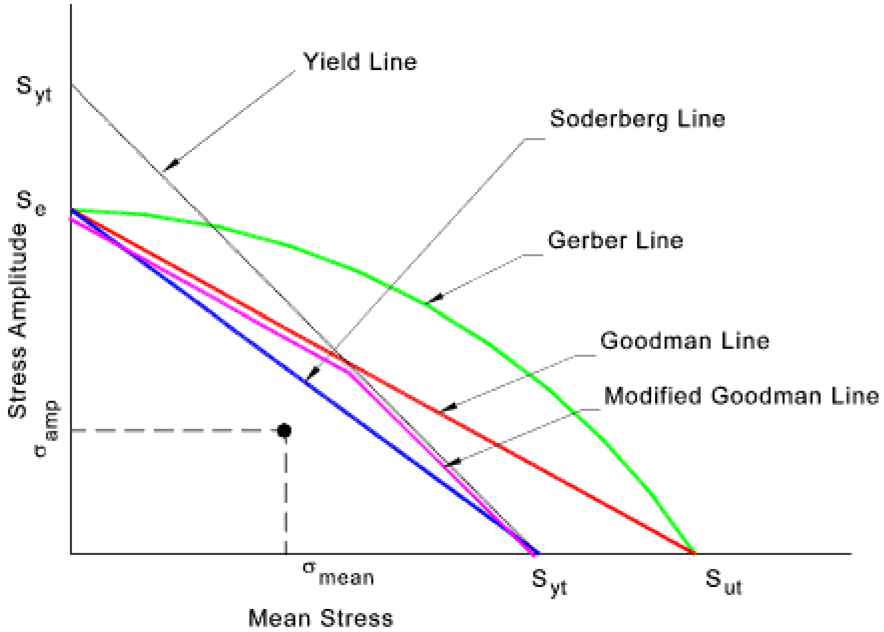
18. What is fluctuating stress?

Variable loading is often characterized by an amplitude component σ_a as ordinate and a steady component σ_m as abscissa. Defined in terms of maximum stress σ_{max} and minimum stress σ_{min} the co-ordinate are as follows:

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

The designer’s fatigue diagram is depicted in fig.



S_e= The modified fatigue strength
S_{ut}=The ultimate tensile strength
S_{yt}=The yield tensile strength
N_f=The factor of safety applicable the fatigue

19. What is repeated stress?

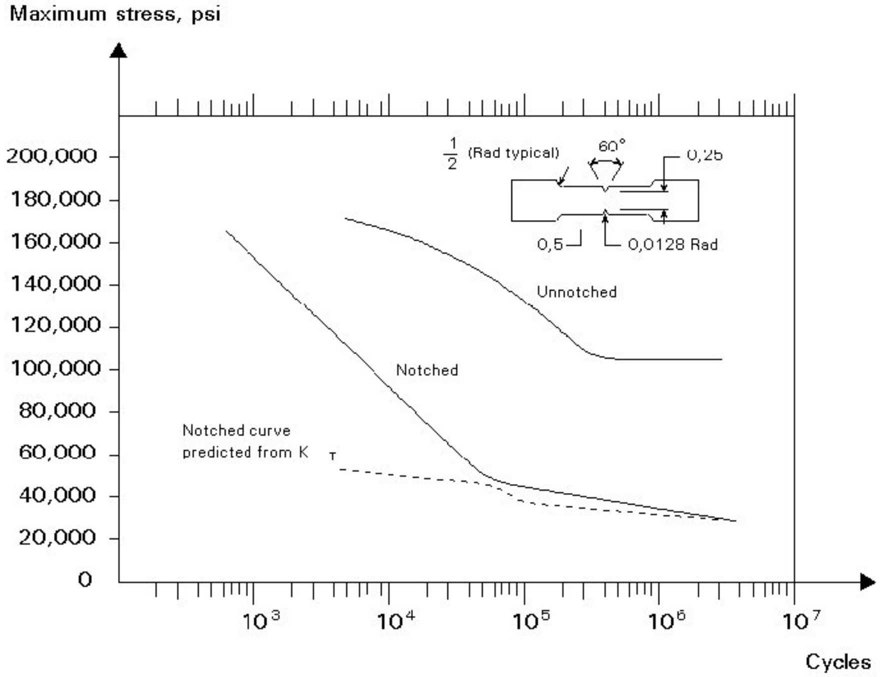
More commonly seen in engineering applications than the reversed cycle, the repeated stress cycle is a sine wave that is asymmetric about the x axis. The max. and min. stresses are NOT equal and opposite in sign.

20. What is Soderberg line?

If the point of the combined stress below the Soderberg line then the component will not fail. To establish the factor of safety relative to the Soderberg criteria. This is a very conservative criteria based on the material yield point S_{yt}.

$$\frac{K_f \sigma_{amp}}{S_e} + \frac{\sigma_{mean}}{S_{yt}} = \frac{1}{N_f}$$

21. Draw the notched S-N curve.



Experimental and calculated S-N curves
for a notched specimen

Unit 2

Statistical Aspects of Fatigue Behaviour

1. What is Miner’s cumulative rule?

Linear Damage rule is commonly known as Miner’s rule. The following terminology will be used in the discussion:

$$\frac{n}{N} = cycle\ ratio$$

Where n is the number of cycles at stress level S and N is the fatigue life in cycles at stress level S.

The damage fraction, D, is defined as the fraction of life used up by an event or a series of events. Failure in any of the cumulative damage theories is assumes to occur when the summation of damage fractions equals 1, or

$$\sum D_i \geq 1$$

2. What do you understand by S-N approach and ε-N approach?

The stress-life (S-N) was the first approach used in an attempt to understand and quantify metal fatigue. The S-N approach is still widely used in design applications where the applied stress is primarily within the elastic range of the material and the resultant lives (cycles to failure) are long, such as power transmission shafts.

The strain life (ε-N) method is based on the observation that in many components the response of the material in critical locations (notches) is strain or deformation dependent. At high load levels, in the low cycle fatigue (LCF) region, the cyclic stress-strain response and the material behaviour are best modeled under strain-controlled conditions.

3. Define the term: ‘Cycle counting’.

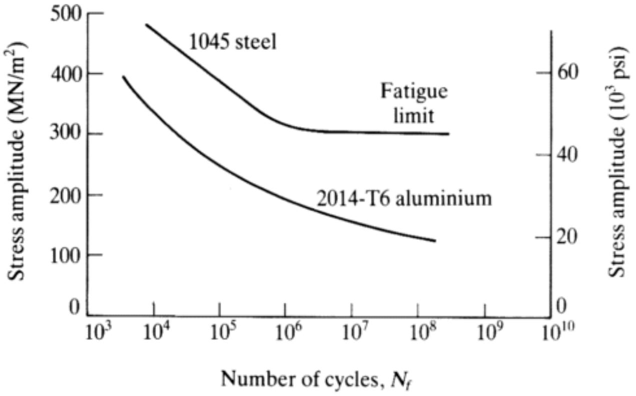
   To predict the life of a component subjected to a variable load history, it is necessary to reduce the complex history into a number of events which can be compared to the available constant amplitude test data. This process of reducing a complex load history into a number of constant amplitude events involved. This is termed as ‘cycle counting’.

4. Define ‘Fatigue Damage’.

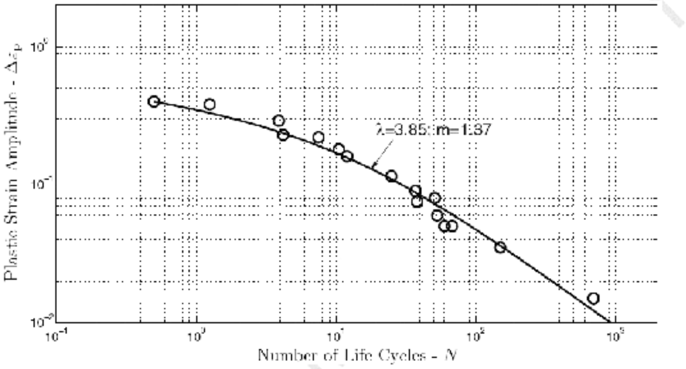
   During the propagation portion of fatigue, damage can be related to crack length. During propagation, damage can be related to an observable and measurable. It has been used to great advantage in the aerospace industry.

   During the initiation phase, the mechanisms of fatigue damage are on the microscopic level. Most damage summing methods for the initiation phase are empirical in nature. It is equal to the formation of a small crack in a large component or structure.

5. With a sketch, explain low and high cycle fatigue.



High cycle fatigue

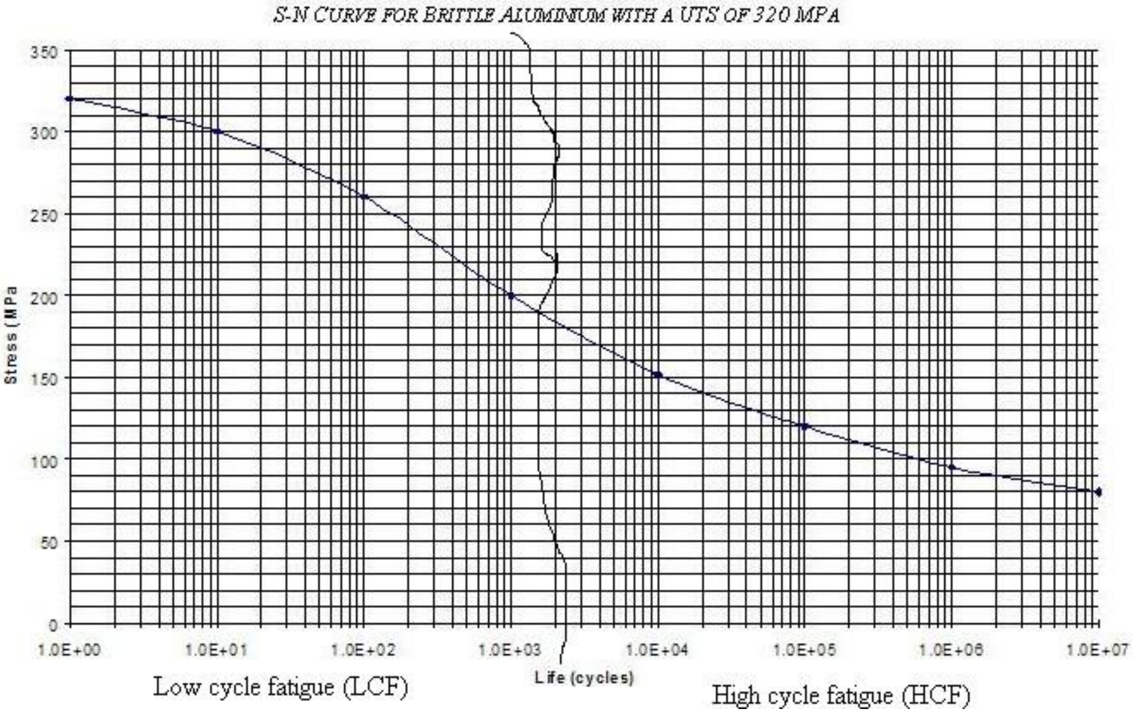


The $\Delta\varepsilon_p$-N relationship for the aluminum alloy 2024-T6. Experimental data from Coffin

Low cycle fatigue

A complete S-N curve may be divided into two regions, namely LCF& HCF. There is no sharp dividing line between the two levels. We might arbitrarily say that 0-above 10^3 or 10^4 cycles (LCF) and above 10^3 or 10^4-10^8 cycles (HCF). Most of the existing fatigue results are for high stress only (HCF). It was realized that for pressure vessels, pressurized fuselages, mechanisms for extending loading gears, controlling wing flaps, missiles, space ship launching equipment etc., only a low fatigue life was required.

6. Draw a typical S-N curve and demarcate it with high cycle and low cycle fatigue.



7. What is the need for cycle counting technique?

To predict the life of a component subjected to a variable load history, it is necessary to reduce the complex history into a number of events which can be compared to the available constant amplitude test data. This process of reducing a complex load history into a number of constant amplitude events involved. This is termed as 'cycle counting'.

8. Discuss the miner's cumulative damage theory.

Linear Damage rule is commonly known as Miner's rule. The following terminology will be used in the discussion:

$$\frac{n}{N} = cycle\ ratio$$

Where n is the number of cycles at stress level S and N is the fatigue life in cycles at stress level S.

The damage fraction, D, is defined as the fraction of life used up by an event or a series of events. Failure in any of the cumulative damage theories is assumes to occur when the summation of damage fractions equals 1, or

$$\sum D_i \geq 1$$

9. What is the need to count the cycles?

To predict the life of a component subjected to a variable load history, it is necessary to reduce the complex history into a number of events which can be compared to the available constant amplitude test data. This process of reducing a complex load history into a number of constant amplitude events involved. This is termed as ‘cycle counting’.

10.Distinguish between low cycle and high cycle fatigue behaviour of structures.

A complete S-N curve may be divided into two regions, namely LCF& HCF. There is no sharp dividing line between the two levels. We might arbitrarily say that 0-above 10^3 or 10^4 cycles (LCF) and above 10^3 or 10^4-10^8 cycles (HCF). Most of the existing fatigue results are for high stress only (HCF). It was realized that for pressure vessels, pressurized fuselages, mechanisms for extending loading gears, controlling wing flaps, missiles, space ship launching equipment etc., only a low fatigue life was required.

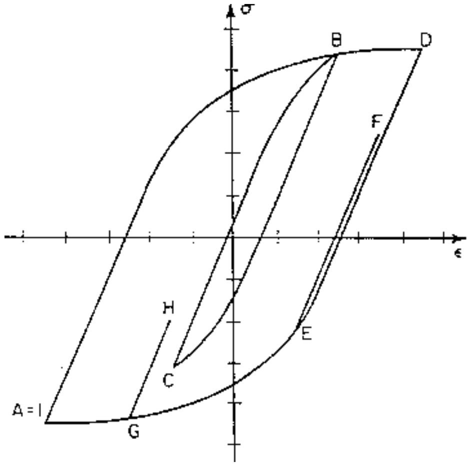
11. What are the transient behaviours of metals?

1. Cyclically harden
2. Cyclically soften
3. Be cyclically stable
4. Have mixed behaviour (soften or harden depending on the strain range)

12. What are the early cycle counting procedures?

(i) Level-crossing counting
(ii) Peak counting
(iii) Simple-range counting

13. What is material memory?



Material stress–strain response to given strain history.

Upon reloading after reaching point B, the material continues to point D along the hysteresis path starting from point A, as though event cycle B-C had never occurred. This behaviour of material “remembering” its prior state of deformation is known as material memory.

14. What is strain life relation?

By using the relation

$$\frac{\Delta\epsilon}{2} = \frac{\Delta\epsilon_e}{2} + \frac{\Delta\epsilon_p}{2}$$

The total strain can now be rewritten by using the equations

$$\frac{\Delta\epsilon_p}{2} = \epsilon_f' \left(2N_f\right)^c$$

and

$$\frac{\Delta\epsilon_e}{2} = \frac{\epsilon_f'}{E} \left(2N_f\right)^b$$

We get the equation

$$\frac{\Delta\epsilon}{2} = \frac{\epsilon_f'}{E} \left(2N_f\right)^b + \epsilon_f' \left(2N_f\right)^c$$

The above equation is the basis of the strain – life method and is termed the strain-life relation.

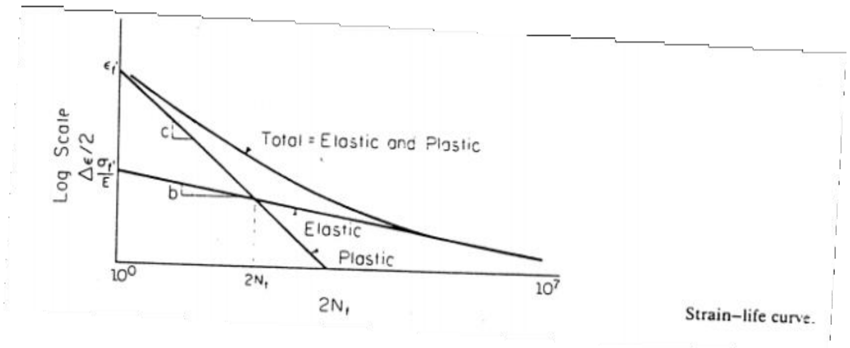
15. What is Rain flow counting? (Falling rain approach)

The first step in implementing this procedure is to draw the strain time history so that the time axis is oriented vertically with increasing time downward. Now imagine that the strain history forms a no. of ‘pagoda roofs’. Cycles are then defined by the manner in which rain is allowed to drip or fall down the roofs. A number of rules are imposed on the dripping rain so as to identify closed hysteresis loops.

16. What is level crossing counting?

In this procedure the strain axis of the strain-time plot is divided into a number of increments. A count is then recorded each time a positively sloped portion of the strain history crosses an increment located above the reference strain. Similarly negatively sloped strain below the reference strain, a count is made. In addition, crossings at the reference strain are also counted.

17. Draw the strain-life curve?



Strain–life curve.

18. What is Peak counting?

The peak counting method is based on the identification of local maximum and minimum strain values. To begin, the strain axis is divided into a number of increments. The positions of all local maximum (peak) strain values above the reference strain are tabulated, as are the positions of all local minimum (valley) strain value below the reference strain.

19. What is simple – range counting?

With this method the strain range between successive reversals is recorded. In determining counts, if both positive ranges (valley followed by peaks) and negative ranges (peaks followed by valleys) are included, each range is considered to form one-half cycle. If just positive or negative ranges are recorded, each is considered to form one full cycle.

20. What is Non-linear damage theory?

Many nonlinear damage theories have been proposed which attempt to overcome the shortcomings of Miner's rule. The following is a general description of a nonlinear damage approach which is currently of research interest and has some applications in design. This method predicts the following relationship between damage fraction, D, and cycle ratio, n/N:

$$D = \left(\frac{n}{N}\right)^{P}$$

where the exponent, P, is a function of stress level. The value of P is considered to fall in the range zero to 1, with the value increasing with stress level. When P=1 the method is equivalent to Miner's rule.

21. What are the applications of strain – life approach?

(i) Applications where plastic strains are significant. This may involves situations where load or stress levels are high, such as root of a notch. It involves material with low yield points such as low strength steels and some stainless steels.

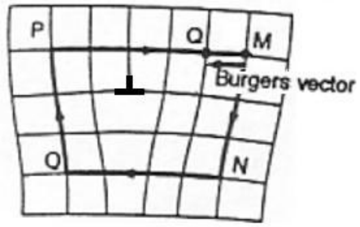
(ii) High temperature applications such as gas turbine engine components where fatigue-creep interaction is important.

Unit 3

Physical Aspects of Fatigue

1. What is dislocation theory?

A dislocation is a crystallographic defect or irregularity, within a crystal structure. The presence of dislocations strongly influences many of the properties of materials. Some types of dislocations can be visualized as being caused by the termination of a plane of atoms in the middle of a crystal. In such a case, the surrounding planes are not straight, but instead bend around the edge of the terminating plane so that the crystal structure is perfectly ordered on either side.

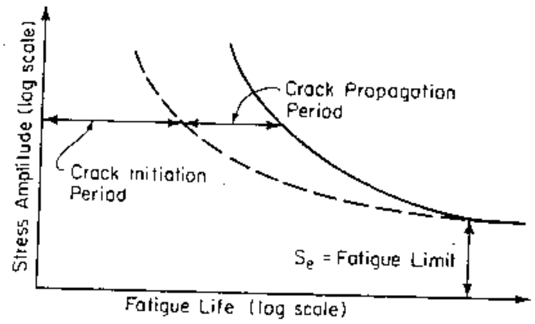


Edge Dislocation

2. What are the different phases of a crack with respect to fatigue life?

The fatigue life of a component is made up of initiation and propagation stages.

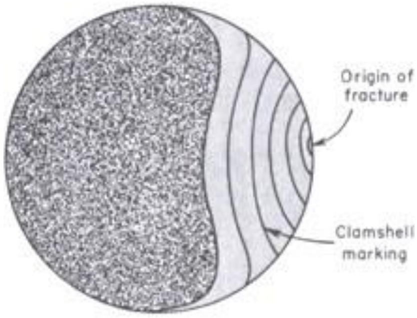
At low strain amplitudes up to 90% of the life may be taken up with initiation, while at high amplitudes the majority of the fatigue life may be spent propagating a crack. Fracture mechanics approaches are used to estimate the propagation life.



Initiation and propagation portions of fatigue life.

3. What are “clam shell markings”?

Features of a fatigue fractures are Beach marks or clamshell marks may be seen in fatigue failures of materials that are used for a period of time, allowed to rest for an equivalent time period and loaded again as in factory usage. Striations are thought to be steps in crack propagation where the distance depends on the stress range. Beach marks contain thousands of striations.



Beachmarks or "clamshell pattern" associated with stress cycles that vary in magnitude and time as in factory machinery

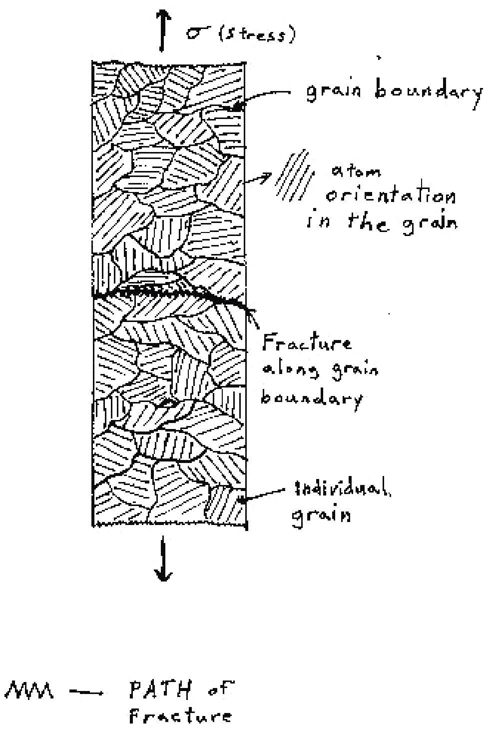
4. What is fatigue crack growth?

The majority of fatigue life may be taken up in the propagation of a crack. By the use of fracture mechanics principles it is possible to predict the no. of cycles spent in growing a crack to some specified length or to final failure.

5. What is inter-granular fracture?

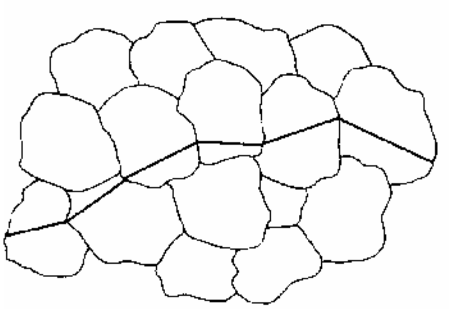
Brittle fracture in crystalline metals can be classified into two broad groups, intergranular and transgranular. A crack of intergranular failure moves along the grain boundaries as shown in fig.

Intergranular Fracture



6. What is transgranular fracture?

Transgranular fracture occurs through fracture within grains. Within grain, cleavage failure occurs along a weak crystallographic plane. Cleavage fracture is the most brittle form of fracture and it hardly damages the fracture surfaces. Once the cleavage crack reaches the grain boundary, it finds another favourable orientation in the next grain.



Transgranular brittle fracture

7. What is brittle fracture?

Some materials are known as brittle because a crack moves easily through the material. From a fractured surface of a brittle fracture, find that material is influenced to a very shallow depth. Rest of the material remains unaffected.

8. What is ductile fracture?

Ductile fracture causes a large amount of plastic deformation to a significant depth. Ductile fracture growth occurs due to plastic deformation and creation of micro voids. The material deforms plastically due to micro-mechanisms such as nucleation and motion of dislocations, formation of twins etc., Tiny voids are formed at the side of these particles under the tensile field of the crack tip.

9. Define: Fatigue fracture surface

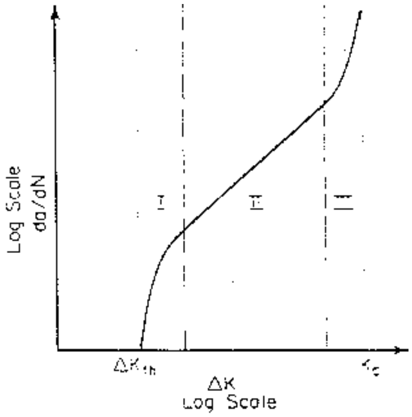
Fatigue failures exhibits very small permanent visual deformation. So fatigue failures are mistakenly often called as brittle failures. This term should however be modified since substantial plastic deformation occurs in small local regions near the fatigue crack tip and at crack initiation sights. Typical fatigue failures exibit the following common aspects.

(iii) Distinct crack initiation sight
(iv) Beach mark or clamshell marks indicative of crack growth
(v) Distinct final fracture region

10. What is torsional fatigue surface?

In torsional fatigue failure the cracks appear on a 45°inclination plane. This is the plane of max. tensile stress which again indicates that fatigue cracks propagate primarily in the plane of max. tensile stress. A smooth semi-elliptical fatigue crack shape is a very common fatigue surface. The final fracture region has a fibrous appearance with radial lines essentially perpendicular to the perimeter of the elliptical fatigue crack. These radial river pattern are often seen on the final fracture surfaces.

11. What are the three regions of crack growth curve?
Region II, the curve is essentially linear. Many structures operate in this region. Finally, in region III, at high ΔK values, crack growth rates are extremely high and little fatigue life is involved.



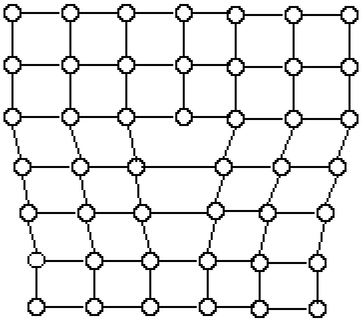
Three regions of crack growth rate curve.

12. What is nucleation?

Nucleation is the extremely localized budding of a distinct thermodynamic phase. Some examples of phases that may form via nucleation in liquids are gaseous bubbles, crystals or glassy regions. Creation of liquid droplets in saturated vapor is also characterized by nucleation. Nucleation of crystalline, amorphous and even vacancy clusters in solid materials is also important, for example to the semiconductor industry. Most nucleation processes are physical, rather than chemical, but a few exceptions do exist. (e.g. electrochemical nucleation).

13. What is edge dislocation?

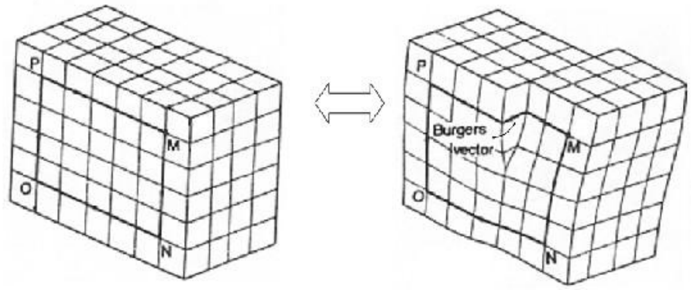
An edge dislocation is a defect where an extra half-plane of atoms is introduced mid way through the crystal, distorting nearby planes of atoms. When enough force is applied from one side of the crystal structure, this extra plane passes through planes of atoms breaking and joining bonds with them until it reaches the grain boundary. A simple schematic diagram of such atomic planes can be used to illustrate lattice defects such as dislocations.



Edge dislocation

14. What is screw dislocation?

A screw dislocation is much harder to visualize. Imagine cutting a crystal along a plane and slipping one half across the other by a lattice vector, the halves fitting back together without leaving a defect. If the cut only goes part way through the crystal, and then slipped, the boundary of the cut is a screw dislocation.



Screw dislocation

15. What is coalescence?

Coalescence is the process by which two or more droplets, bubbles or particles merge during contact to form a single daughter droplet, bubble or particle. It can take place in many processes, ranging from meteorology to astrophysics. For example, it is both involved in the formation of raindrops as well as planetary and star formation.

16.What are the factors influencing fatigue crack growth?

(i) Stress ratio effects

(ii) Environmental effects

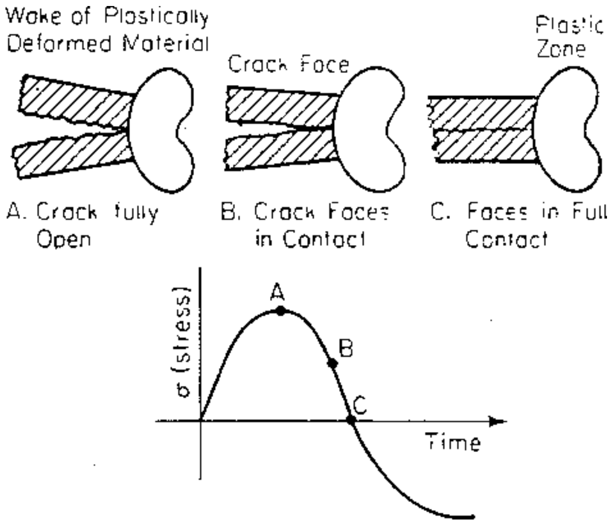
(iii) Frequency of loading

(iv)Temperature effects

(v)Waveform of loading cycle

17. What is crack closure?

Crack closure occurs as a result of crack tip plasticity that a plastic zone develops around the crack tip as the yield stress of the material is exceeded. As the crack grows, a wake of plastically deformed material is developed while the surrounding body remains elastic. As the component is unloaded the practically "stretched" material causes the crack surfaces to contact each other before zero load is reached.



Crack closure phenomenon

18. What is the relationship between fatigue life and crack size?

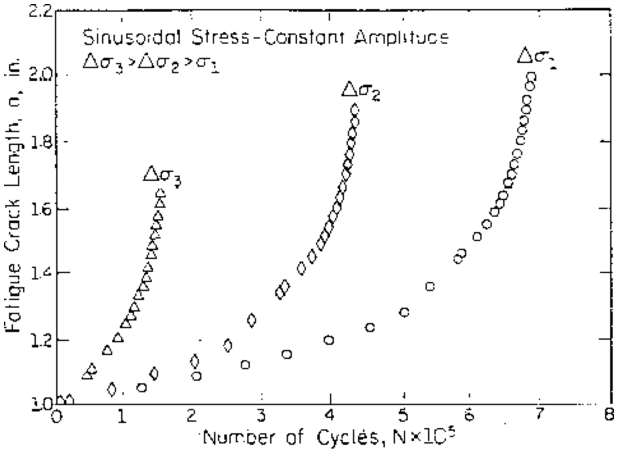
The fatigue life estimate is strongly dependent on the initial crack size, a_i. Large changes in the estimate of final crack size, a_f, result in only small changes in the life estimate.

19. By using crack growth rate, how cycles to failure may be calculated?

The fatigue crack growth rate can be related to the stress intensity factor range. From this, cycles to failure may be calculated.

20. Define: Fatigue crack growth.

Typical constant amplitude crack propagation data as shown in fig. The crack length, a, is plotted versus the corresponding number of cycles, N, at which the crack was measured. As shown, most of the life of the component is spent while the crack length is relatively small. In addition, the crack growth rate increases with increased applied stress.



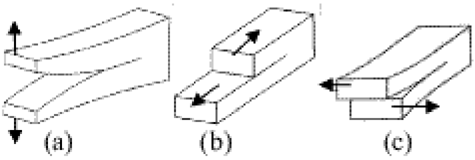
Constant amplitude crack growth data

Unit 4
Fracture Mechanics

1. Explain the fracture failure in terms of energy.

When fracture failure occurs, two important quantities are invoked: (i) how much energy is released when a crack advances and (ii) minimum energy required for the crack the crack to advance in forming two new surfaces. The first quantity is measured with a parameter, energy release rate, denoted by the symbol G. The energy release rate is defined as energy release per unit increase in area during crack growth. Rate is defined here with respect to change in crack area. The second quantity is the surface energy required to create two new surfaces.

2. What are the three modes of loading in fracture mechanics? Explain with neat sketches.



(a) mode-I opening mode; (b) mode-II sliding or shear mode; (c) mode- III tearing mode.

Three loading modes

There are generally three modes of loading, which involve different crack surface displacements as shown in figure. The three modes are

Mode I : opening or tensile mode (the crack faces are pulled apart)
Mode II : sliding or in-plane shear (the crack surfaces slide over each other)
Mode III: tearing or anti-plane shear (the surfaces move parallel to the leading edge of the crack and to each other)

3. What are the disadvantages of using LEFM theory?

It must be noted that LEFM can be extended to cope with only limited crack tip plasticity. i.e. When the crack tip plastic zone is small compared to the crack size and the cracked body still behaves in an elastic manner. If this is not the case the problem has to be treated elasto plastically. However the concepts of EPFM (Elastic-plastic fracture Mechanics) are not nearly so well defined as LEFM.

4. Explain the fracture based approach to estimate the fatigue life.

The LEFM approach is the only method that deals directly with the propagation of fatigue cracks. It also provides a method to characterize final failure due to fracture of the remaining cracked section.

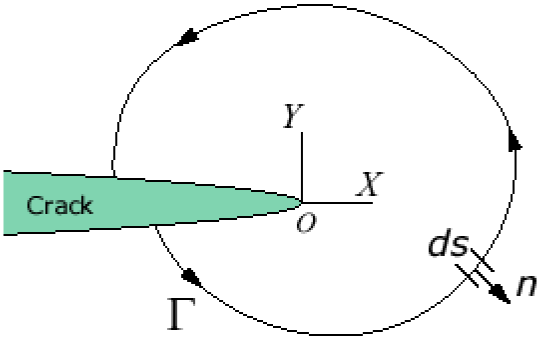
Since crack length gives a physical measure of damage, crack growth rates can be incorporated with nondestructive inspection techniques to find the “safe life” of cracked components.

5. What is meant by fracture toughness?

As the stress intensity factor reaches a critical value,K_c, unstable fracture occurs. This critical value of the stress intensity factor is known as the fracture toughness of the material. The fracture toughness can be considered the limiting value of stress intensity just as the yield stress might be considered the limiting value of applied stress.

6. Explain the significance of J-integral.

Like other parameters (G and K) J-integral is also a parameter to characterizing a crack. In fact, G is a special case of J-integral: that is G is usually applied only to linear elastic materials whereas J-integral is not only applicable to linear and non-linear elastic materials but is found very useful to characterize materials exhibiting elastic-plastic behaviour near the crack tip.



Path Γ around the crack tip with outward normal n

7. What is stress intensity factor?

The stress intensity factor, K defines the magnitude of the local stresses around the crack tip. This factor depend on loading crack size, crack shape and general form given by

$K = f(g)\sigma\sqrt{\pi a}$

σ = Remote stress applied to the component

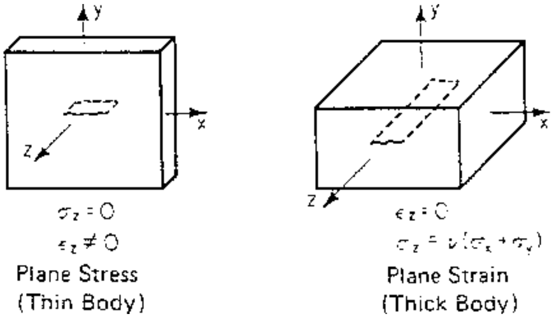
a=crack length

f (g)= correction factor that depends on specimen and crack geometry\

8. Explain: Plane stress and plane strain conditions.

In a thin body, the stress through the thickness (σ_z) cannot vary appreciably due to the thin section. Because there can be no stresses normal to free surface, σ_z=0 throughout the section and a bi-axial state of stress results. This is termed a plane stress condition.

In a thick body, the material is constrained in the z direction due to the thickness of the cross section $\epsilon_z=0$, resulting in a plane strain condition.



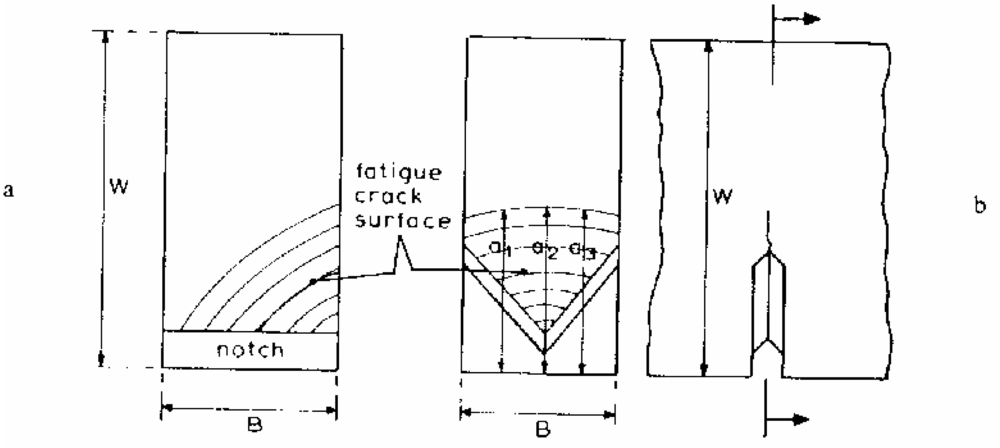
Plane stress and plane strain conditions.

9. What are the recommended specimens for plane strain fracture toughness testing?

(i) Three point Bend specimen
(ii) Four point bend specimen
(iii) Compact tension specimen
(iv) C – specimen

10. Define: Chevron notch

The specimens have to provide with a fatigue crack. In order to ensure that cracking occurs at the right place, the specimens contain a starter notch. In thick members, fatigue cracks starts at a corner such cracking behaviour results in an irreproducible, curved crack front not suitable for a standard test. It can be avoided by providing the specimens with a chevron notch. This notch forces initiation of crack in the centre, which enhance the probability of a relatively straight crack front. It has the additional advantage that, the fatigue crack starts almost immediately upon fatigue cycling.



Starter notch
a. Normal edge notch; b. Chevron notch

11. Define; Griffith’s theory.

Griffith realized that a crack in a body will not extend unless in a body will not extend unless energy is released in the process to overcome the energy needs of forming two new surfaces one below and one above the crack plane. The surface energy of a material depends on the material properties.

12. What are the advantages of stress - life approach?

(i) The analysis and estimation of material constants necessary for this method ore quite simple.

(ii) This method works well for designs involving long life, constant amplitude histories.

13. What are the disadvantages of stress –life approach?

(i) This method is completely empirical in nature and lacks the physical in rights into the mechanisms of fatigue given by the other methods.

(ii) The S-N approach does not distinguish between initiation and propagation. This gives limited insights into the concept of damage.

14. State the applications of stress-life approach.

(i) The S-N method can be used in almost any situation to get a rough estimate of life.

(ii) The best examples of application of this method are in the design of various machine elements such as power transmission shafts, valve springs and gears.

15. Explain the advantages of strain – life (Є-N) approach.

(i) Plastic strain, the mechanism that leads to crack initiation, is accurately modeled. This method can be used in high strain/low cycle situations.

(ii) This method can be more easily extrapolated to situations involving complicated geometries.

(iii) This method can be used in high temp. applications where fatigue creep interaction is critical.

16. Explain the disadvantages of strain – life (Є-N) approach.

(i) This method involves more complicated level of analysis. The life calculation involves numerical iterations which are best handled with computers.

(ii) This method only accounts for initiation life and cannot be used to predict propagation life.

17. Define: LEFM approach

This method is the only method that deals directly with the propagation of fatigue cracks. It also provides a method to characterize final failure due to fracture of the remaining cracked section.

18. What are the advantages of LEFM approach?

(i) Since crack length gives a physical measure of damage, crack growth rates can be incorporated with non-destructive inspection techniques to find the safe life of cracked components.

(ii) It provides a method to deal with non propagating cracks and crack arrest behaviour due to over loads.

(iii) State the applications of LEFM approach?

(i) To measure the crack growth from an assumed initial existing flaw. Examples of applications are in the aerospace and nuclear reactor pressure vessel industries, where the consequences of fatigue are significant.

(ii) To determine the life at components with sharp notches, where only a small fraction of life involves initiation.

19. Define: Energy release rate.

Energy release rate, denoted by the symbol G, which is defined as energy release per unit increase in area during crack growth. In fracture mechanics rate is defined here with respect to change in crack area.

20. What is meant by ‘change in compliance approach’?

In fracture mechanics, it is easier to deal with compliance which is inverse of stiffness. Thus compliance of a body increases with increase in the crack length.

Unit5

Fatigue Design and Testing

1. What is Fretting Fatigue?

A surface wear phenomenon occurring between two contacting surfaces having oscillatory relative motion of small amplitude. e.g. Riveted, bolted & pinned joints, lug fasteners, shrink fits and press fits, splines, keyways, clamps, universal joints, bearings/housing/shafts interfaces, gear/shafts interfaces, oscillatory bearings, fittings, leaf springs and wire ropes.

Fatigue corrosion is both material and atmospheric dependant.

2. Explain safe life design with examples.

   Safe life : The structure is designed to have a minimum life during which it is known that no damage will occur. At the end of this life the structure must be replaced even though there may be no detectable signs of fatigue.

   (e.g) Landing gear, major wing joints, wing-fuselage joints and hinges on all-moving tail planes or on variable geometry wings.

3. Explain safe design with examples.

   Fail safe : The failure of a member in a redundant structure does not necessarily lead to the collapse of the complete structure, provided that the remaining members are able to carry the load shed by the failed members and can withstand further repeated loads until the presence of the failed member is discovered.

   (e.g) Wing skins which are stiffened by stringers and fuselage skins which are stiffened by frames and stringers; the stringers and frames prevent skin cracks.

4. List down the factors that are to be considered while designing the components to avoid fatigue failure.

(i) Choice of materials
(ii) Sharp corners
(iii) Cut outs
(iv) Rivets should not be used.
(v) Stiffeners should be bonded not by rivets
(vi) Material thickness at bolt holes
(vii) Holes in bolted joints should be reamed to get good surface finish.

5. Define: Infinite life design

   It requires design stresses to below the pertinent fatigue limit for parts subjected to many millions of almost uniform cycles, like engine valve springs. This is still a good design criteria.

6. What is 'Damage tolerant Design'

This philosophy is a refinement of failsafe philosophy. It assumes that cracks will exist caused either by processing (or) fatigue –uses fracture mechanics analysis and tests whether such cracks will grow large enough to produce failure before they are sure to be detected by periodical inspection. This philosophy looks for materials with slow crack growth and high fracture toughness. In pressure vessel design leak before burst is an expression for this philosophy.

7. What are the factors affecting fatigue?

   (i) Micro structure (Grain size, texture)
   (ii) Processing (Deformation, history, manufacturing)
   (iii) Local spectrum (sign, magnitude, rate, history)
   (iv) Geometry of component(Surface finish, notches, weld connection, Thickness)
   (v) Environment (Temperature, corrosive medium)

8. What are the required sequences for fatigue life prediction?

(i) Load spectrum-Structural system-Stress and strain histories-prediction model- N_i,N_f, da/dN.

(ii) Material selection- Monotonic and cyclic properties - prediction model- N_i,N_f, da/dN.

9. Write the procedures followed while designing against fatigue?

   (i) Design to keep stress below the threshold of fatigue limit (infinite life time concept)
   (ii) Design for a fixed life after which the user is instructed to replace the part with a new one (safe life design)
   (iii) Instruct the user to inspect the part periodically for cracks and replace the part once a crack exceeds a critical length.(Damage tolerant design)

10. What is sonic fatigue?

One of the problems in super sonic transport (SST) is the sonic or acoustic fatigue. The extremely powerful engines that will be used on SST creates intense level of sound. During take off and landing, the energy from the sound waves will be reflected from the concrete runways on to the under surface of the wing and fuselage. These sound waves can set up very high frequency vibration in such surfaces causing fatigue failure of the outer skin or in the case of honey comb panels the internal structure. In order to obtain adequate fatigue life such structures exposed to high acoustic forces must be strengthened by thicker surfaces and properly positioned interior members which can absorb and dampen the induced vibrational energies.

11. Define: ‘Effect of geometry in fatigue life’.

The fatigue life of a notched component is generally less than that of an un notched one. While the surface roughness of a component could be improved by polishing, notches cannot be avoided in most industrial components because of the functional requirements.

12. What is the influence of grain size in fatigue life?

For most metals, smaller grains, yields longer fatigue lives.

13. Define: ‘Effect of processing techniques in fatigue life’.

Processing techniques such as forging, rolling and extrusion produce directional properties in material due to the grain orientation. Fatigue life is generally enhanced in the oriented direction and is lower in the transverse direction.

Heat treatment, case hardening, cold and hot working, surface coating, plating, cladding etc., can all influence fatigue life. Manufacturing process could produce residual stresses. Compressive residual stresses at the external surfaces generally enhance fatigue life, while tensile ones are detrimental. Process such as shot peening, cold rolling and static preloading are employed to induce compressive residual stresses.

14. What are the factors involved in the successful operation of the aircraft in the severe environment?

To design and construct efficient fatigue resistant supersonic transport (SST) and subsonic transport structures reliable for 50,000 hrs in the severe environment (particularly in Mach 3 condition) is a considerable challenge. The successful operation of the aircraft in this severe environment generally depends on the following factors.

(i) Temp. variation of above -56˚C to 2000˚C.
(ii) A combination of mechanical and thermal stresses
(iii) Up to 50000 hrs of operating life under the above conditions.

15. Explain: ‘Designing against fatigue in aircraft’

An estimation of the number of frequency and magnitude of the fluctuating loads ao aircraft encounters necessary.

During taxiing the aircraft may be manoeuvring over uneven ground with a full payload so that wing stresses, for example are greater than in the static case. We shall see experiences a greater number of gusts than during the cruise.

16. Define: Step method.

First test a specimen at a stress level corresponding to a survival of about 70-90% of the specimen survives the prescribed long life, say 10^7cycles, the stress is increased about 5% of the estimated fatigue strength and test continues for further 10^7 cycles, if the specimen does not fail. The stress is increased after each survival until the specimen fails. Specimens may be used in this method, single specimen. The estimate of a fatigue strength from result of one specimen.

17. Define: Stair-case method

The first specimen is tested at the estimated fatigue strength (at say 10^7 cycles) for the prescribed no. of cycles. If this specimen fails, the next specimen is tested at a stress level i.e. one increment below the first stress level. If the first specimen does not fail, the second specimen is tested at a high stress level, i.e. one increment above the first stress level. The third specimen is tested at a stress level higher (or) lower than the stress level for the second specimen, depending on whether the second specimen survives or fails.

18. What is limit load in aircraft?

The limit load, which is the maximum load that the aircraft is expected to experience in normal operation.

19. What is proof load?

The proof load, which is the product of the limit load and proof factor (1.0 to 1.25).

20. What is ultimate load?

The ultimate load, which is the product of the limit load and the ultimate factor (usually 1.5).

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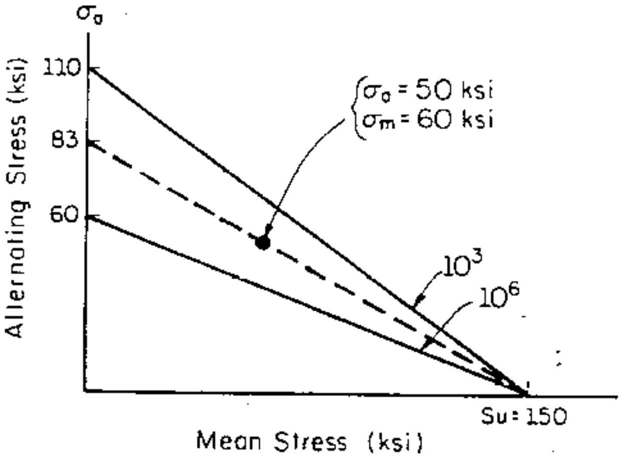
16 Marks Questions (Solution - Key only)

Unit 1

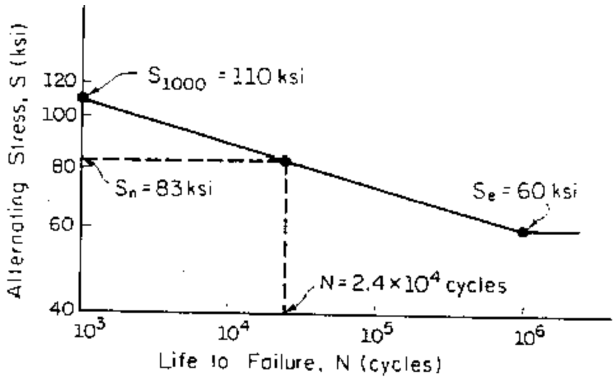
Fatigue of structures

1.A component undergoes a cyclic stress with a maximum value of 10 ksi and a minimum value of 10 ksi. The component is made from a steel with ultimate strength, S_u, of 150 ksi, an endurance limit, S_e of 60 ksi and a fully reversed stress at 1000 cycles, S_{1000} of 110 ksi. Using the Goodman relationship, determine the life of the component.

Solution



Haigh diagram.



S–N diagram.

Ans.: Life of the component = 2.4×10^4 cycles

2. Bending stress in a structural member fluctuates between a tensile stress of 280 MPa and compressive stress of 140 MPa. What should be the minimum ultimate stress for the member to carry this fluctuation? Assume the factor of safety to be 2 and the endurance strength to be 50% of the ultimate strength of the material?

Solution

$$\sigma_a = \frac{\sigma_{max} - \sigma_{min}}{2}$$

= 210 MPa

$$\sigma_m = \frac{\sigma_{max} + \sigma_{min}}{2}$$

=70 MPa.

Goodman relation

$$\frac{\sigma_{amp}}{S_e} + \frac{\sigma_{mean}}{S_{ut}} = \frac{1}{N_f}$$

S_{ut}= 980 MPa.

Ans.: S_{ut}= 980 MPa.

3. Determine the thickness of a 120mm wide uniform plate for a safe continuous operation if it is subjected to a varying maximum tensile load of 250 kN and a minimum of 100 kN for a factor of safety 1.5. The endurance strength and yield strength of the material are 225 MPa and 300 MPa respectively.

Solution

Mean or average load, W_m=175 kN

Mean stress,σ_{mean} =175/0.12t kN/m^2

Variable load or amplitude load, W_{amp} = 75 kN

Amplitude stress, σ_{amp}=75/0.12t kN /m^2

According to Soderberg's formula

$$\frac{K_f \sigma_{amp}}{S_e} + \frac{\sigma_{mean}}{S_{yt}} = \frac{1}{N_f}$$

t = 11.439 mm

4. Under Soderberg method, determine the required diameter of a solid circular rod of ductile material having endurance strength has 265MPa and a tensile yield strength has 350 MPa. The rod is subjected to varying axial load from 300 kN compression to 700 kN tension. The stress concentration factor is 1.8 and factor of safety is 2.0.

Solution

Area=0.7854 d^2mm^2

W_m=200x10^{3}N

Mean stress, f_m=254.6x10^3/d^{2}mm^2

Variable load,w_v=500x10^{3}N

F_v=W_v/A=636.5x10^3/d^2 N/mm^2

According to Soderberg’s formula

$$\frac{K_f\sigma_{amp}}{S_e}+\frac{\sigma_{mean}}{S_{yt}}=\frac{1}{N_f}$$

d = 100.5 mm

Ans.: d= 100.5 mm

5. What is the need for using factor of safety in the design of components?

<u>Solution</u>

Factor of safety

Choosing design factors

For aircraft

6. A component undergoes a cyclic stress with a maximum value of 750 MPa and a minimum value of 75 MPa. The component made from steel with an ultimate stress of 1000 MPa and endurance limit stress of 400 MPa and undergoes fully reversed stress at 1000 cycles. Using the Goodmann’s relationship determine the life of the component.

<u>Solution</u>

$$\sigma_a=\frac{\sigma_{max}-\sigma_{min}}{2}$$

= 337.5 MPa

$$\sigma_m=\frac{\sigma_{max}+\sigma_{min}}{2}$$

=412.5 MPa.

Goodman relation

$$\frac{\sigma_{amp}}{S_e}+\frac{\sigma_{mean}}{S_{ut}}=1$$

S_e=574 MPa

Ans.: N_f = 3.33 x 10^4 cycles

<u>Unit 2</u>

<u>Statistical Aspects of Fatigue Behaviour</u>

1.Write briefly on:

Linear Damage rule

<u>Solution</u>

Linear Damage rule is commonly known as Miner’s rule. The following terminology will be used in the discussion:

$$\frac{n}{N}=cycle\ ratio$$

Where n is the number of cycles at stress level S and N is the fatigue life in cycles at stress level S.

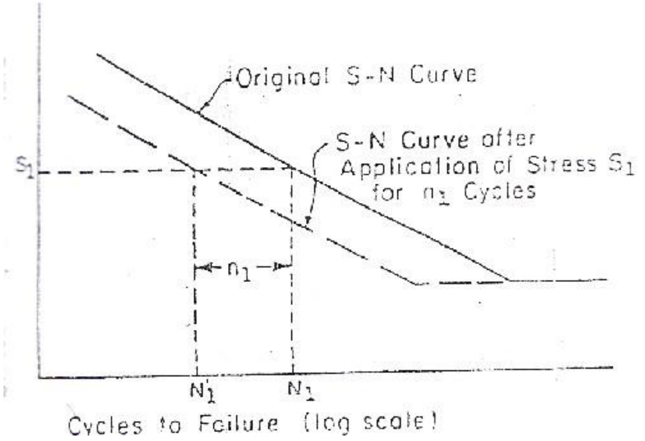
The damage fraction, D, is defined as the fraction of life used up by an event or a series of events. Failure in any of the cumulative damage theories is assumes to occur when the summation of damage fractions equals 1, or

$$\sum D_i \geq 1$$

The linear damage rule states that the damage fraction, D_i , at stress level S_i is equal to the cycle ratio, n_i / N_i. For example, the damage fraction, D, due to one cycle of loading is 1/N. In other words, the application of one cycle of loading consumes 1/N. The failure criterion for variable amplitude loading can now be stated as

$$\sum \frac{n_i}{N_i} \geq 1$$

The life to failure can be estimated by summing the percentage of life used up at each stress level. This method is very simple.



Effect of Miner's rule on S–N curve

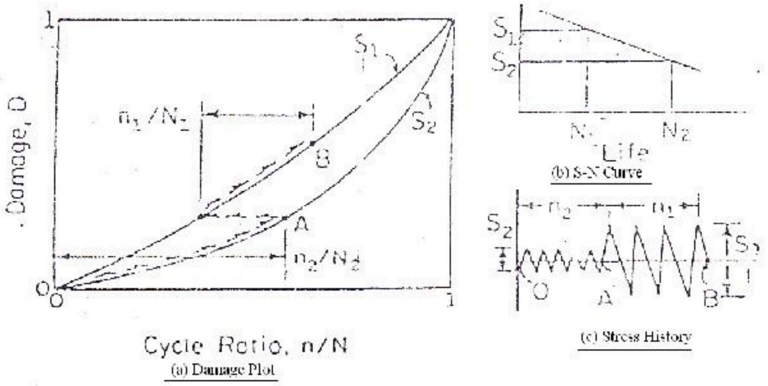
Miner's rule can also be graphically by showing its effects on the S-N curve. If n_1 cycles are applied at stress level S_1, the S-N curve is shifted so that it goes through the new life value, N_1^l. In this procedure N_1^l is N_1-n_1, and N_1 is the original life to failure at stress level S_1. The S-N curve retains its original slope but is shifted to the left.

2. Write briefly on:
Non Linear Damage Rule

Solution

Many nonlinear damage theories have been proposed which attempt to overcome the shortcomings of Miner's rule. The following is a general description of a nonlinear damage approach which is currently of research interest and has some applications in design. This method predicts the following relationship between damage fraction, D, and cycle ratio, n/N:

$$D = \left(\frac{n}{N}\right)^P$$



(a) Damage Plot (b) S-N Curve (c) Stress History

Demonstration of nonlinear damage theory

where the exponent, P, is a function of stress level. The value of P is considered to fall in the range zero to 1, with the value increasing with stress level. When P=1 the method is equivalent to Miner's rule.

The use of this method is shown in Fig. (a) which is a plot of damage fraction versus cycle ratio for two stress levels, where $S_1>S_2$. Figure (c) shows a low-high stress history, where S_2 is applied for n_2 cycles and then stress S_1 is applied for n_1 cycles. The values N_1 and N_2 are the lives to failure corresponding to S_1 and S_2 on the S-N curve [Fig.(b)]. Corresponding points on the stress history [Fig.(c)] and damage plot [Fig.(a)] are labeled O,A and B. When the stress level is changed, a transfer is made to the damage curve corresponding to the new stress level, S_1 by following a horizontal line of constant damage. This procedure is continued until failure is predicted ($D\geq1$).

Note that if the stress blocks are reversed in Fig.(c) giving a high-low test, there will be a an increase in the total damage predicted on the damage plot [Fig.(a)]. Therefore this method includes both sequence and stress level effects.

3. Explain Coffin – Manson theory.

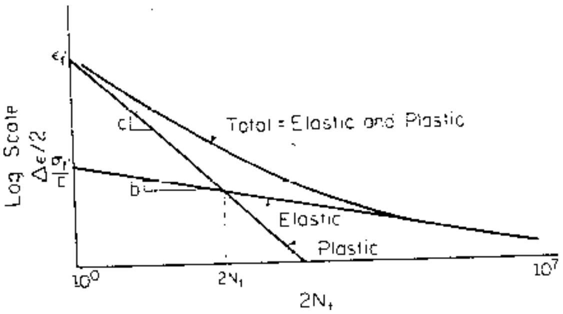
<u>Solution</u>

Plastic strain can be related by a power law function:

$$\frac{\Delta\epsilon_p}{2} = \epsilon_f \left(2N_f\right)^c$$

Where ϵ_p – Plastic
strain N- Life

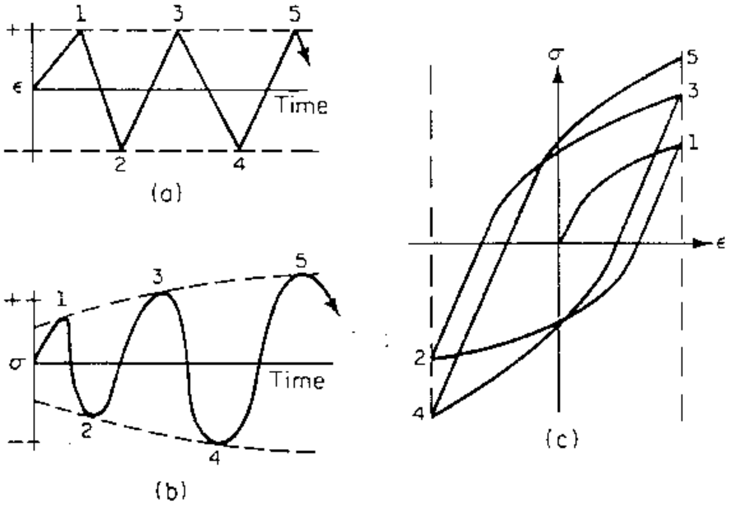
$$\frac{\Delta\epsilon}{2} = \underbrace{\frac{\sigma_f'}{E}(2N_f)^b}_{\text{elastic}} + \underbrace{\epsilon_f'(2N_f)^c}_{\text{plastic}}$$



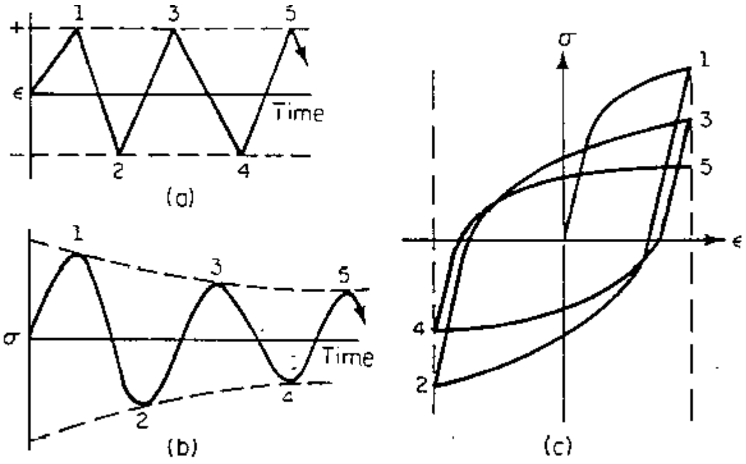
Strain–life curve.

4..Explain the terms cyclic strain hardening and softening with a sketch.

Solution
The stress-strain response of metals is often drastically altered due to repeated loading.



Cyclic hardening: (a) constant strain amplitude; (b) stress response (increasing stress level); (c) cyclic stress–strain response.

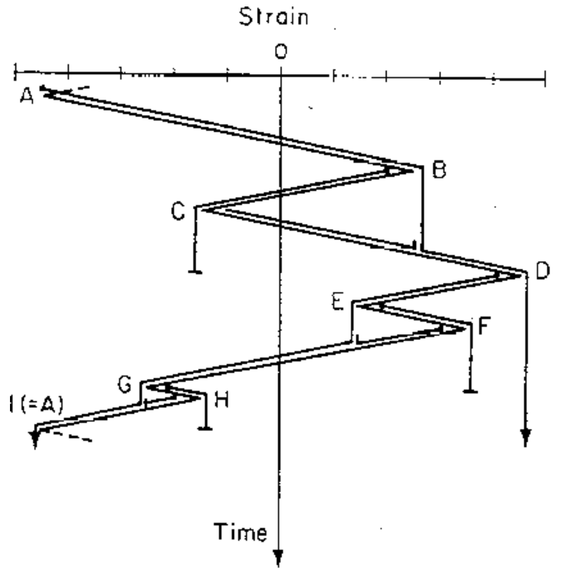


Cyclic softening: (a) constant strain amplitude; (b) stress response (decreasing stress level); (c) cyclic stress–strain response.

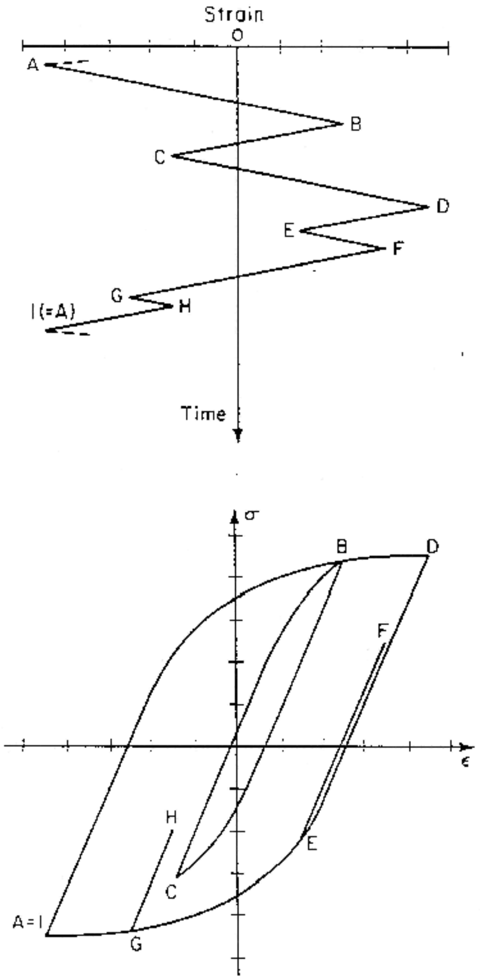
5.Explin the Rainflow counting method with suitable sketches.

Solution

‘Rain flow counting’ that describes any cycle counting method which attempts to identify closed hysteresis loops in the stress-strain response of a material subjected to cyclic loading. It is also called ‘falling rain’ approach.



Rainflow counting ("falling rain" approach).



Material stress–strain response to given strain history.

Unit III
Physical Aspects of Fatigue

1. Explain the mechanism of fatigue crack growth.

Solution

Under the action of cyclic loads cracks can be initiated as a result of cyclic plastic deformation.

Several equivalent models have been proposed to explain the initiation of fatigue cracks by local plastic deformation. The model is depicting in the below figure. In the falling-load part, slip takes place in the reverse direction on a parallel slip plane, since slip on the first plane is inhibited by strain hardening and by oxidation of the newly created free surface. This first cyclic slip can give rise to an extrusion or an intrusion in the metal surface. An intrusion can grow into a crack by continuing plastic flow during subsequent cycles (Fig.). If the fatigue loading is cyclic tension-tension this mechanism can still work since the plastic deformation occurring at increasing load will give rise to residual compressive stresses during load release.

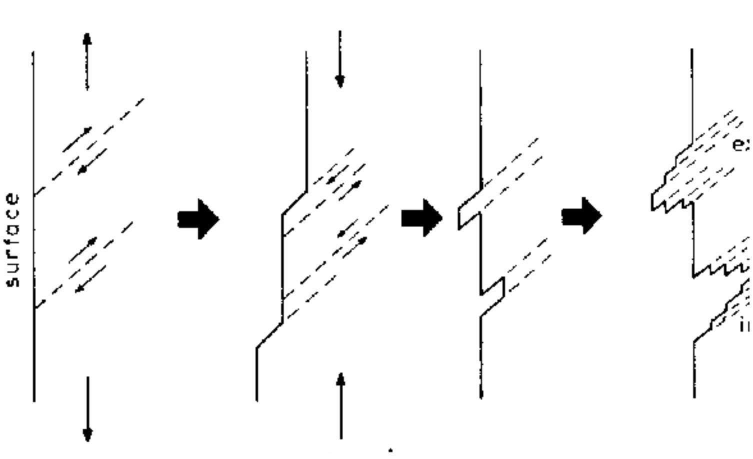
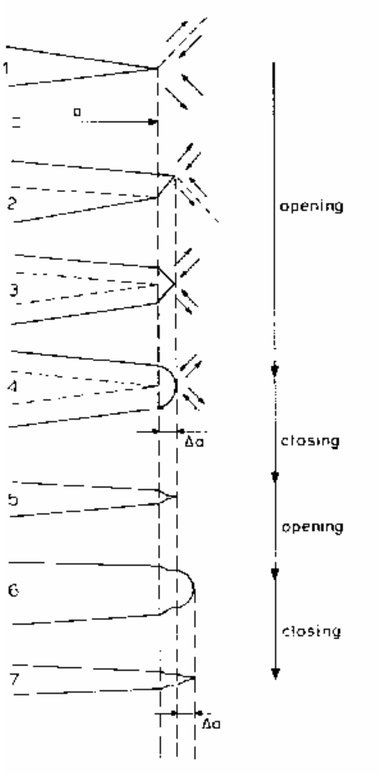


Fig I model for fatigue crack initiation

A fatigue crack, once started, can also grow by a mechanism of reversed slip. Several stages of fatigue crack growth are shown in the following figure. A sharp crack in a tension field causes a large stress concentration at its tip where slip can occur fairly easily. The material above the crack (stages 1 and 2 in figure) may slip along a favourable slip plane in the direction of maximum shear stress. Due to that slip the crack opens but it also extends in length. Slip can now occur on another plane (stage 3). Work hardening and increasing stress will finally activate other parallel slip planes, which leads to a blunt crack tip (stage 4). During the rising load part of the cycle the crack has propagated by an amount Δa.



Possible model for fatigue crack growth

Plastic deformation has occurred in a small region embedded in elastic surroundings. During load release the elastic surroundings will contract and the plastically deformed region, which has become too large, does not fit any more in its surroundings. In order to make it fit the elastic material will exert compressive stresses on the plastic region during the decreasing load part of the cycle. These compressive stresses will be above yield again, at least at the crack tip. This means that reversed plastic deformation occurs, which will close and re-sharpen the crack tip, as is shown in stage 5 of above figure.

The cyclic opening and closing of the crack (stages 1 – 5 and 6 – 7) will develop a typical pattern of ripples, every new cycle adding a new ripple. These ripples show up on the fracture surface in the electron microscope, they are called fatigue striations.

Striations represent the successive positions of the crack front during crack propagation. This can be deduced and showing an electron micrograph of a fatigue specimen that was subjected to a programme fatigue test. The load history can easily be recognized in the electron micrograph patches of 5 fine striations are interspersed with wide striations resulting from the periodic cycles at a higher amplitude. This proves that one striation is formed in each cycle and that the spacing of the striations is a measure of the rate of crack propagation per cycle.

The formation of regular striations requires:

a. Many available slip systems and easy cross slip to accommodate the (usually curved) crack front and to facilitate continuity of the crack front through adjacent grains.

b. Preferably more than one possible crystallographic plane for crack growth.

If these requirements are met, the slip occurring during opening and closing of the crack can adjust to the conditions of the crack front, allowing well-developed striations to be formed.

If the above requirements are not fulfilled, slip will be irregular and find periodic striations cannot develop. The orientation of a particular grain may be suitable for the generation of regular striations, but the limited possibilities for slip may prevent striation formation over some length along the crack front in adjacent grains of other orientation. In such cases poorly defined striations will usually be observed in a few isolated grains with only tangled slip marks in the surrounding grains.

The question arises whether inclusions and second phase particles have an influence on fatigue cracking. As far as initiation of fatigue cracks is concerned they must be expected to have an influence. In the case of smooth specimens the inclusions are sites of stress concentration. At such locations the required plastic deformation (Fig.1) can occur.

For the same reason it must be expected that particles have little influence on fatigue crack propagation. Indeed, at low crack rates their influence is very limited. The effect of a fairly large particle, which apparently remained intact until the crack front had approached it very closely, the last striation before the particle being still straight. At that moment the particle cleaved, as may be concluded from the faint river pattern on its fracture surface. Due to cleavage of the particle, the crack had a locally advanced front where propagation occurred slowly, as can be observed from the closely spaced striations in front of the particle.

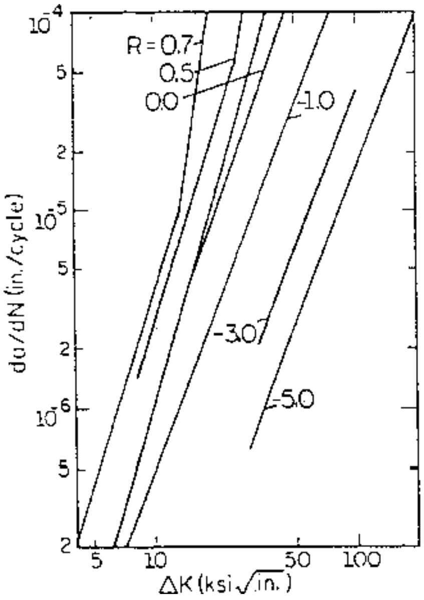
High growth rates are a result of a high stress intensity at the crack tip (large crack or high loads). Due to the higher stress concentration, particle in front of the crack tip may cleave or lose coherence with the matrix, thus initiating a (large) void. The remaining material between the void and the crack tip now may rupture by ductile tearing, thus producing a local jump of the crack front.

The influence of particles on fatigue crack propagation is limited to high crack propagation rates. This means that it is limited to the very last and small part of the crack propagation life.

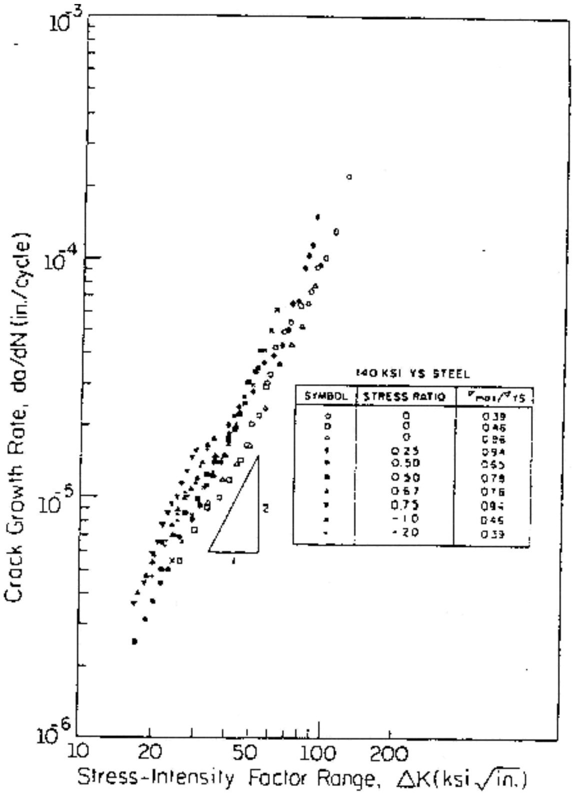
2. What are the various factors that influences the fatigue crack growth?

Solution

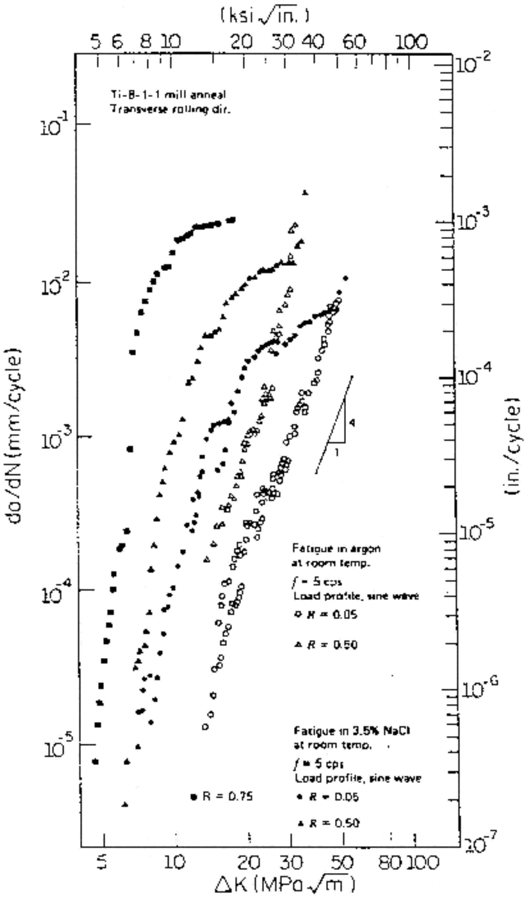
(i) Stress ratio effects
(ii) Environmental effects
(iii) Frequency of loading
(iv) Temperature effects
(v) Wave form of loading cycle



Influence of R on fatigue crack growth in Ti–6Al–4V.



No major influence of *R* on fatigue crack growth in 140-ksi yield strength steel.

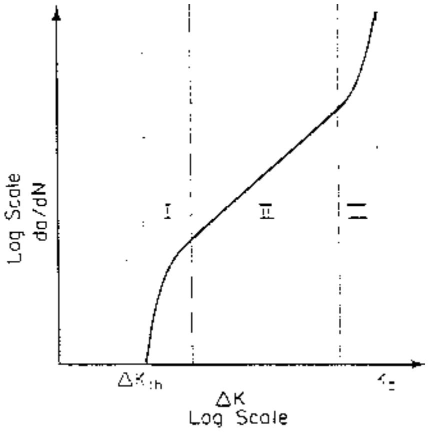


Effect of load ratio *R* on fatigue crack propagation in Ti–8Al–1Mo–1V alloy. Tests conducted in 3.5% NaCl solution and in argon.

3.With neat sketches, explain different modes of crack growth.

Solution

A plot of log da/dN versus log ΔK, a sigmoidal curve is shown in figure.



Three regions of crack growth rate curve.

In the mid-region, Region II, the curve is essentially linear. Many structures operate in this region. Finally, in Region III, at high ΔK values, crack rates are extremely high and little fatigue life is involved. These three regions are discussed in detail in the following sections.

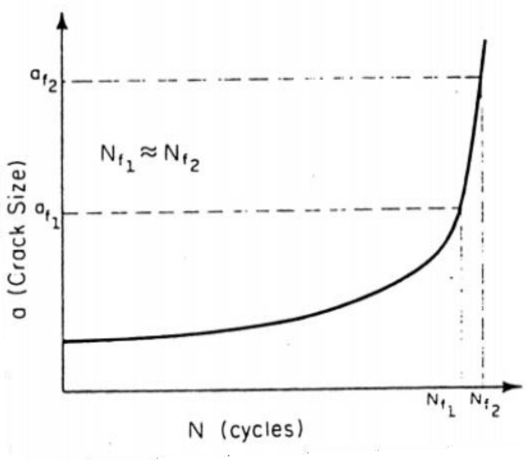
Region II

$$K = f(g)\sigma\sqrt{\pi a}$$

$$a_f = \frac{1}{\pi}\left[\frac{K_c}{\sigma f(g)}\right]^2$$

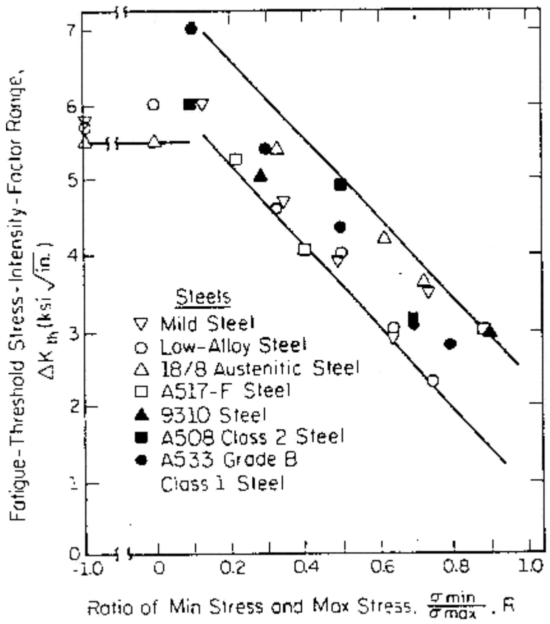
$$= \frac{1}{\pi}\left(\frac{K_c}{1.12\sigma_{\max}}\right)^2$$

Where a_f = final crack size



Effect of final crack size on life.

Region I



Dependence of fatigue-threshold stress-intensity-factor range on stress ratio.

$$\frac{da}{dN} = \frac{C\,\Delta K^m}{(1 - R)K_c - \Delta K}$$

4.Explain the informations you may gat about the materials from the fatigue fracture surfaces.

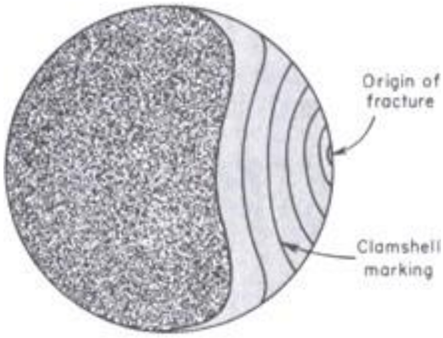
Solution

Fatigue fracture surfaces

Fatigue failures exhibits very small permanent visual deformation.

Typical fatigue failures exibit the following common aspects.

(i) Distinct crack initiation sight
(ii) Beach mark (or) clamshell marks indicative of crack growth
(iii) Distinct final fracture region



Beachmarks or "clamshell pattern" associated with stress cycles that vary in magnitude and time as in factory machinery

Torsional fatigue surface

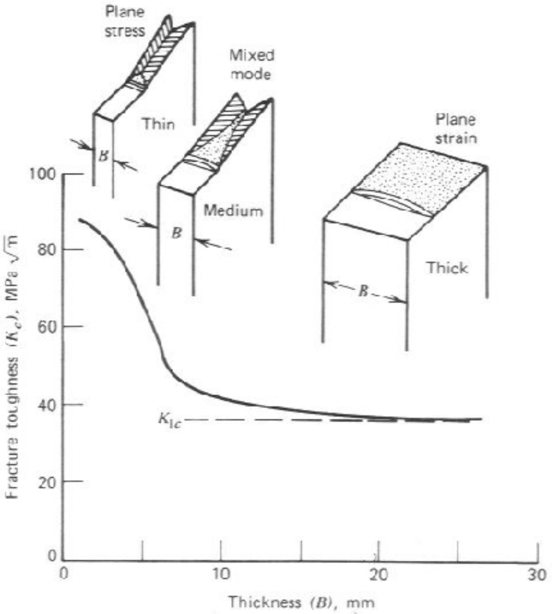
In torsional fatigue failure the cracks appear on 45˚ inclination plane.This is the plane of max. tensile stress which again indicates that fatigue cracks propagate primarily in the plane of max. tensile stress.

5. What are the various factors influencing fatigue behaviour?
   (i) Type and nature of loading
   (ii) Size of component and stress strain distribution
   (iii) Surface finish and directional properties
   (iv) Mean stress or Є
   (v) Stress or strain concentrations
   (vi) Environmental effects
   (vii) Metallurgical factors and properties
   (viii) Strain rate and frequency effects

Unit 4

Fracture Mechanics

1. Explain the effect of thickness on fatigue toughness. (8 Marks)



Effect of specimen thickness on fracture toughness.

The general relationship between fracture toughness, K_C, and thickness is shown in Fig. The appearance of the fracture surface accompanying the different thicknesses is also shown schematically for single edge notch specimens. The beach markings at the initial crack tip represent fatigue pre-cracking at a low cyclic stress intensity factor range to ensure a sharp crack tip. The fracture toughness values would be higher for dull or notch type crack fronts. It is seen that thin parts have a high K_c value accompanied by appreciable" shear lips" or slant fracture. This type of fracture is high energy fracture. As the thickness is increase, the percentage of shear lips or slant fracture decreases, as does K_C. This type of fracture appearance is called "mixed mode", implying both slant and flat fractures. For thick parts essentially the entire surface is flat and K_C approaches an asymptotic minimum value. Any further increases in thickness does not decreases the fracture toughness, nor does it alter the fracture surface appearance. The minimum value of fracture toughness is called "plane strain fracture toughness", K_{Ic}. The subscript I refers to the fact that these fracture occur almost entirely by the mode I crack opening. The term "plane strain" is incorporated here since flat fractures best approach a true plane strain constraint through out most of the crack tip region. For thin sections where appreciable shear lips occur, the crack tip region most closely experiences a plane strain situation. Thus, plastic zone sizes at fracture are much larger in thin parts in thick parts. Plane strain fracture toughness K_{Ic} is considered a true material property because it is independent of thickness. However in order for a plane strain fracture toughness value to be considered valid, it is required that

$$a \text{ and } t \geq 2.5\left(\frac{K_{Ic}}{S_y}\right)^2$$

2. Derive an expression for plane stress and plane strain using Griffith's theory.

(OR)

Derive the expression for fracture stress using Griffith's theory.

<u>Solution</u>

$$Released\ energy, E_R = (Volume\ of\ triangle) \times \left(\frac{\sigma^2}{2E}\right)$$

$$\sigma_c = \left[\frac{2E_\gamma}{\pi(1-\vartheta^2)a}\right]^{1/2}$$

3. Starting from first principles prove the equivalence of j integral and g, the strain energy release rate.

<u>Solution</u>

Potential Energy,

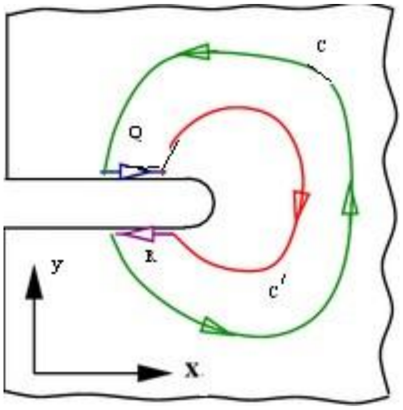
$$\emptyset = \int_A W dA - \int_{\Gamma_T} T_i\, u_i d_s$$

$$G = \int_\Gamma W d_{x2} - \int_\Gamma T_i \frac{\partial u_i}{\partial x_1} d_s$$

The right hand side has the expression of J which is path independent and therefore G=J

Thus, for linear elastic materials, expression of J-integral is restatement of G.

4. starting from J-integral prove the equivalence of J=G, strain energy release rate for plane stress and strain conditions.



$$J = \int_C \left(W dy - T\frac{\partial u}{\partial x} ds\right)$$

Where x,y=rectangular coordinates normal to the crack front (see Fig.)

ds = increment along contour C

T= stress vector acting on the contour

u= displacement vector

W= strain energy density= $\int \sigma_{ij}\, d\epsilon_{ij}$

J= crack driving force

Therefore Plane stress

$$J = G = \frac{K^2}{E}$$

Plane strain

$$J = G = \frac{K^2}{E}(1-\vartheta^2)$$

5. Obtain the strain energy release rate on a plate with crack subjected to a tensile load.

(OR)

Derive the strain energy release rate for and edge cracked plate under uniaxial tensile load.

<u>Solution</u>

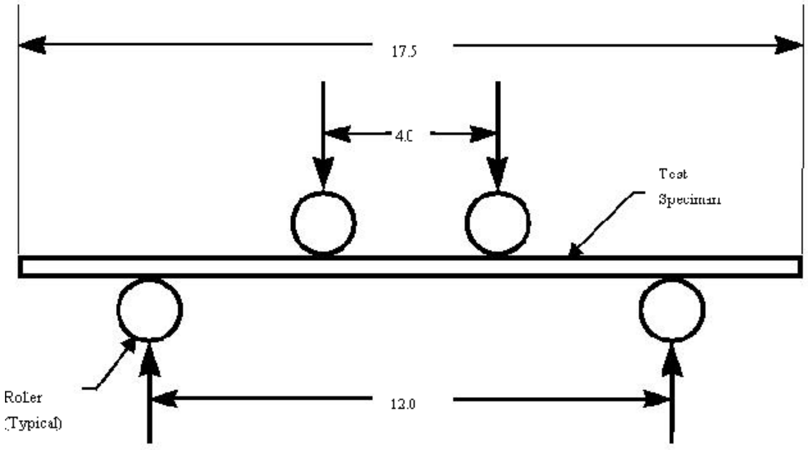
Available energy $G\Delta A=\Delta W_{ext}-\Delta U$

$$G = \frac{P^2}{2B}\frac{dC}{da}$$

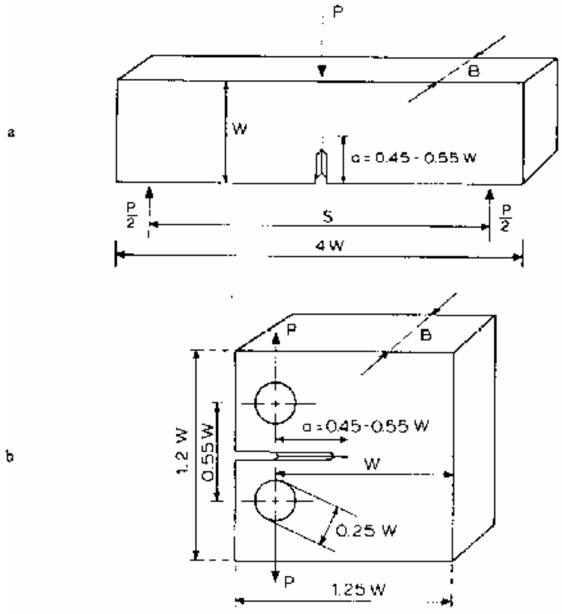
Unit5

Fatigue Design and Testing

1. Sketch the following specimens geometry as per standards.
(a) Three point bend specimen
(b) Four point bend specimen
(c) Compact tension specimen

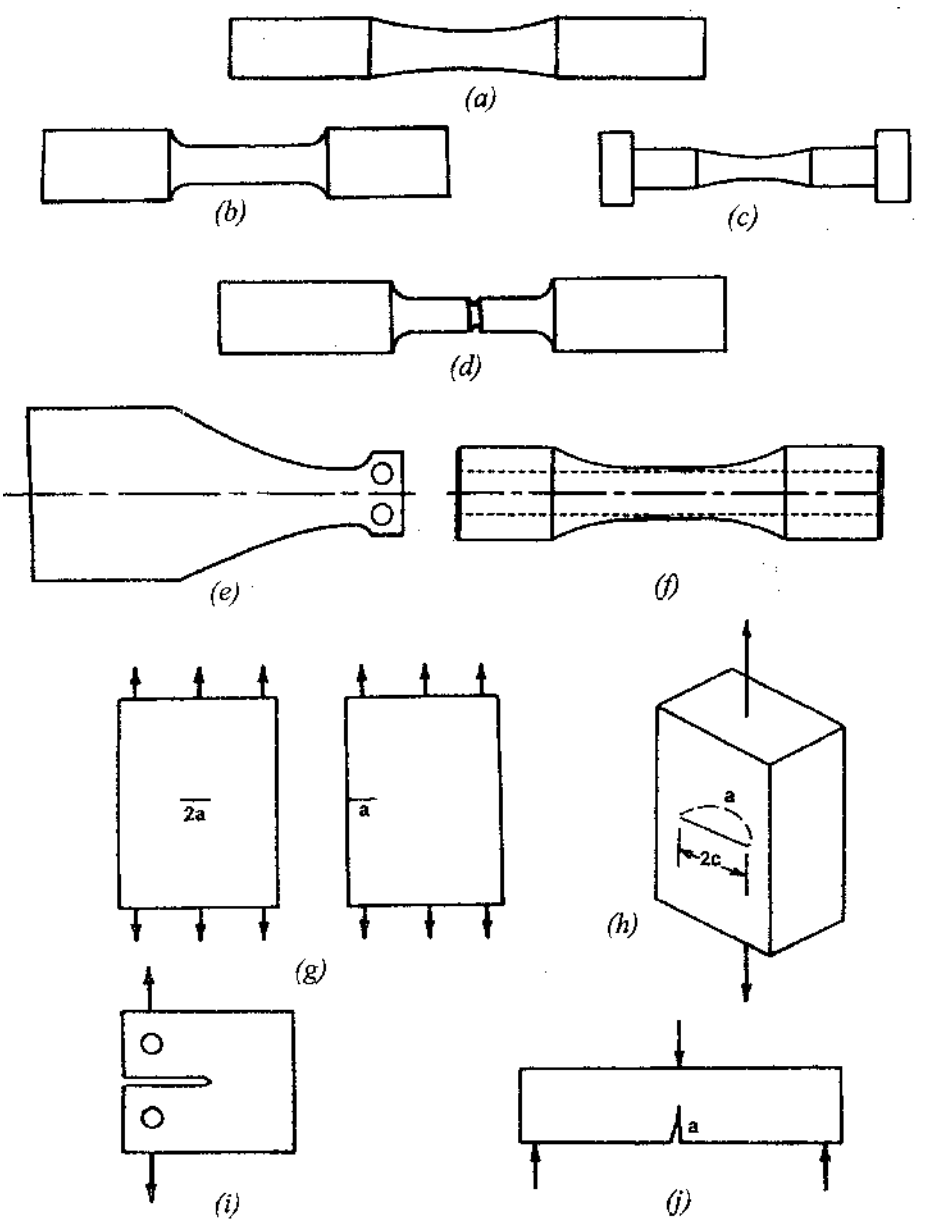


Schematic illustration of center notch four-point bend test(Dimensions in inches).



Standard specimens
a. Bend specimen; b. Compact tension specimen

2.Sketch a few fatigue test specimens.



Fatigue test specimens. (*a*) Rotating bending. (*b*) Axial uniform. (*c*) Axial hourglass. (*d*) Axial or bending with circumferential groove. (*e*) Cantilever flat sheet/plate. (*f*) Tubular combined axial/torsion with or without internal/external pressure. (*g*) Axial cracked sheet/plate. (*h*) part-through crack. (*i*) Compact tension. (*j*) Three-point bend.

3.Discuss the following:
(i) Probit method
(ii) PROT method

Solution

Probit method :This method one in which several specimens are tested for a given no.of cycles at each of4 or 5 stress values near the stress of interest first the desired life is determined, for example 10^7cycles.

PROT method : Test a no. of specimens starting from a low stress of about 60% to 70% of its estimated fatigue strength stress in increased continuously in each cycle at a constant rate will fracture, then test a no. of specimens at a different rate of increase in loading, starting from the same low stress.

4. What is the need for Fracture mechanics in design of aircraft components?

An estimation of the number of frequency and magnitude of the fluctuating loads an aircraft encounters is necessary.

During taxiing the aircraft may be manoeuvring over uneven ground with a full payload so that wing stresses for example are greater than in the static case.

The loads corresponding to these various phases must be calculated before the associated stresses can be obtained.

To determine the no. of load fluctuations during a ground-air-ground cycle carried by standard operations.

Since an aircraft is subjected to the greatest no. of load fluctuations during taxi-take off-climb and descent standoff- landing while little damage is carried during cruise, fatigue life does not depend by flying hours but on the no. of flights.

Operated requirements of aircraft is differ from class to class.

5. Discuss a few fatigue problems encountered in subsonic and supersonic aircraft.

To design and construct efficient fatigue resistant supersonic transport (SST) and subsonic transport structures reliable for 50,000 hrs in the severe environment (particularly in Mach 3 condition) is a considerable challenge. The successful operation of the aircraft in this severe environment generally depends on the following factors.

(i) Temp. variation of above -56˚C to 2000˚C.
(ii) A combination of mechanical and thermal stresses
(iii) Up to 50000 hrs of operating life under the above conditions.

As a first step in providing optimum fatigue resistance as well as many other functional requirements data on

(i) Service loads
(ii) Temp
(iii) Atmosphere conditions

In addition to these general problems, several examples of more specific fatigue data are involved in the design of SST.

(a) Conventional; fatigue data at above -56˚C, 30˚C and 1000˚C.

(b) Extrapolation of short and long term fatigue since 3000 hrs and 5000 hrs.
(c) Simultaneous load and temperature effects
(d) Fail safe designs, crack arrestors and redundant structures for high reliability.
(e) At cruising altitudes (about 17,000 ft) the ozone atmosphere may impact the fatigue resistance of metals.
(f) Because the wing skin acts more like a 2D plane than a 1D beam surface and the fuel tanks and the fuselage are internally pressurized there are bi-axial stresses fatigue data under bi-axial stresses are required.
(g) Prevention of explosive failure of pressurized fuselage.
(h) Control of cracking due to sonic fatigue.
(i) Prevention of cumulative damage.

* * *