AERODYNAMICS 2 LECTURE NOTES @ WINGS OF AERO

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UNIT TEST I

PART-A

1. What is isentropic and isothermal compressibility?

   If the temperature of the fluid element is held constant by some heat transfer mechanism, then that compressibility is defined as isothermal compressibility (1)

$$\beta_T = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_T$$

   If no heat is added or taken away from the fluid element and if the friction is ignored, the compression of the fluid element takes place isentropically and that compressibility is defined as isentropic compressibility (1)

$$\beta_S = -\frac{1}{V}\left(\frac{\partial V}{\partial p}\right)_S$$

2. Define characteristic Mach number and what is the maximum value of it?

   It is defined as the ratio between the velocity of the object and the critical velocity of sound (1)

$$M^* = \frac{\text{velocity of the object}}{\text{critical velocity of sound}}$$

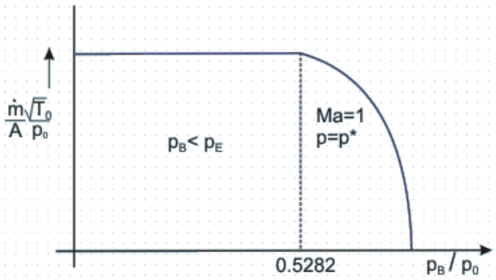
   Maximum value for $M^* = \sqrt{\frac{\gamma+1}{\gamma-1}}$

   For $\gamma = 1.4,\ M^*$ =2.45 (1)

3. Distinguish between thermally perfect gas and calorically perfect gas?
   - If the specific heat capacity is a constant value, the gas is said to be calorically perfect gas. (1)
   - A gas that follows the ideal equation of state is said to be thermally perfect gas (1)

4. Why is a convergent divergent nozzle required to expand a flow from stagnation condition to supersonic velocity?
   - Converging section – stagnation velocity to sonic velocity
   - Diverging section- large decrease in density of the fluid which makes acceleration in the divergent section to achieve mach numbers >1 (2)

5. Explain the phenomenon of choking in a nozzle?
   - Mass flow rate remains constant even for further decrease in the back pressure.(1)



(1)

6. Define nozzle efficiency in terms of enthalpies?

   The nozzle efficiency, η_n, is defined as the ratio of the actual enthalpy drop to the isentropic enthalpy drop

   (1)

$$\eta = \frac{\text{actual enthalpy drop}}{\text{isentropic enthalpy drop}}$$

$$\eta = \frac{h_{01} - h_2}{h_{01} - h_{2s}}$$

   h_{01}- stagnation enthalpy at the nozzle inlet
   h_2- enthalpy at the exit for actual nozzle
   h_{2s}- enthalpy at the exit for nozzle under isentropic conditions
   (1)

7. Write the one-dimensional energy equation for an adiabatic compressible steady flow?

   Integral form

   (1)

$$\iiint \rho q dv + Qvis + \iiint \rho fV dv - \iint PVds + Wvis = \iint \rho Vds(e + \frac{v^2}{2}) + \frac{\partial}{\partial t} \iiint \rho dv(e + \frac{v^2}{2})$$

   Differential form

   (1)

$$\rho q + Qvis + \rho fv - \nabla Pv + Wvis = \nabla \rho v\left(e + \frac{v^2}{2}\right) + \frac{\partial \rho}{\partial t}\ (e + \frac{v^2}{2})$$

8. Write down the Bernoulli's equation for compressible flow?

$$\frac{v^2}{2} + \left(\frac{\gamma}{\gamma - 1}\right)\frac{p}{\rho} = \left(\frac{\gamma}{\gamma - 1}\right)\frac{p_0}{\rho_0}$$

   Where P_0 is the total pressure
   ρ_0 is the total density
   (2)

9. Write the Area Mach number relation?

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\left\{\frac{2}{\gamma+1}\right\}\left\{1 + \frac{(\gamma-1)}{2}Ma^2\right\}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

   Where A is the area of the nozzle at any section
   A* is the area at the nozzle throat
   M is the local Mach number
   (2)

10. Derive the relation $\frac{T_0}{T} = \left[1 + \left(\frac{\gamma-1}{2}\right)\right] M^2$

$$h_0 = h + \frac{V^2}{2}$$

$$T_0 = T + \frac{V^2}{2Cp}$$

Sub for Cp, a^2 and M

$$\frac{T_0}{T} = \left[1 + \left(\frac{\gamma-1}{2}\right)\right] M^2 \qquad (2)$$

PART-B

11. A. Starting from the energy equation for an adiabatic process, show that

$$M^2 = \frac{2}{\left(\frac{\gamma+1}{M*2}\right) - (\gamma-1)}$$

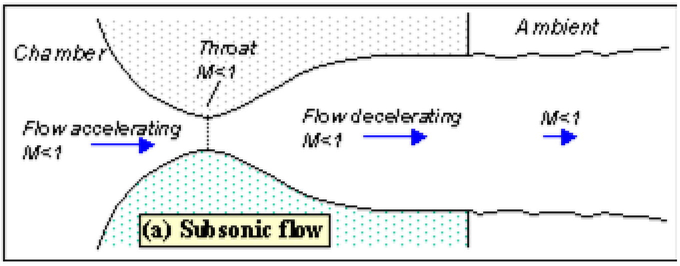
- ❖ Definition
  (2)
- ❖ Energy equation
  (1)
- ❖ $M^2 = \frac{2}{\left(\frac{\gamma+1}{M*2}\right) - (\gamma-1)}$
  (5)

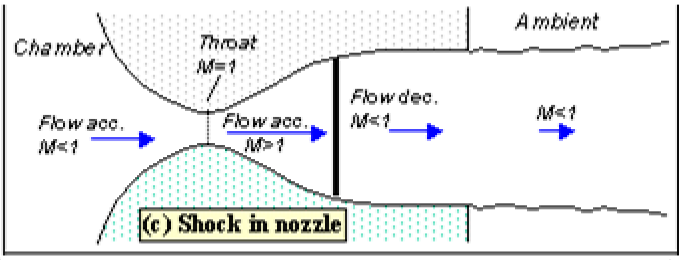
B. Derive energy equation for one-dimensional steady compressible flow from first principle.

- ❖ Principle and Rate of heat added
  (2)
- ❖ Rate of work done
  (2)
- ❖ Rate of change of internal energy
  (2)
- ❖ Integral form and Differential form
  (2)

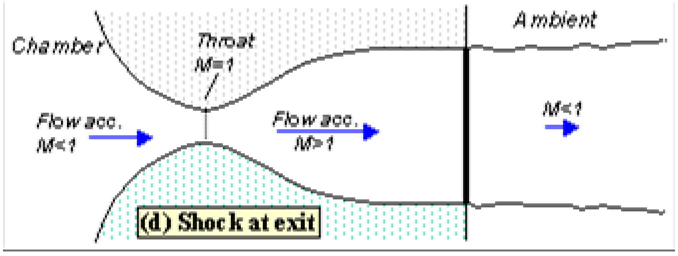
12. Explain the term choking in a C-D nozzle and the flow conditions leading to the same. Hence describe with illustrations a) An under expanded b) Correctly expanded c) Over expanded nozzles

- ❖ CD nozzle introduction and overall performance curve
  (4)
- ❖ Nozzle performance (subsonic, supersonic)
  (2)



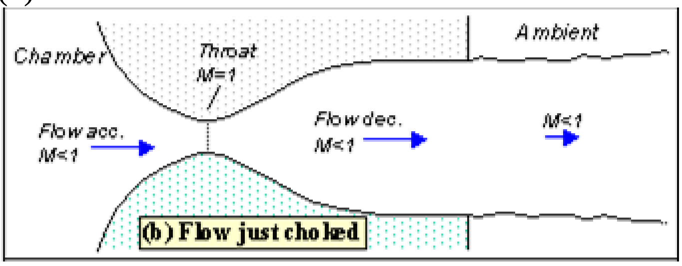


(c) Shock in nozzle



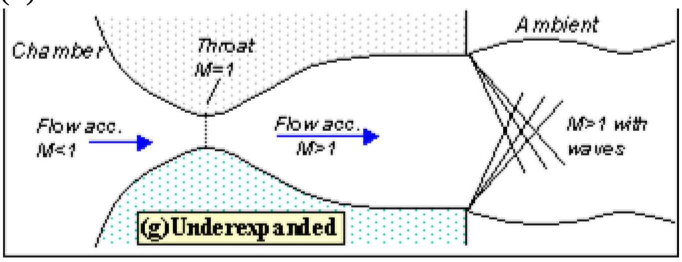
(d) Shock at exit

❖ Choking condition
(2)



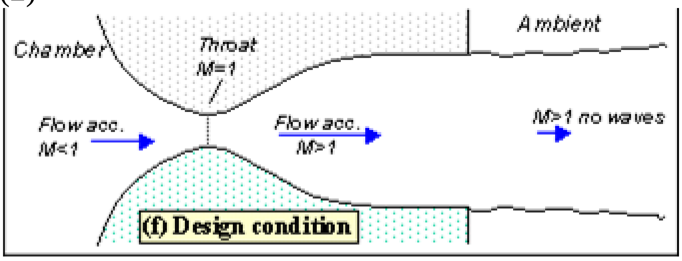
(b) Flow just choked

❖ under expanded nozzles
(3)



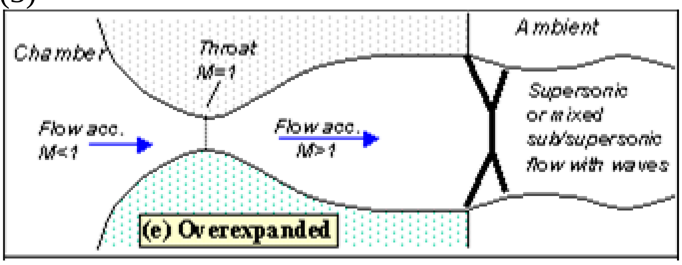
(g) Underexpanded

❖ Correctly expanded nozzles
(2)



(f) Design condition

❖ Over expanded nozzles
(3)



(e) Overexpanded

13. A. When the stagnation properties are constant show that for isentropic flow of air through a duct, the maximum mass flow can be given by $m_{max} = \frac{0.6847\ P_0 A^*}{\sqrt{RT_0}}$

- Nozzle diagram
  (1)
- Find density and velocity
  (2)
- Maximum mass flow rate
  (4)
- $m_{max} = \frac{0.6847\ P_0 A^*}{\sqrt{RT_0}}$
  (1)

B. A De Laval Nozzle has to be designed for an exit Mach number of 1.5 with exit diameter of 200mm. Find the ratio of throat area to exit area necessary. The reservoir conditions are given as P_0 = 1 atm; T_0 = 200C. Find also the maximum mass flow rate through the nozzle. What will be the exit pressure and temperature?

GIVEN

P_0 = 1 atm
T_0 = 200°C
M_2 =1.5
D_2 =200mm

TO FIND

$\frac{A*}{A2}$, maximum mass flow rate

SOLUTION (1)

From gas tables

P_2=2.75*10^5N/m^2
T_2=352.8K (2)
A_2=0.0314 m^2
A*=0.0267 m^2 (3)
M_{max}=5.027 kg/s (2)

RESULT

14. A. A supersonic nozzle expands air from P_o=25bar and T_o=1050K to an exit pressure of 4.35bar; the exit area of the nozzle is 100 cm2. Determine a) Throat area, b) Pressure and temperature at the throat c) Temperature at exit d) Exit velocity e) Mass flow rate

GIVEN

P_o=25bar
T_o=1050K
P_2=4.35bar
A_2=100 cm^2

TO FIND

Throat area, Pressure and temperature at the throat, Temperature and velocity at exit, Mass flow rate

SOLUTION

(1)

$\frac{P_2}{P_{02}} = 0.174$

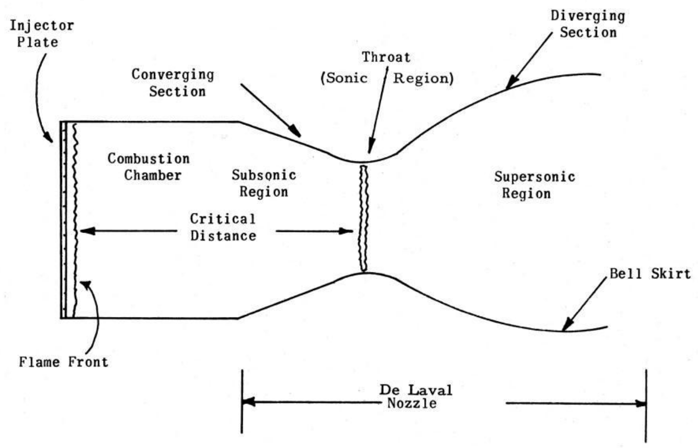
From Gas Tables

M_2=1.80
T_2=637.35K
A*=6.949*10^{-3} m^2
(1)
P*=13.2*10^5 N/m^2
T*=875.7 K (2)
m=21.59 kg/s (2)
$\frac{C_2}{C_{max}} = 0.0523$ (2)

RESULT

B. Define De Laval Nozzle and derive the Area Mach number relation.

De Laval Nozzle is a tube used to accelerate a hot, pressurized gas passing through it to a supersonic speed, and upon expansion, to shape the exhaust flow so that the heat energy propelling the flow is converted into directed kinetic energy.



(2)

- Continuity equation (1)
- Density and velocity ratio (2)

$$\frac{A}{A^*} = \frac{1}{Ma}\left[\left\{\frac{2}{\gamma+1}\right\}\left\{1 + \frac{(\gamma-1)}{2}Ma^2\right\}\right]^{\frac{\gamma+1}{2(\gamma-1)}}$$

(3)

15. A. Air flows isentropically through a convergent divergent nozzle of inlet area 12cm^2 at a rate of 0.7 kg/s. the conditions at the inlet and exit of the nozzle are 8 kg/m^3 and 400K and 4kg/m^3 and 300K respectively. Find the cross sectional area, the pressure and the Mach number at the nozzle exit

GIVEN

A_1=12cm^2
m=0.7 kg/s
$\rho_1 = 8$ kg/m^3
T_1=400K
ρ_2=4kg/m^3
T_2=300K

TO FIND

Cross sectional area, pressure and Mach number at exit

SOLUTION (1)

V_1= 72.916 m/s
M_1=0.1818

T_{01}=402.414 K
P_1=918.4*10^3 N/m^2
P_{01}=939.059*10^3 N/m^2
A*=3.66*10^{-4} m^2
$\frac{T_2}{T_{02}} = 0.745$ (3)

From Gas Tables

M_2=1.31
P_2=0.356*939.059*10^3 N/m^2
A_2=1.071*3.66*10^{-4} m^2 (4)

B. Obtain an expression for velocity of sound on terms of specific heats and local temperature in air medium from one dimensional continuity, momentum and energy equations.

- Definition and diagram (1)
- $a = \sqrt{\frac{dp}{d\rho}}$ (3)
- $a = \sqrt{\gamma RT}$ (3)
- Dependant factors (1)

UNIT TEST II
PART-A

1. Explain why shocks cannot occur in subsonic flows?

In subsonic flows, the velocity of the object is less than the velocity of sound, and the object will not have much pressure to compress the fluid, so shocks cannot occur in subsonic flows

(2)

2. Explain zone of action and zone of silence for a body moving at a speed of sound?

The region on the left of the wave front i.e. the region downstream of the source of disturbance is called as zone of silence because the shock waves cannot reach this zone. The region on the right of the wave front i.e. the region upstream of the source of disturbance is called as zone of action because the shock waves will be created in this zone.

(1)

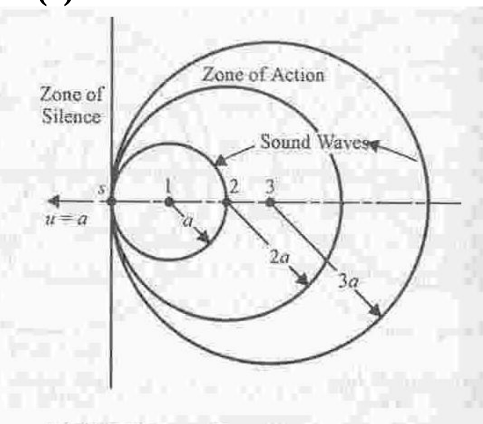


FIGURE 5.4 (c) : Sonic flow ($a = u$, $M = 1.0$) **(1)**

3. Write the Hugonoit equation and draw a typical Rankine-Hugoniot curve and explain it?

$$e_2 - e_1 = \frac{1}{2}(P_1 + P_2)(\frac{1}{\rho_1} - \frac{1}{\rho_2})$$

$$= \frac{1}{2}(p_1 + p_2)(v_1 - v_2)$$

(1)

The curve which shows the relation between pressure and specific volume of the compression process is called as Rankine-Hugoniot curve

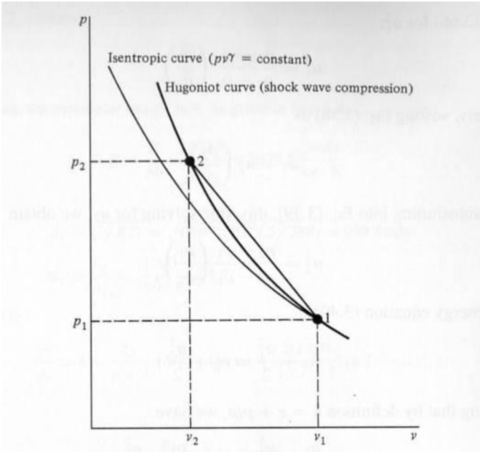


Figure 3.11 | Hugoniot curve; comparison with isentropic compression.

(1)

4. What is the need for a correction to the Pitot static tube readings in supersonic flow and write Rayleigh supersonic Pitot formula?

When a pitot static tube is immersed in a supersonic flow, shock waves will be formed in front of the nose of the tube. The pressure measurement will not be accurate. So corrections should be done.

(1)

$$\frac{p_{0,2}}{p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right)^{\gamma/(\gamma-1)} \frac{1-\gamma+2\gamma M_1^2}{\gamma+1}$$

(1)

5. How is flow over a cone different from flow over a wedge?
   - Flow over a cone is 3D and in wedge its 2D
   - Pressure over the surface of the cone is less but in wedge, pressure is more **(1)**

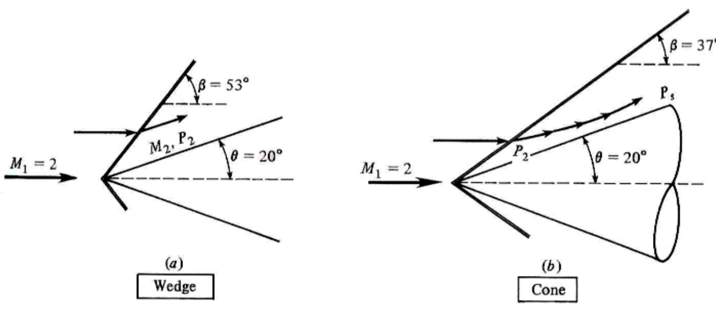


Figure 4.11 | Comparison between wedge and cone flow; illustration of the three-dimensional relieving effect.

(1)

6. Give the oblique shock relation in terms of flow angle and wave angle?

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta - 1}{M_1^2(\gamma + \cos 2\beta) + 2}\right]$$ **(1)**

Where θ is the flow deflection angle

β is the shock angle

M is the Mach number **(1)**

7. What is shock polar? Draw the shock polar for different Mach numbers?

The locus of all the points for θ values ranging from zero to maximum representing all possible velocities behind the shock wave is called as shock polar. **(1)**

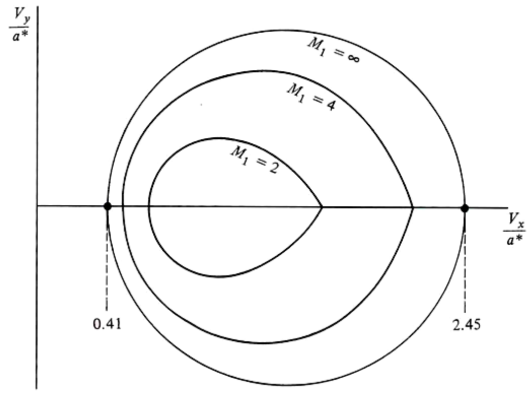


Figure 4.17 | Shock polars for different Mach numbers.

(1)

8. Explain why a supersonic airplane is not given a blunt nose?

When a blunt nose airplane is flying in air at supersonic speeds, bow shocks will be formed in front of the nose which will reduce the velocity of the aircraft. So for this reason, supersonic airplane is not given a blunt nose.

(2)

9. Define pressure turning angle and Hodograph Plane?

The locus of all possible static pressures behind an oblique shock wave as a function of deflection angle for any given upstream conditions is called as pressure turning angle. **(1)**

The plane which uses velocity components as the coordinates of the system is referred to as Hodograph Plane. **(1)**

10. Define the strength of a shock wave? Explain the shocks of vanishing strength?

It is the ratio between the difference in downstream and upstream pressure to the upstream pressure.

$$\xi = \frac{P_y - P_x}{P_x}$$ **(1)**

When the strength of the shock wave is equal to zero, that shock is called as the shocks of vanishing strength which happens at M=1. **(1)**

PART-B

11. Derive the continuity, momentum and energy equation for one dimensional flow through a normal shock wave. Using these equation, derive the Prandtl relation in the form $M_2{}^* = \frac{1}{M^*}$ where subscripts 1 and 2 refers to upstream and downstream of the

shock and M_1 and M_2 are the corresponding speed ratios
[16]

a) Introduction, Diagram, Assumptions **(2)**

b) Continuity equation **(2)**

$$\rho_1 u_1 = \rho_2 u_2$$

c) Momentum equation **(2)**

$$p_1 + \rho_1 {u_1}^2 = p_2 + \rho_2 {u_2}^2$$

d) Energy equation **(2)**

$$h_1 + \frac{{u_1}^2}{2} = h_2 + \frac{{u_2}^2}{2}$$

e) Prandtl equation

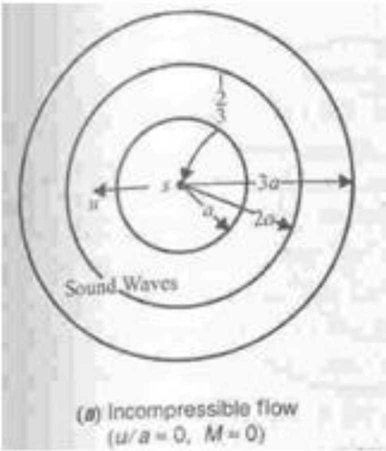
i. Introduction, Diagram, basic equations **(1)**

ii. $u_2 - u_1 = a^2$ **(6)**

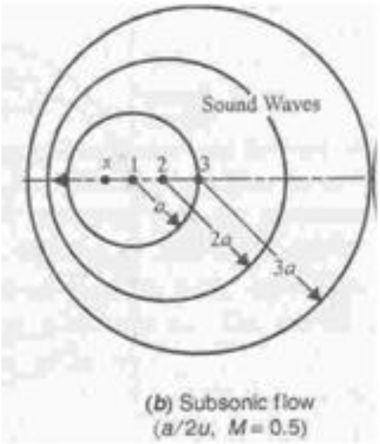
iii. ${M_1}^* {M_2}^* = 1$ **(1)**

12. A. With a neat diagram of wave propagation, explain mach angle and mach cone.
[8]

a) Incompressible flow explanation



b) Subsonic flow explanation



(b) Subsonic flow
(a/2u, M = 0.5)

(2)

c) Sonic flow explanation **(2)**

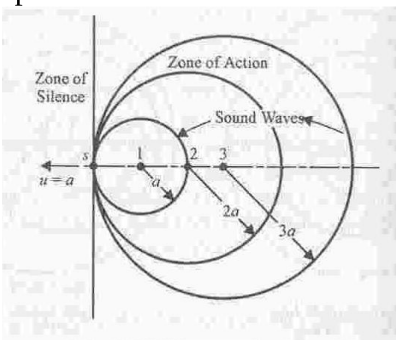
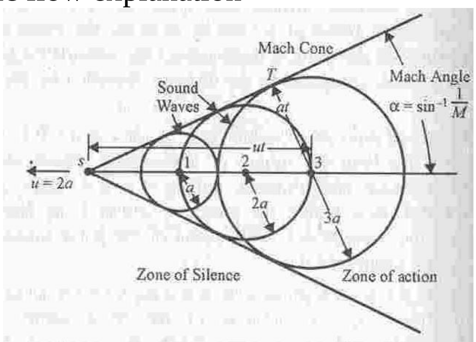
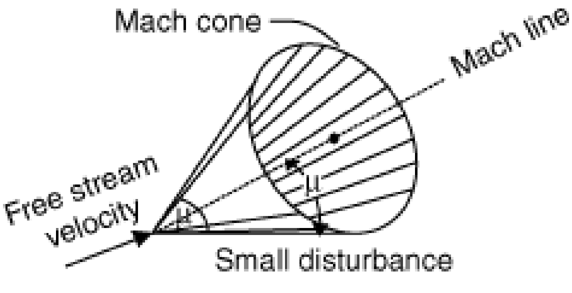


FIGURE 5.4 (c) : Sonic flow (a = u, M = 1.0)

d) Supersonic flow explanation **(2)**



The area bounded by the sides of the cone-shaped shock wave produced by a sharp pointed object moving through the atmosphere at a speed greater than Mach 1 is called as Mach cone.



The angle created by the mach cone with the horizontal line is called as mach angle.

$$\mu = \arcsin\left(\frac{1}{M}\right),$$ **(2)**

B. Derive the Rankine Hugonoit pressure density relationship for the shocks [8]

a) Definition and basic equations **(2)**

b) Derivation

$$e_2 - e_1 = \frac{1}{2}(P_1 + P_2)\left(\frac{1}{\rho_1} - \frac{1}{\rho_2}\right)$$ **(6)**

c) R-H curve $= \frac{1}{2}(p_1 + p_2)(v_1 - v_2)$

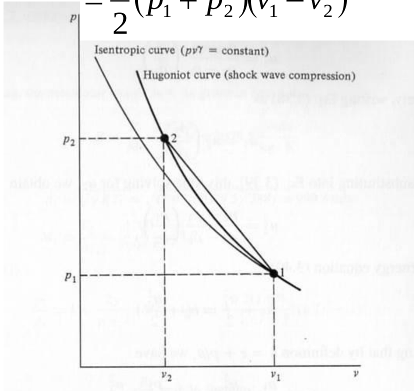
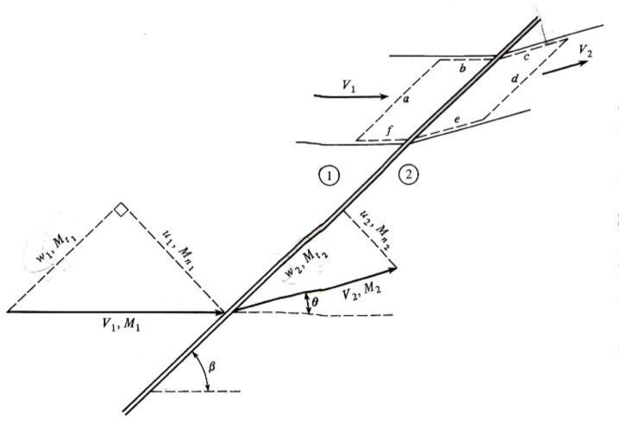


Figure 3.11 | Hugoniot curve; comparison with isentropic compression.

(2)

13. Derive a relation connecting flow turning angle, shock angle and free stream Mach number for oblique shock waves. [16]

a) Oblique shock introduction and diagram **(2)**



b) basic equations **(4)**

i. Continuity equation

$$\therefore \rho_1 u_1 = \rho_2 u_2$$

ii. Momentum equation

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

iii. Energy equation

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

c) $$\therefore \frac{\tan\beta}{\tan(\beta-\theta)} = \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)Mn_1^2}{(\gamma-1)Mn_1^2+2} = \frac{(\gamma+1)M_1^2\sin^2\beta}{(\gamma-1)M_1^2\sin^2\beta+2}$$ **(3)**

d) $$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}\right]$$ **(3)**

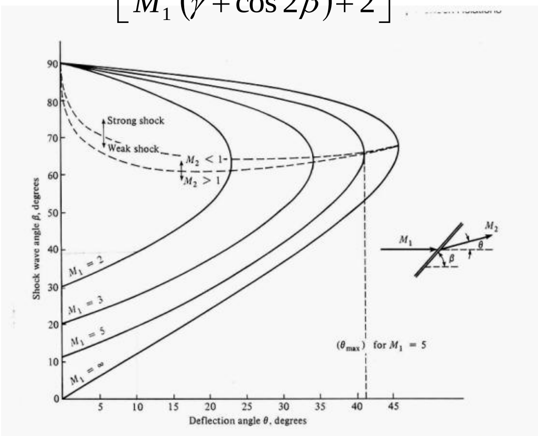


Figure 4.8 | θ-β-M curves. Oblique shock properties. *Important:* See front end pages for a more detailed chart.

e) Inference of the result **(4)**

i. Straight and curved shocks
ii. Attached and detached shocks
iii. Strong and weak shocks
iv. Increasing upstream mach number
v. Increasing deflection angle

14. A. Sketch a Pitot-static tube and explain how you will measure subsonic and supersonic velocities. Derive an expression for the correction factor to be applied to Pitot-static probe readings in compressible subsonic flows.
[8]

a) Purpose and diagram **(2)**

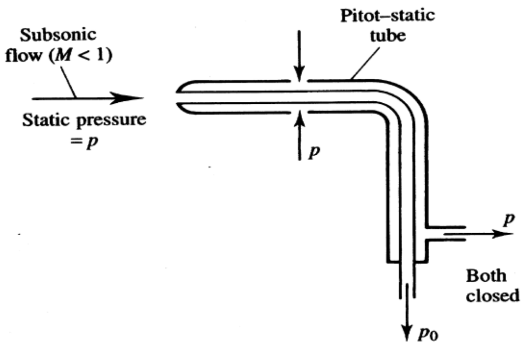


FIGURE 4.10
Flow over a pitot-static tube.

b) Subsonic compressible flow **(2)**

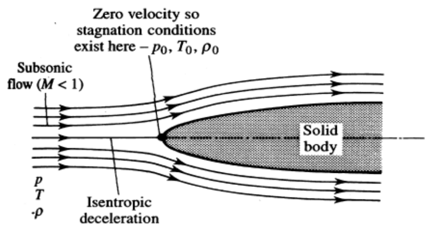


FIGURE 4.7
Stagnation conditions at leading edge of submerged body.

c) Supersonic compressible flow **(3)**

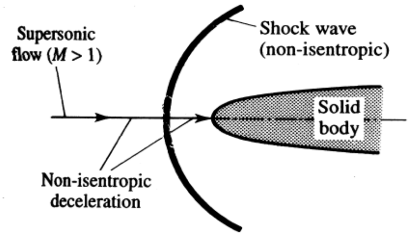


FIGURE 4.8
Supersonic flow near leading edge of submerged body.

Rayleigh correction formula

$$\frac{p_{0,2}}{p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right)^{\gamma/(\gamma-1)} \frac{1-\gamma+2\gamma M_1^2}{\gamma+1}$$

d) Limitations **(1)**

1. Don't work well at low speeds because pressure difference is very small
2. At supersonic speeds, shock waves are formed in front of the tube
3. Ice will be formed at low speeds

B. A normal shock wave is standing in the test section of a supersonic wind tunnel. Upstream of the wave, M_1=4, P_1= 0.4atm and T_1= 210K. Find M_2, T_2, T_{02}, P_2 and u_2 downstream of the wave.
[8]

GIVEN DATA
M_1=4
P_1= 0.4atm
T_1= 210K
TO FIND
M_2, T_2, T_{02}, P_2 and u_2
RESULT **(1)**

SOLUTION
M_2 = 0.435 **(1)**
T_2 = 849.87 K
P_2 = 7.4 atm **(3)**
u_2 = 254.19 m/s
T_{02} = 882.03 K **(3)**

15. A. Briefly explain the Shock Polar
[8]

a) Definition **(1)**

b) Physical plane and hodograph plane **(1)**

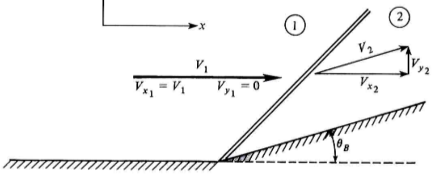


Figure 4.13 | The physical (xy) plane.

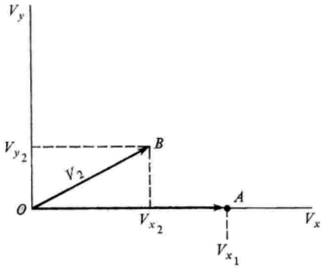


Figure 4.14 | The hodograph plane.

c) V_x Vs V_y graph **(1)**

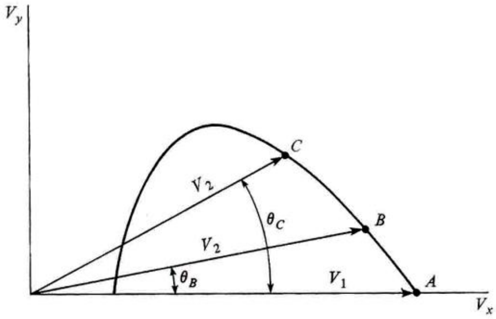


Figure 4.15 | Shock polar for a given V_1.

d) Using M* **(1)**

e) Shock polar equation **(1)**

$$\left(\frac{V_y}{a^*}\right)^2 = \frac{\left(M_1^* - V_x/a^*\right)^2\left[\left(V_x/a^*\right)M_1^* - 1\right]}{\frac{2}{\gamma+1}\left(M_1^*\right)^2 - \left(\frac{V_x}{a^*}\right)M_1^* + 1}$$

f) M_x^* Vs M_y^* **(1)**

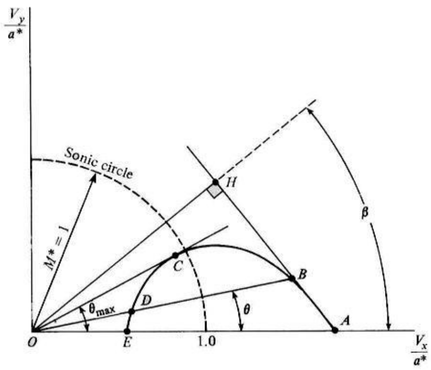


Figure 4.16 | Geometric constructions using the shock polar.

g) Inference of the results **(1)**

h) Various mach numbers **(1)**

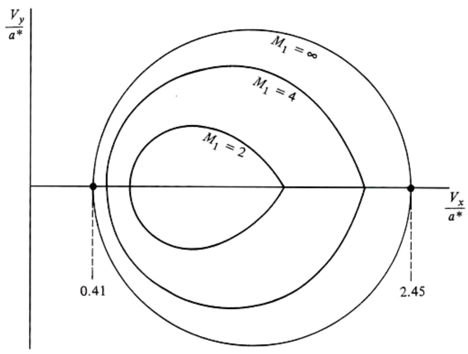


Figure 4.17 | Shock polars for different Mach numbers.

B. The following data refers to a supersonic wind tunnel

Nozzle throat area = 200cm^2

Test section cross section = 337.5cm^2

Working fluid: gases [$\gamma = 1.4, R = 0.287\ KJ/kgk$]

Determine the test section Mach number and the diffuser throat area if a normal shock is located in the test section

[8]

GIVEN DATA

A_x*= 200cm^2

A_x= 337.5cm^2

$$\gamma = 1.4, R = 0.287\ KJ/kgk$$

TO FIND

Test section Mach number

Diffuser throat area

RESULT **(1)**

SOLUTION

Find $\frac{A_x}{A_x{}^*}$ **(1)**

From isentropic table Mx = 2

From normal shock table $\frac{P_{oy}}{P_{ox}} = 0.721$ **(2)**

$$A_x{}^* P_{ox} = A_y{}^* P_{oy}$$

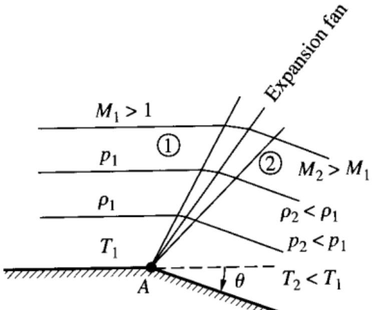
$A_y{}^* = 0.02773\ m^2$ **(4)**

UNIT TEST III

1. With a neat sketch, illustrate Prandtl Meyer expansion round a convex corner?

   For a convex corner, the wall must be deflected downwards through an angle of θ. The flow at the wall must be tangent to the wall, so that the streamlines are also deflected through an angle of θ. When a supersonic flow is deflected away from itself, an expansion wave will occur. Across this wave, Mach number increases and all other flow properties decreases.

   (1)



(b) Convex corner **(1)**

2. Define Mach Reflection and regular reflection?

   When an oblique shock wave is intercepted by a frictionless surface, the shock wave will be reflected and that is called as regular reflection **(1)**

   When an attached oblique shock wave cannot be formed at the wall, the shock wave becomes normal to the wall and "curves out" to become tangent to the incident shock wave. Such reflection is called as Mach Reflection. The shock wave from the wall to the point G is called as a mach shock wave. **(1)**

3. Can we use the method of characteristics to determine the contour of a supersonic nozzle? How?

   When the flow inside the nozzle is assumed to be one dimensional, it won't give any information about the contour of the nozzle. But the actual nozzle flow is two dimensional and the contour should be proper, otherwise shock waves will be formed inside the nozzle. Method of characteristics is used to design shock free expansion. Contour can be designed by using wall points and internal points assuming the nozzle to be symmetrical.

   (2)

4. Differentiate like reflection and unlike reflection? **(1+1=2)**

S.No	Like reflection	Unlike reflection
1	Reflection of an incident shock wave from a solid boundary is called as like reflection.	Reflection of an incident shock wave from a free boundary is called as unlike reflection.
2	In a like reflection, shock wave reflects as shock wave and expansion wave reflects as expansion waves.	In an unlike reflection, shock wave reflects as expansion waves and expansion wave reflects as shock wave.

5. Define characteristic lines and limiting characteristics?

A characteristic line is defined as the path of propagation of a physical disturbance. For supersonic flow, the disturbances are propagated through mach lines, so mach lines are the characteristic lines. **(1)**

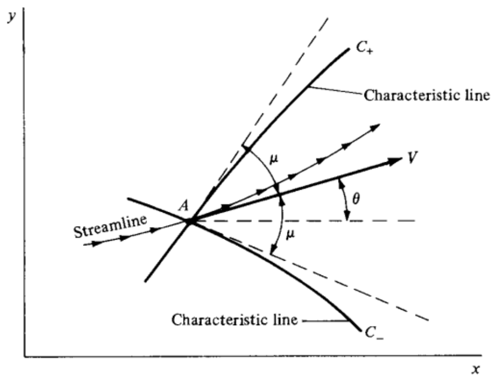
The characteristic line which emanates from the object and intersects the shock wave at the point where the sonic line also intersects the shock waves is called as the limiting characteristics. It prevents the intersection of any characteristic line originating downstream with the sonic line.

(1)

6. What are right running and left running waves in supersonic flow?

   The waves which run to the left of the flow field when it is viewed upstream of the flow is called as left running waves. It is denoted by C_+. The waves which run to the right of the flow field when it is viewed upstream of the flow is called as right running waves. It is denoted by C_-

   (1)



(1)

7. Show the heating and cooling processes in a Fanno curve for subsonic and supersonic flow?

   If the flow is subsonic, heating causes the flow Mach number to increase and the corresponding static pressure to decrease. Cooling causes the flow Mach number to decrease and the static pressure to increase.

   (1)

   If the flow is supersonic, heating causes the flow Mach number to decrease and the corresponding static pressure to increase. Cooling causes the flow Mach number to increase and the static pressure to decrease.

   (1)

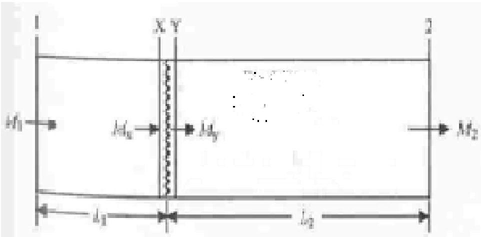
8. Find out the length of the pipe for fanno flow, if the Mach number changes from 2.8 at the entry to 1.0 at the exit. Take the friction factor for the pipe surface to be 0.0025?

   GIVEN DATA

   $M_1 = 2.8$

   $M_2 = 1.0$

   $f = 0.0025$



TO FIND

Length of the pipe

SOLUTION

$$\frac{4\bar{f}L}{D} = \left(4\bar{f}\frac{L_{max}}{D}\right)_{M_1} - \left(4\bar{f}\frac{L_{max}}{D}\right)_{M_2}$$

L = **(2)**

9. Bring out two important differences between Rayleigh Flow and Fanno Flow? **(1+1=2)**

S.No	Rayleigh Flow	Fanno Flow
1	Flow in a constant area duct with heat transfer and without friction is called as Rayleigh flow	Flow in a constant area duct with friction and without heat transfer and work transfer is called as fanno flow
2	There is no friction	It has friction

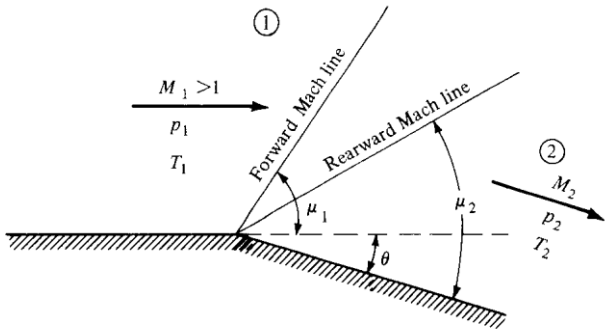
10. Distinguish between mach lines and compression waves? **(1+1=2)**

S.No	Shock wave	Mach lines
1	A compression wave is a large-amplitude compression wave which carries energy and can propagate through a medium.	Mach lines are the weak waves which are produced in supersonic flow due to the sharp leading edge.
2	Across a shock there is always a rapid rise in pressure, temperature and density of the flow.	Across a mach wave, air velocity increases, temperature and pressures are reduced.

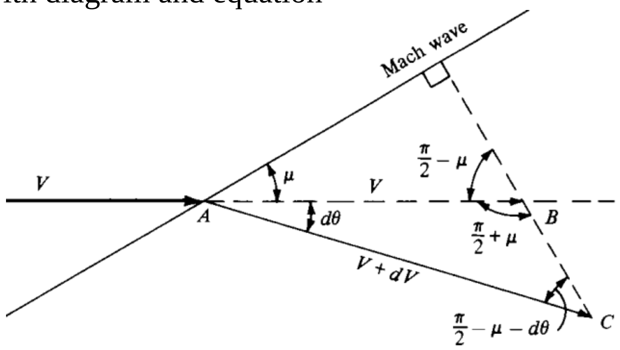
PART-B

11. Derive an expression for the Prandtl Meyer function of expansion waves and show that the maximum possible deflection angle is 130.5° **(16)**

a) Introduction with diagram **(2)**



b) Law of sines with diagram and equation **(2)**



$$\frac{V + dV}{V} = \frac{\sin(\pi/2 + \mu)}{\sin(\pi/2 - \mu - d\theta)}$$

c) Flow deflection **(3)**

$$d\theta = \sqrt{M^2 - 1}\frac{dV}{V}$$

d) Change in velocity **(4)**

$$\frac{dV}{V} = \frac{1}{1 + [(\gamma - 1)/2]M^2}\frac{dM}{M}$$

e) Prandtl meyer function **(4)**

$$\nu(M) = \sqrt{\frac{\gamma + 1}{\gamma - 1}}\tan^{-1}\sqrt{\frac{\gamma - 1}{\gamma + 1}(M^2 - 1)} - \tan^{-1}\sqrt{M^2 - 1}$$

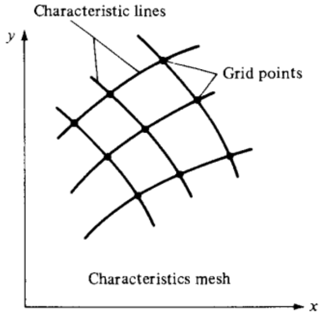
$$\theta = \nu(M_2) - \nu(M_1)$$

f) $v_{max} = \frac{\pi}{2}\left(\sqrt{\frac{\gamma+1}{\gamma-1}} - 1\right)$ **(1)**

If γ=1.4, then v_{max} =130.5°

12. Discuss in detail the philosophy of the method of characteristics by deriving the necessary equations
(16)

a) Introduction and definition **(3)**



Procedure

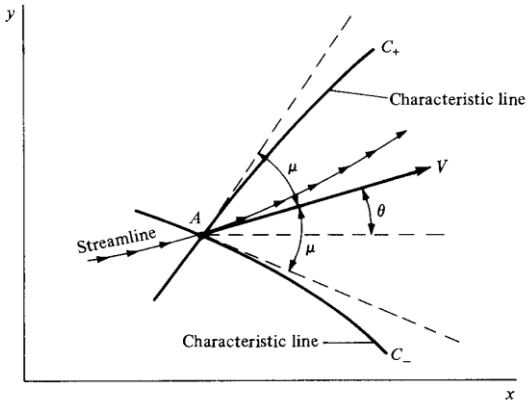
- Find characteristic line
- Convert partial differential equation to ordinary differential equation
- Solve the compatibility equation

b) 2 dimensional supersonic flow till Cramers rule **(3)**

$$\frac{\partial^2 \phi}{\partial x \, \partial y} = \frac{\begin{vmatrix} 1 - \frac{u^2}{a^2} & 0 & 1 - \frac{v^2}{a^2} \\ dx & du & 0 \\ 0 & dv & dy \end{vmatrix}}{\begin{vmatrix} 1 - \frac{u^2}{a^2} & -\frac{2uv}{a^2} & 1 - \frac{v^2}{a^2} \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix}} = \frac{N}{D}$$

c) Slope **(3)**

$$\left(\frac{dy}{dx}\right)_{\text{char}} = \tan(\theta \mp \mu)$$



d) Compatibility equation **(3)**

$$d\theta = \mp\sqrt{M^2 - 1}\frac{dV}{V}$$

$$\theta + \nu(M) = \text{const} = K_- \qquad \text{(along the } C_- \text{ characteristic)}$$

$$\theta - \nu(M) = \text{const} = K_+ \qquad \text{(along the } C_+ \text{ characteristic)}$$

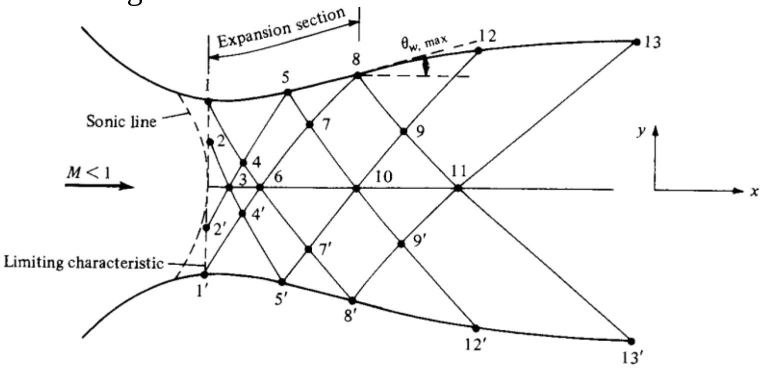
e) Grid points **(2)**

Initial data line, internal points, wall points, shock points

f) Region of influence, domain of dependence **(2)**

13. A. Explain briefly the procedure to be followed for the design of a supersonic nozzle using method of characteristics. **(8)**

   a) Introduction and need for contour design **(2)**
   b) Nozzle diagram **(2)**

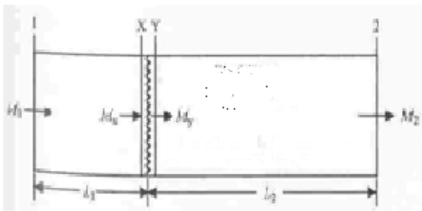


   c) Construction of the nozzle **(2)**
      Sonic line
      Limiting characteristic
      Expansion section
      Straightening section
   d) Procedure **(2)**

B. Air at a pressure of 0.685bar and temperature 310K enters a 60cm diameter duct at a mach number 3. The flow passes through a normal shock wave at a section L_1 meters downstream of the entry where the Mach number is 2.5. The Mach number at the exit (at a distance L_2 meters downstream of the shock) is 0.8. The mean coefficient of skin friction is 0.005. Determine: (i) The length L_1 and L_2, (ii) State of air at exit (iii) Mass flow rate through the duct. Take $\gamma = 1.3$ and R=0.287 KJ/KgK **(8)**

GIVEN DATA

$P_1 = 0.685$bar
$T_1 = 310$K
D = 60cm
$M_1 = 3$
$M_x = 2.5$
$M_2 = 0.8$
f = 0.005



TO FIND

Length L_1 and L_2
State of air at exit
Mass flow rate **(1)**

SOLUTION

From isentropic table $P_1/P_{01} = 0.025$
$T_1/T_{01} = 0.426$

P_{01}= 2740000 N/m^2

T_{01} = 727.7 K **(1)**

From normal shock table

M_y = 0.493

P_y/P_x= 6.935

T_y/T_x = 1.869

$$\frac{4\bar{f}L}{D} = \left(4\bar{f}\,\frac{L_{max}}{D}\right)_{M_1} - \left(4\bar{f}\,\frac{L_{max}}{D}\right)_{M_2}$$

L_1 = 3.42 m

L_2 = 32.73 m **(2)**

T_2 = 665 K

P_2 = 376309.012 N/m^2 **(2)**

m = 222.386 kg/s **(2)**

RESULT

14. A. Write short notes on Rayleigh flow and Fanno flow with the effects of cooling and heating on subsonic and supersonic flows **(8)**

a) Rayleigh flow definition with assumptions **(1)**

b) Rayleigh curve **(1)**

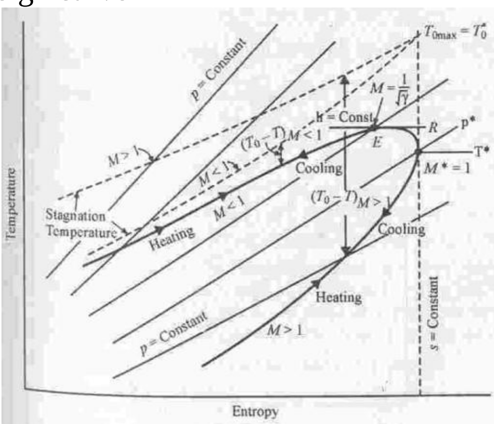


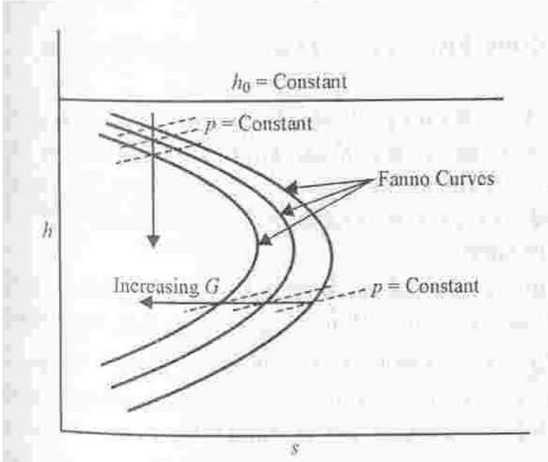
FIGURE 9.2 : Various flow parameters on Rayleigh line

c) Effect of Rayleigh flow in subsonic and supersonic flows **(2)**

Property	Heating		Cooling	
	Subsonic	Supersonic	Subsonic	Supersonic
M	increases	decreases	decreases	increases
p	decreases	increases	increases	decreases
F	constant	constant	constant	constant
p_0	decreases	decreases	increases	increases
c	increases	decreases	decreases	increases
ρ	decreases	increases	increases	decreases
T_0	increases	increases	decreases	decreases
T	1. increases for $M < \frac{1}{\sqrt{\gamma}}$ 2. decreases for $M > \frac{1}{\sqrt{\gamma}}$	increases	1. decreases for $M < \frac{1}{\sqrt{\gamma}}$ 2. increases for $M > \frac{1}{\sqrt{\gamma}}$	decreases
s	increases	increases	decreases	decreases

d) Fanno flow definition with assumptions **(1)**

e) Fanno curve **(1)**



f) Effect of Fanno flow in subsonic and supersonic flows **(2)**

Property	$M < 1$	$M > 1$
dp/p	−	+
dt/t	−	+
$d\rho/\rho$	−	+
dP/P	−	−
dV/V	+	−
dM/M	+	−
$d\mathscr{F}/\mathscr{F}$	−	−
ds/c_p	+	+

B. A mach 2 air stream passes over a 10° compression corner. The oblique shock from the corner is reflected from a flat wall which is parallel to the free stream. Compute the angle of the reflected shock wave relative to the flat wall and the Mach number downstream of the reflected shock.

(8)

GIVEN DATA

$M_1 = 2$

$\theta_1 = 10°$

DIAGRAM

TO FIND

Angle of the reflected shock wave

Mach number downstream of the reflected shock **(2)**

SOLUTION

From oblique shock chart

$\beta_{12} = 39.3$

$M_{1n1} = 1.267$

From normal shock table

$M_{2n1} = 0.807$

$P_2/P_1 = 1.685$

$M_2 = 1.65$ **(2)**

From oblique shock chart

$\beta_{23} = 49.4$
$M_{2n2} = 1.253$
From normal shock table
$M_{3n1} = 0.813$
$P_3/P_2 = 1.655$
$M_3 = 1.281$ **(2)**
$\beta_r = 39.4$ **(2)**

RESULT

15. A. The Mach number at the exit of a combustion chamber is 0.9. The ratio of stagnation temperatures at exit and entry is 3.74. If the pressure and temperature of the gas at exit are 2.5 bar and 1000°C resp. determine (a) mach number, pressure and temperature of the gas at entry, (b) heat supplied per kg of gas, (c) maximum heat that can be supplied. Take $\gamma = 1.3$ and $C_p = 1.218$ KJ/KgK **(8)**

GIVEN DATA

$M_2 = 0.9$
$\frac{T_2}{T_{02}} = 3.74$
$P_2 = 2.5$ bar
$T_2 = 1000°C$

TO FIND

Mach number at entry
Pressure at entry
Temperature at entry
Heat supplied per kg of gas
Maximum heat **(1)**

SOLUTION

From Rayleigh table for M2 = 0.9
$P/P^* = 1.12$
$T/T^* = 1.017$
$T_0/T_0^* = 0.991$ **(2)**
$M_1 = 0.26$
$P_1 = 4.718$ bar
$T_1 = 378$ K **(2)**
$Q = 1273.48$ KJ/kg **(1)**
$Q_{max} = 1287.18$ KJ/kg **(2)**

RESULT

B. Consider a supersonic flow with an upstream mach number of 3 and pressure of $10^5 N/m^2$. This flow is first expanded around an expansion corner with equal angle of $\theta = 15°$ so that it is returned to its original upstream direction. Calculate the Mach number and pressure downstream of the compression corner. **(8)**

GIVEN DATA

$M_1 = 3$
$P_1 = 10^5 N/m^2$
$\theta = 15°$

TO FIND

Mach number

Pressure **(1)**

SOLUTION

$\theta_1 = 15°$ and $M_1 = 3$

From Prandtl Meyer table,

$v_1 = 49.757$

$v_2 = 64.757$

$M_2 = 3.92$

$P_2 = 27683.824\ N/m^2$ **(3)**

From shock wave table

$\theta_3 = 15$

$\beta_3 = 27$

$M_3 = 2.987$

$P_3 = 97723.899\ N/m^2$ **(4)**

RESULT

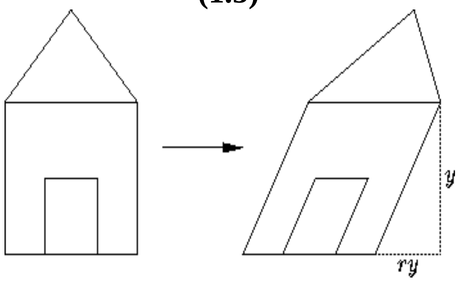
UNIT TEST IV

PART -A

1. What do you mean by affine transformation?

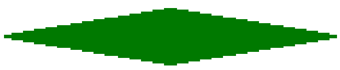
   If all the coordinates of the system are changed by uniform ratio, then that transformation is termed as affine transformation. It is the transformation that preserves lines and parallelism. It preserves straight lines and ratios of distances between points lying on a straight line. The midpoint of a line segment remains the midpoint after transformation.

   (1.5)



(0.5)

2. Sketch the different types of supersonic profiles?
   1. Diamond wedge airfoil **(2)**

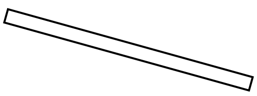


Double Wedge Airfoil

   2. Biconvex airfoil

Bi-Convex Airfoil

   3. Flat plate



3. What are the assumptions of small perturbation potential theory? **(2)**
    i. Small perturbations
    ii. Slender bodies (thin)
    iii. Small angle of attack
    iv. Subsonic and supersonic flows

4. Give the compressibility correction given by Karman-Tsien and Laitone?

Karman-Tsien compressibility correction

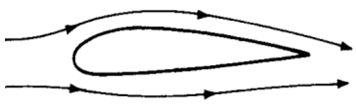
$$C_P = \frac{C_{P0}}{\sqrt{1-M^2} + \frac{C_{P0}}{2}\left(M^2/(1+\sqrt{1-M^2})\right)}$$ **(1)**

Laitone compressibility correction

$$C_p = \frac{C_{p0}}{\sqrt{1-M^2} + \left[M_\infty^2\left(1+\frac{\gamma-1}{2}M_\infty^2\right)/2\sqrt{1-M_\infty^2}\right]C_{p0}}$$ **(1)**

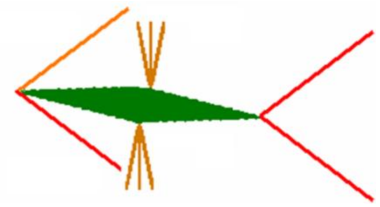
5. What are subsonic and supersonic leading edges? Explain with sketches?

The subsonic leading edge will have a round nose and a sharp tail section. When this airfoil moves in the air at supersonic speeds, bow shocks will be formed in front of the leading edge.



(1)

The supersonic leading edge will have a sharp nose and a sharp tail section. When this airfoil moves in the air at supersonic speeds, shock waves and expansion waves will be formed in front of the leading edge.



(1)

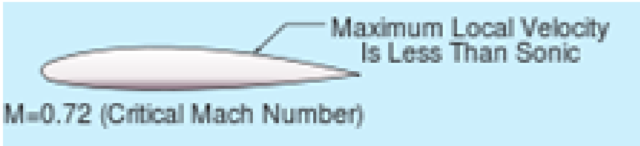
6. State Prandtl-Glauert rule?

Prandtl-Glauert rule states that if we know the incompressible pressure distribution over an airfoil, then the compressible pressure distribution can be found using Prandtl-Glauert rule. **(1)**

$$c_p = \frac{c_{p0}}{\sqrt{1-M^2}}.$$ **(1)**

7. Define critical Mach number of an airfoil? What are the types of critical Mach number?

Critical Mach number (M_{cr}) of an aircraft is the lowest Mach number at which the airflow over some point of the aircraft reaches the speed of sound



(1)

- Upper critical mach number
- Lower critical mach number **(1)**

8. Explain the phenomena of lift divergence and drag divergence?

Lift divergence is a departure from the smooth increase in lift with increasing airspeed that occurs just above the critical Mach number. **(1)**

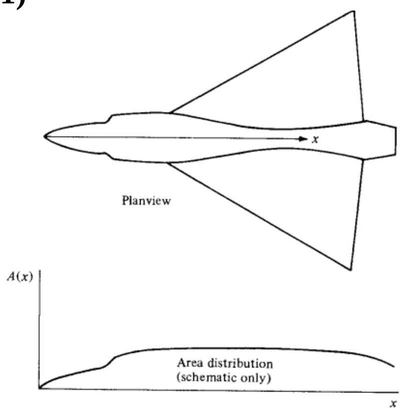
Drag divergence is the phenomenon at which the aerodynamic drag on an airfoil begins to increase rapidly as the Mach number continues to increase. **(1)**

9. Why is there a sudden drag rise in transonic flow?

When the aircraft changes its speed from subsonic to supersonic speeds, the speed of the airflow over some parts of the wing reaches the speed of sound and other parts will remain subsonic. When this happens, the aircraft is said to have reached its critical Mach number and marks the beginning of transonic speed range. At a particular point after this critical Mach number, the drag will be increased suddenly. That is the reason for a sudden drag rise in transonic flow. **(2)**

10. Explain the "coke bottle fuselage design" given by whitcomb?

In an area ruled aircraft, the fuselage cross section should be decreased in the wing and tail regions to compensate for the addition of wing and tail cross sectional area and this shape is called as the coke bottle fuselage design. **(1)**



(1)

PART –B

11. A. Based on small perturbation theory, derive the linearised velocity potential equation for compressible flows. **(12)**

   a) Introduction with diagram **(1)**

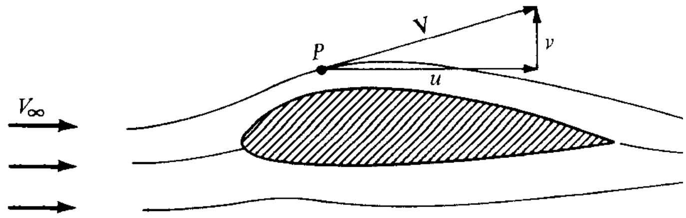


Figure 11.2 Uniform flow and perturbed flow.

b) Perturbation velocity potential equation **(3)**

$$\left[a^2 - \left(V_\infty + \frac{\partial \hat{\phi}}{\partial x}\right)^2\right]\frac{\partial^2 \hat{\phi}}{\partial x^2} + \left[a^2 - \left(\frac{\partial \hat{\phi}}{\partial y}\right)^2\right]\frac{\partial^2 \hat{\phi}}{\partial y^2} - 2\left(V_\infty + \frac{\partial \hat{\phi}}{\partial x}\right)\left(\frac{\partial \hat{\phi}}{\partial y}\right)\frac{\partial^2 \hat{\phi}}{\partial x\, \partial y} = 0$$

c) Irrotational isentropic flow **(3)**

$$\begin{aligned}(1 - M_\infty^2)\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} = {} & M_\infty^2\left[(\gamma + 1)\frac{\hat{u}}{V_\infty} + \frac{\gamma + 1}{2}\frac{\hat{u}^2}{V_\infty^2} + \frac{\gamma - 1}{2}\frac{\hat{v}^2}{V_\infty^2}\right]\frac{\partial \hat{u}}{\partial x} \\ & + M_\infty^2\left[(\gamma - 1)\frac{\hat{u}}{V_\infty} + \frac{\gamma + 1}{2}\frac{\hat{v}^2}{V_\infty^2} + \frac{\gamma - 1}{2}\frac{\hat{u}^2}{V_\infty^2}\right]\frac{\partial \hat{v}}{\partial y} \\ & + M_\infty^2\left[\frac{\hat{v}}{V_\infty}\left(1 + \frac{\hat{u}}{V_\infty}\right)\left(\frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x}\right)\right]\end{aligned}$$

Applications and limitations

Linearized velocity potential equation

$$(1 - M_\infty^2)\frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0$$

d) Linearized Pressure Coefficient **(4)**

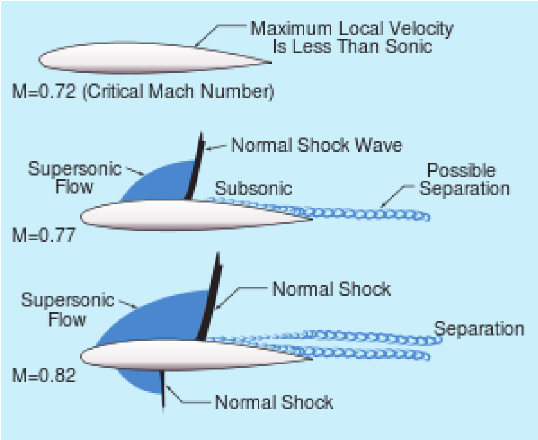
$$C_p = -\frac{2\hat{u}}{V_\infty}$$

e) Flow tangency condition **(1)**

$$\frac{\partial \hat{\phi}}{\partial y} = V_\infty \tan\theta$$

B. Explain the significance of critical Mach number? **(4)**

a) Definition with diagram **(1)**



b) Factors influencing critical Mach number **(1)**

- Thickness to chord ratio
- Aspect ratio
- Wing sweep

c) Types of critical Mach number **(1)**

- Upper critical mach number
- Lower critical mach number

d) Equation **(1)**

$$c_p = \frac{2}{\gamma M_\infty^2}\left[\left(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_{\infty_{cr}}^2}\right)^{\frac{\gamma}{1-\gamma}}-1\right]$$

12. A. Derive the Linearised two-dimensional supersonic flow theory
(12)

a) Introduction with diagram **(2)**

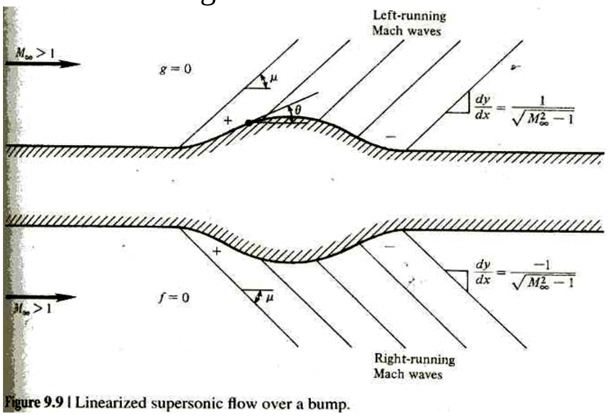


Figure 9.9 | Linearized supersonic flow over a bump.

b) Governing equation **(2)**

$$\lambda^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} - \frac{\partial \hat{\phi}}{\partial y^2} = 0$$

Laplace equation

$$\lambda^2 f'' - \lambda^2 f'' = 0$$

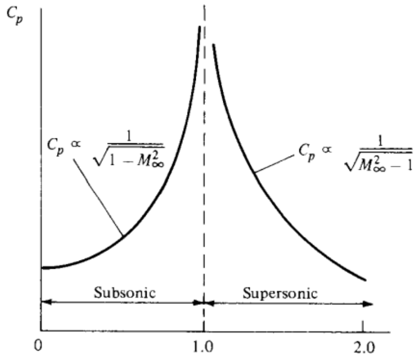
c) Slope of mach line **(2)**

$$\tan \mu = \frac{1}{\sqrt{M_\infty^2 - 1}}$$

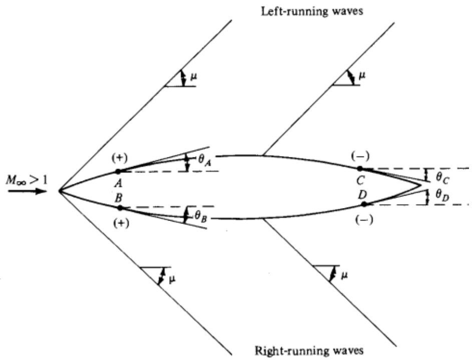
d) Coefficient of pressure **(2)**

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

e) Curve **(2)**



f) Application to biconvex airfoil **(2)**

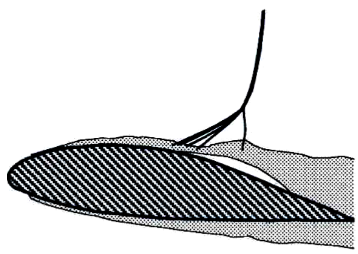


$$C_{p,A} = \frac{2\theta_A}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p,B} = \frac{2\theta_B}{\sqrt{M_\infty^2 - 1}}$$

$$C_{p,C} = -\frac{2\theta_C}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p,D} = -\frac{2\theta_D}{\sqrt{M_\infty^2 - 1}}$$

B. Explain with neat sketch about the shock induced separation.
(4)

a) Definition **(1)**



b) Effects of shock induced separation **(1)**

- External flow

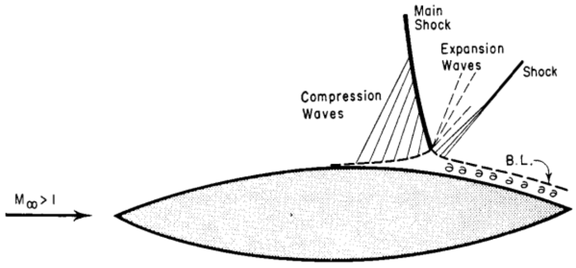
        - Increased aerodynamic drag, loss of lift
        - Aerodynamic heating
        - Increased instabilities such as inlet
    - Internal flow
        - Total pressure loss and unsteadiness
        - Loss of flow control performance

c) shock boundary layer interactions **(1)**

- Weak interaction
- Moderate interaction
- Strong interaction
- Very strong interaction

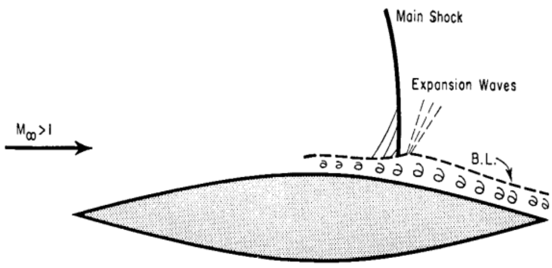
d) Various flows **(1)**

Laminar



(a)

Turbulent



(b)

13. A. Determine the flow field around a symmetric double wedge of 30° included angle kept at 10° angle of attack in a supersonic flow of Mach number 3.0.
(12)

GIVEN DATA

$M = 3.0$

$\theta = 15°$

$\alpha = 10°$

DIAGRAM

TO FIND **(2)**

P/P_0, T/T_0

SOLUTION

$M_1 = 3.0$

$P_1/P_{01} = 0.0272$

$T_1/T_{01} = 0.3571$

$M_2 = 2.75$

$P_2/P_{02} = 0.0397$

$T_2/T_{02} = 0.398$ **(2)**

$M_3 = 4.77$

$P_3/P_{03} = 2.49*10^{-3}$

$T_3/T_{03} = 0.1803$

$M_4 = 2.51$

$P_4/P_{04} = 0.0227$

$T_4/T_{04} = 0.446$ **(2)**

$M_5 = 1.72$

$P_5/P_{05} = 0.134$

$T_5/T_{05} = 0.629$ **(2)**

$M_6 = 2.93$

$P_6/P_{06} = 0.0206$

$T_6/T_{06} = 0.3686$ **(2)**

$M_7 = 2.625$

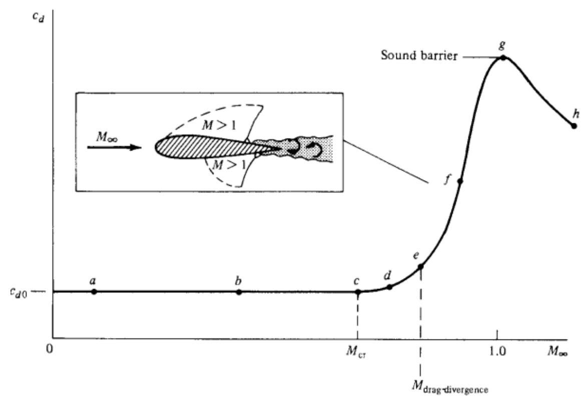
$P_7/P_{07} = 0.0306$

$T_7/T_{07} = 0.413$ **(2)**

RESULT

B. Define drag divergence Mach number and show the variation of C_D with Mach number for various thickness ratios **(4)**

a) Definition with diagram **(2)**

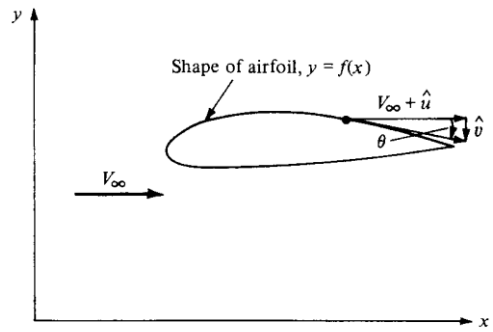


b) Explain the points on the curve **(2)**

14. A. Explain Prandtl-Glauert compressibility correction for compressible flows. What are the other methods for compressibility correction?

(10)

a) Introduction with diagram **(1)**



b) Velocity potential equation **(3)**

$$\beta^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0$$

Laplace equation

$$\frac{\partial^2 \bar{\phi}}{\partial \xi^2} + \frac{\partial^2 \bar{\phi}}{\partial \eta^2} = 0$$

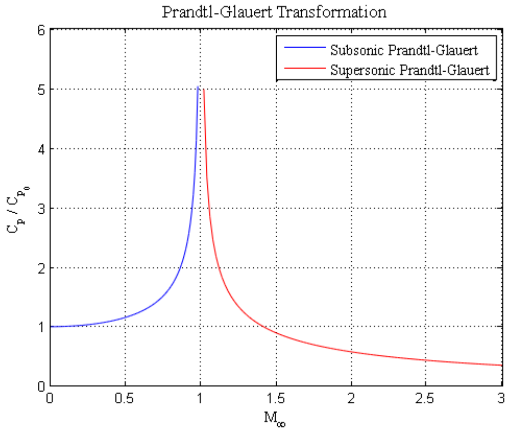
c) Shape of physical and transformed planes **(2)**

$$\frac{df}{dx} = \frac{dq}{d\xi}$$

d) Coefficient of pressure **(2)**

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}$$

e) Curve **(1)**



f) Other methods of correction **(1)**

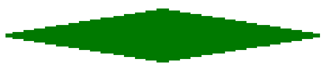
i. Laitone correction

ii. Karman and Tsien correction

B. Determine the various supersonic profiles and show the flow field around such profiles for various angles of attack

(6)

1. Diamond wedge airfoil
   a) Explain for various angles of attack **(2)**



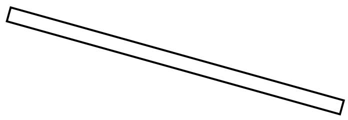
Double Wedge Airfoil

2. Biconvex airfoil
   b) Explain for various angles of attack **(2)**

Bi-Convex Airfoil

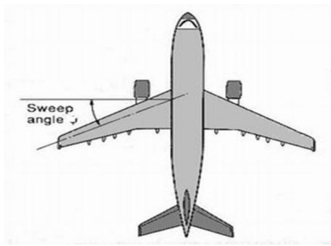
3. Flat plate

   c) Explain for various angles of attack **(2)**

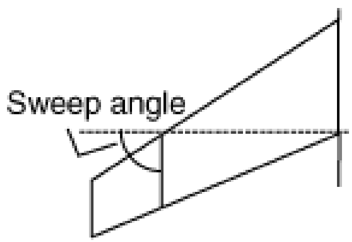


15. A. What is the role of sweep back and sweep forward for aircraft wings? Explain the aerodynamic characteristics of swept wings **(8)**

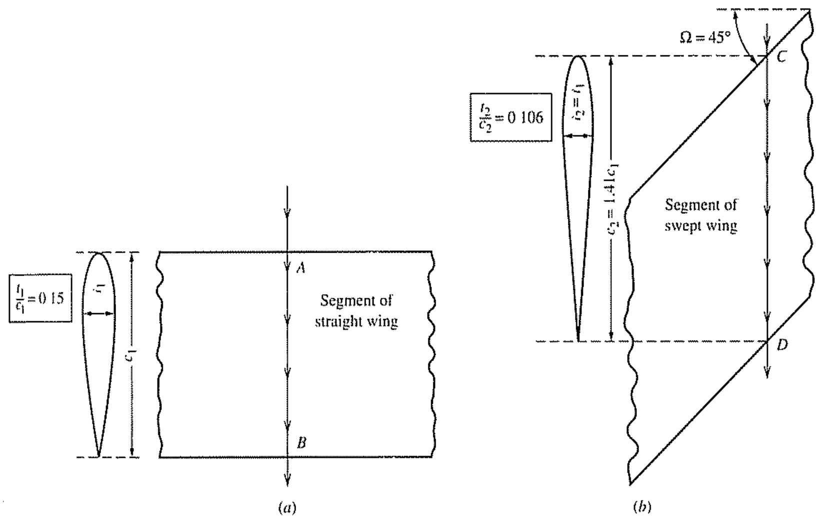
   a) swept wing plan form **(2)**



Sweep angle



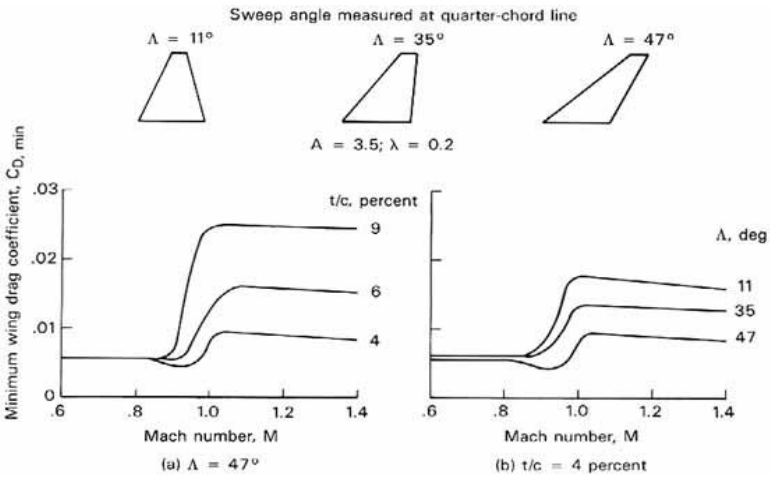
   b) Straight wing Vs swept wing **(2)**



   c) Types **(2)**

   - Forward swept
   - Backward swept
   - Variable swept

   d) Curve **(2)**

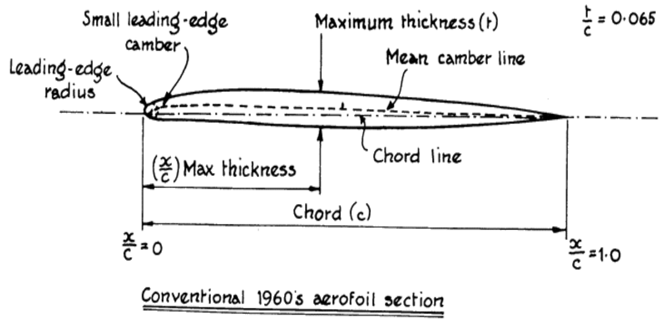


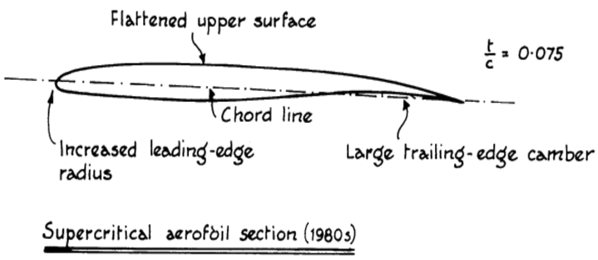
B. Write short notes on **(8)**

(i) Supercritical airfoils

a) Definition with benefits **(2)**

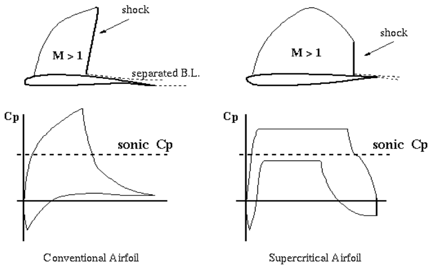
b) Comparison between supercritical and traditional airfoils **(2)**





Pressure distribution

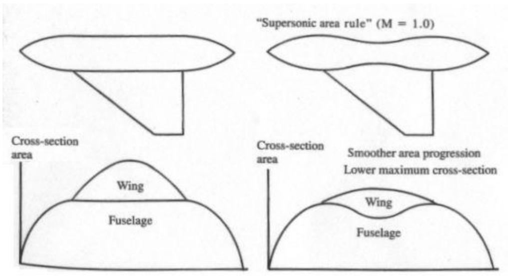
Conventional vs. Supercritical Airfoils



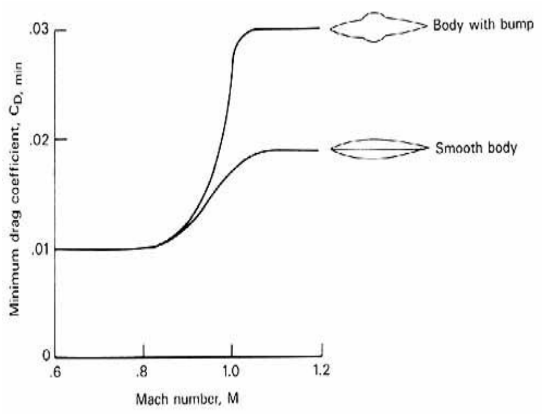
(ii) Transonic area rule

a) Introduction **(2)**

Area ruled Vs non area ruled aircraft



b) Mach number Vs drag curve **(2)**



Applications

MODEL TEST I

PART -A

1. Why is a converging diverging passage required to go from subsonic to supersonic flow?
   - ❖ Converging section – stagnation velocity to sonic velocity
   - ❖ Diverging section- large decrease in density of the fluid which makes acceleration in the divergent section to achieve mach numbers >1 (2)

2. Define characteristic Mach number and what is the maximum value of it?
   It is defined as the ratio between the velocity of the object and the critical velocity of sound
   (1)

$$M^* = \frac{\text{velocity of the object}}{\text{critical velocity of sound}}$$

Maximum value for $M^* = \sqrt{\frac{\gamma+1}{\gamma-1}}$

For $\gamma = 1.4,\ M^* = 2.45$
(1)

3. Define hodograph and pressure turning angle?
   The locus of all possible static pressures behind an oblique shock wave as a function of deflection angle for any given upstream conditions is called as pressure turning angle. **(1)**
   The plane which uses velocity components as the coordinates of the system is referred to as Hodograph Plane. **(1)**

4. Define shock polar? Sketch the shape of shock polar for $M_1^* = 2.45$
   The locus of all the points for θ values ranging from zero to maximum representing all possible velocities behind the shock wave is called as shock polar. **(1)**

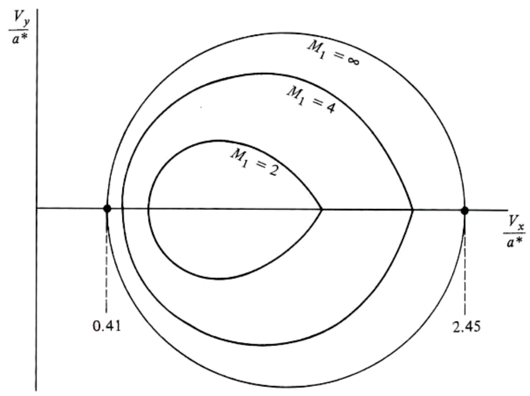
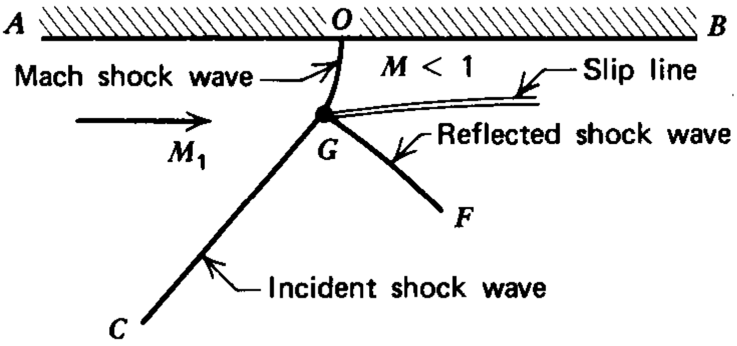


Figure 4.17 | Shock polars for different Mach numbers.

(1)

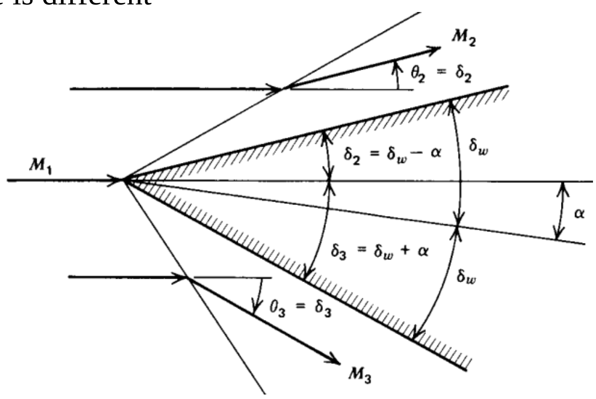
5. What is meant by mach reflection?
   When an attached oblique shock wave cannot be formed at the wall, the shock wave becomes normal to the wall and “curves out” to become tangent to the incident shock wave. Such reflection is called as Mach Reflection. The shock wave from the wall to the point G is called as a mach shock wave. **(1)**



6. What is meant by expansion hodograph?
   The hodograph which shows the expansion characteristics of a Prandtl Meyer flow is called as expansion hodograph. The hodograph characteristics for a uniform steady two dimensional planar isentropic flow are epicycloids which is a curve generated by rolling a circle of radius (b-1)/2 on the circumference of a circle of radius M*=1.

7. An unsymmetrical diamond airfoil at zero angle of attack is kept in supersonic flow. Sketch the wave pattern and the streamlines?

   If the wedge is unsymmetrical, the flow over the top and bottom surfaces will be unsymmetrical and so the flow pattern should be studied separately because the flow deflection angle is different



8. By linearised theory, what are the expressions for the lift and drag coefficients for a symmetric bi convex profile?

9. What is the effect of sweep back on compressibility?

   - Delay the drag rise
   - Increase the critical mach number
   - Requires a longer chord length for a given span
   - Allows all parts of the aircraft that create lift to remain in subsonic flow,
   - Creates less lift for a given airspeed than a non-swept wing would.

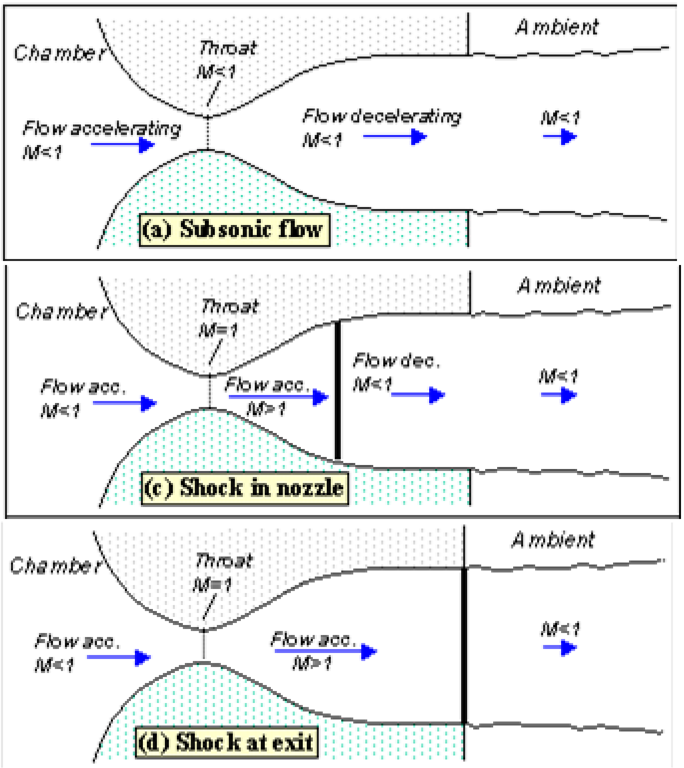
10. Why is that airfoil designed for a high critical mach number must have a thin profile
    In thin airfoils,

1. critical Mach number is increased
2. lower minimum pressure
3. lift coefficient will decrease
4. drag rise small
5. Can fly at high free stream Mach number

PART -B

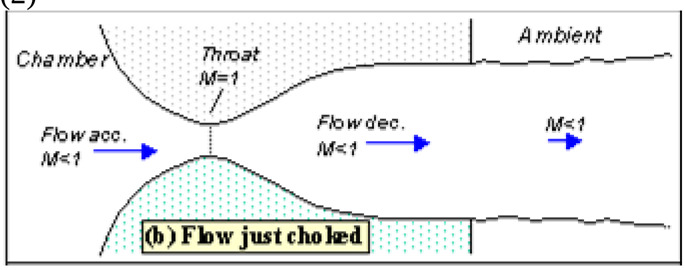
11. A. (i) A convergent divergent nozzle is operated at a fixed pressure at the entrance and decreasing back pressures at the exit. Sketch the variation of pressure and velocity along the nozzle.
(8)

- ❖ CD nozzle introduction and overall performance curve
(4)
- ❖ Nozzle performance (subsonic, supersonic)
(2)



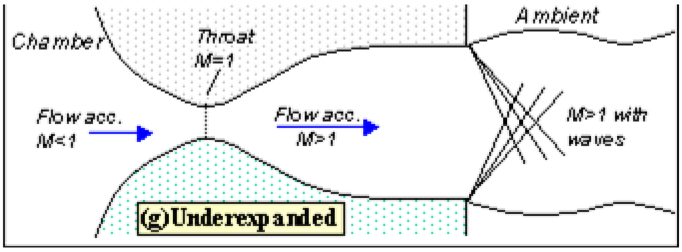
(a) Subsonic flow
(c) Shock in nozzle
(d) Shock at exit

- ❖ Choking condition
(2)

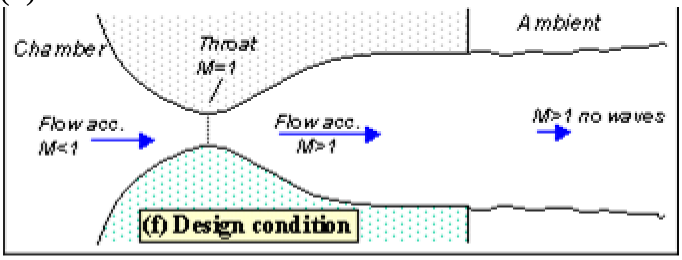


(b) Flow just choked

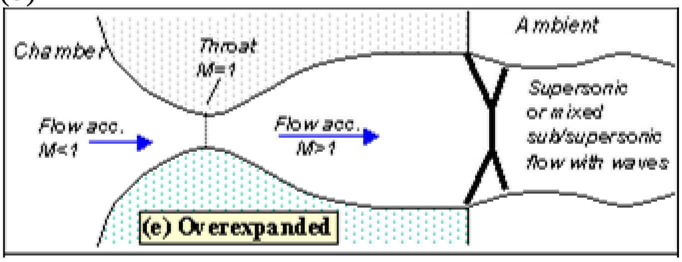
- ❖ under expanded nozzles
(3)



- Correctly expanded nozzles
(2)



- Over expanded nozzles
(3)



(ii) A converging- diverging nozzle is connected to a reservoir of pressure and temperature 10×10^5 N/m$_2$ and 300K respectively. There are two locations in the nozzle where A/A*=6, one in the converging section and other in the diverging section. At each section, calculate the Mach number, static pressure, static temperature and velocity. (8)

OR

B. (i) (ii) An airplane flies at 1800Kmph at an altitude where the pressure is 1/3 of that of sea level value and the temperature is −40 degree centigrade. Derive the necessary equations and calculate the pressure, density and temperature at the wing leading edge which may be assumed as stagnation point

(8)

(ii) A De Laval nozzle has to be designed for an exit mach number of 1.5 with exit diameter of 200mm. Find the ratio of throat area/exit area necessary. The reservoir conditions are given as P_o = 1 bar, T_o = 20°C. Find also the maximum mass flow rate through the nozzle. What will be the exit pressure and temperature?
(8)

12. A. (i) Air flows at M=2, P=80 KPa passes through an oblique shock and turned by 4°. The emerging flow then passes through a normal shock. Estimate the total pressure behind the normal shock. (8)

(ii) Write the continuity, momentum and energy equation for one dimensional flow through a normal shock wave. Using these equation, derive the Prandtl relation in the form $M_2^* = \frac{1}{M_1^*}$ where subscripts 1 and 2 refers to upstream and downstream of the shock and M_1^* and M_2^* are the corresponding speed ratios. (8)

a) Introduction, Diagram, Assumptions **(2)**

b) Continuity equation **(2)**

$$\rho_1 u_1 = \rho_2 u_2$$

c) Momentum equation **(2)**

$$p_1 + \rho_1 u_1^2 = p_2 + \rho_2 u_2^2$$

d) Energy equation **(2)**

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

e) Prandtl equation

i. Introduction, Diagram, basic equations **(1)**

ii. $u_2 - u_1 = a^2$ **(6)**

iii. $M_1^* M_2^* = 1$ **(1)**

OR

B. (i) A supersonic stream of air at M=3 and 1atm passes through a sudden convex and then a sudden concave corner of turning angle 15° each. Determine Mach number and pressure of flow downstream of the concave corner. (8)

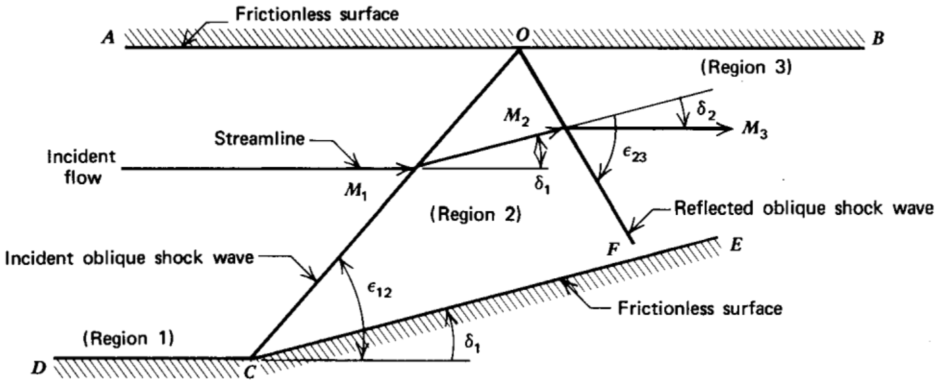
(ii) Explain what are the limiting values of shock angle in oblique shocks? (4)

(iii) A point in a supersonic flow has static pressure of 0.4atm when a Pitot tube is inserted in the flow at this point; the pressure measured by the Pitot tube is 3atm. Calculate the Mach number at this point. (4)

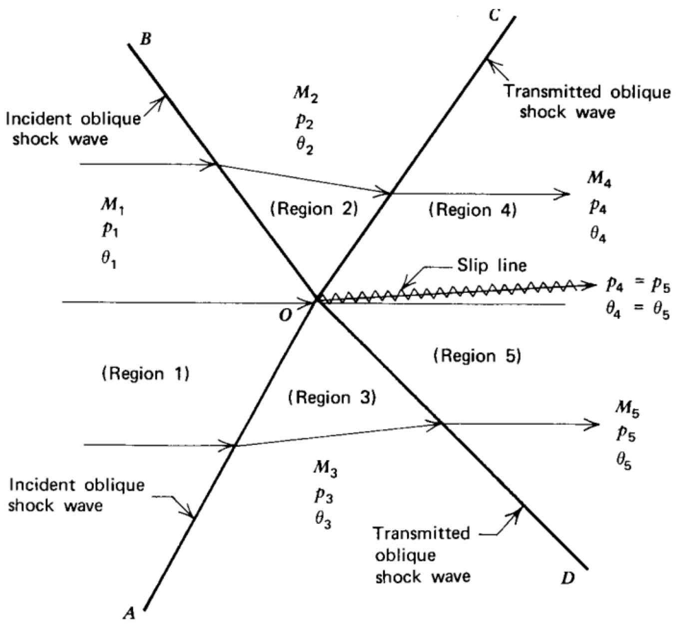
13. A. (i) Air flowing with a mach number of 2.5 with a pressure of 60KPa and temperature of $-20°C$ passes over a wedge which turns the flow through an angle of 4° leading to the generation of an oblique shock wave. The oblique shock wave impinges on a flat wall, which is parallel to the flow upstream of the wedge, and is reflected from it. Find the pressure and velocity behind the reflected shock wave and also the angle of the reflected shock wave. (8)

(ii) Explain the concept of reflection and intersection of shock waves in a supersonic flow field (8)

a) reflection of shock wave on a solid boundary



b) intersection of shock waves



OR

C. B. (i) Air at a pressure of 0.685bar and temperature 310K enters a 60cm diameter duct at a mach number 3. The flow passes through a normal shock wave at a section L_1 meters downstream of the entry where the Mach number is 2.5. The Mach number at the exit (at a distance L_2 meters downstream of the shock) is 0.8. The mean coefficient of skin friction is 0.005. Determine:

(i) The length L_1 and L_2
(ii) State of air at exit
(iii)Change in entropy
(iv)Mass flow rate through the duct (8)

(ii) Briefly explain the method of characteristics (8)

14. A. (i) Derive the differential equation for steady, 2-d compressible flows and by the assumptions of small perturbations, show that the equipotential lines in supersonic flows are mach waves.
(8)

a) Introduction with diagram **(1)**

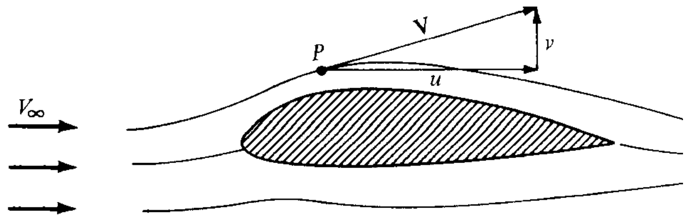


Figure 11.2 Uniform flow and perturbed flow.

b) Perturbation velocity potential equation **(3)**

$$\left[a^2 - \left(V_\infty + \frac{\partial \hat{\phi}}{\partial x}\right)^2\right]\frac{\partial^2 \hat{\phi}}{\partial x^2} + \left[a^2 - \left(\frac{\partial \hat{\phi}}{\partial y}\right)^2\right]\frac{\partial^2 \hat{\phi}}{\partial y^2} - 2\left(V_\infty + \frac{\partial \hat{\phi}}{\partial x}\right)\left(\frac{\partial \hat{\phi}}{\partial y}\right)\frac{\partial^2 \hat{\phi}}{\partial x\,\partial y} = 0$$

c) Irrotational isentropic flow **(3)**

$$\begin{aligned}(1 - M_\infty^2)\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} = M_\infty^2 &\left[(\gamma + 1)\frac{\hat{u}}{V_\infty} + \frac{\gamma + 1}{2}\frac{\hat{u}^2}{V_\infty^2} + \frac{\gamma - 1}{2}\frac{\hat{v}^2}{V_\infty^2}\right]\frac{\partial \hat{u}}{\partial x}\\ &+ M_\infty^2\left[(\gamma - 1)\frac{\hat{u}}{V_\infty} + \frac{\gamma + 1}{2}\frac{\hat{v}^2}{V_\infty^2} + \frac{\gamma - 1}{2}\frac{\hat{u}^2}{V_\infty^2}\right]\frac{\partial \hat{v}}{\partial y}\\ &+ M_\infty^2\left[\frac{\hat{v}}{V_\infty}\left(1 + \frac{\hat{u}}{V_\infty}\right)\left(\frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x}\right)\right]\end{aligned}$$

Applications and limitations

Linearized velocity potential equation

$$(1 - M_\infty^2)\frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0$$

d) Linearized Pressure Coefficient **(4)**

$$C_p = -\frac{2\hat{u}}{V_\infty}$$

e) Flow tangency condition **(1)**

$$\frac{\partial \hat{\phi}}{\partial y} = V_\infty \tan\theta$$

g) Introduction with diagram **(2)**

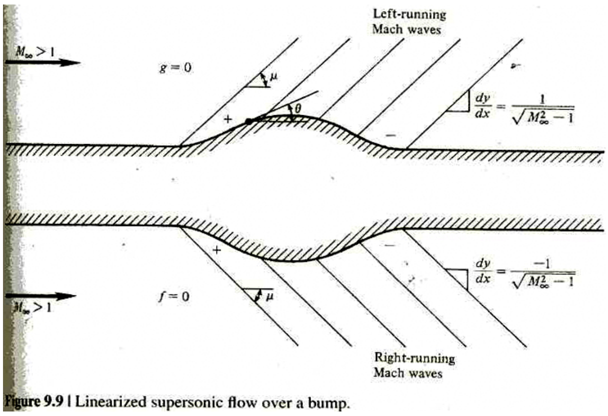


Figure 9.9 | Linearized supersonic flow over a bump.

h) Governing equation **(2)**

$$\lambda^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} - \frac{\partial \hat{\phi}}{\partial y^2} = 0$$

Laplace equation

$$\lambda^2 f'' - \lambda^2 f'' = 0$$

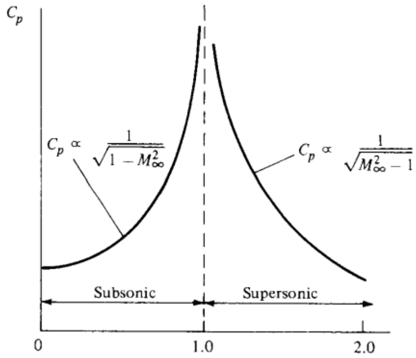
i) Slope of mach line **(2)**

$$\tan \mu = \frac{1}{\sqrt{M_\infty^2 - 1}}$$

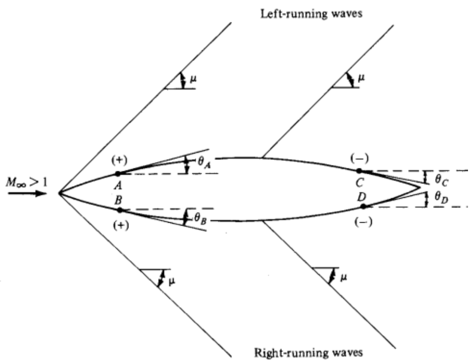
j) Coefficient of pressure **(2)**

$$C_p = \frac{2\theta}{\sqrt{M_\infty^2 - 1}}$$

k) Curve **(2)**



l) Application to biconvex airfoil **(2)**

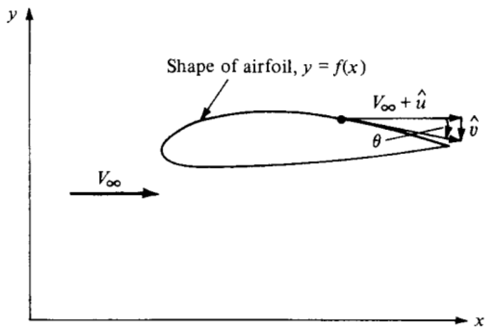


$$C_{p,A} = \frac{2\theta_A}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p,B} = \frac{2\theta_B}{\sqrt{M_\infty^2 - 1}}$$

$$C_{p,C} = -\frac{2\theta_C}{\sqrt{M_\infty^2 - 1}} \quad \text{and} \quad C_{p,D} = -\frac{2\theta_D}{\sqrt{M_\infty^2 - 1}}$$

(ii) Explain the need for the Prandtl Glauert (P-G) transformation used in high subsonic flows. State the P-G rules (8)

a) Introduction with diagram **(1)**



b) Velocity potential equation **(3)**

$$\beta^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0$$

Laplace equation

$$\frac{\partial^2 \bar{\phi}}{\partial \xi^2} + \frac{\partial^2 \bar{\phi}}{\partial \eta^2} = 0$$

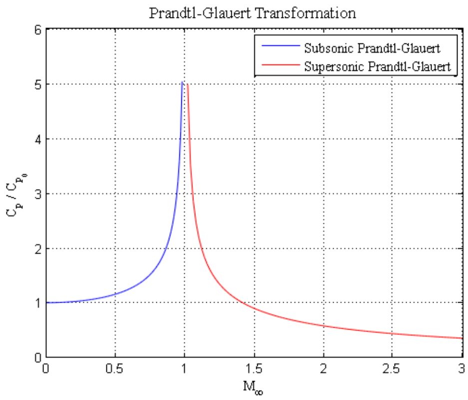
c) Shape of physical and transformed planes **(2)**

$$\frac{df}{dx} = \frac{dq}{d\xi}$$

d) Coefficient of pressure **(2)**

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}$$

e) Curve **(1)**



OR

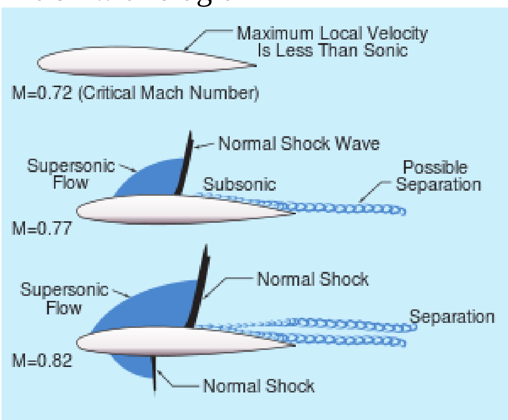
B. (i) A symmetric diamond profile of semi angle 5 deg is kept at an angle of attack of 12 deg in a flow of Mach number 2.4 in sea level air. Using the shock expansion theory, calculate the mach numbers and pressures over the entire flow field. (12)

(ii) What are subsonic and supersonic leading edges? Explain with sketches (4)

15. A. Discuss the following (4*4=16)

a) Critical mach numbers

a) Definition with diagram **(4)**



b) Factors influencing critical Mach number
- Thickness to chord ratio
- Aspect ratio
- Wing sweep

c) Types of critical Mach number
- Upper critical Mach number
- Lower critical Mach number

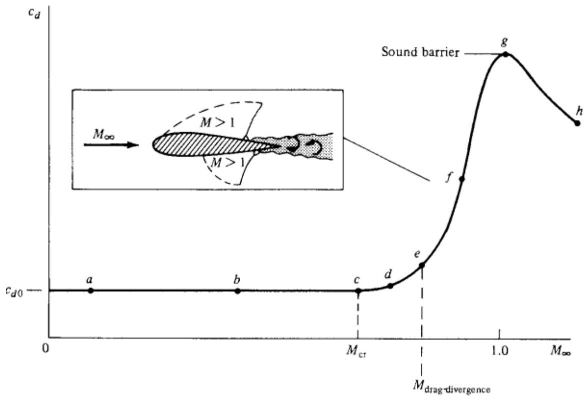
d) Equation

$$c_p = \frac{2}{\gamma M_\infty^2}[(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_{\infty_{cr}}^2})^{\frac{\gamma}{1-\gamma}} - 1]$$

b) Lift and drag divergence **(4)**

a. drag divergence definition

b. curve

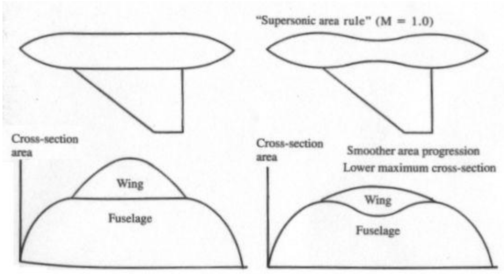


c. Lift divergence

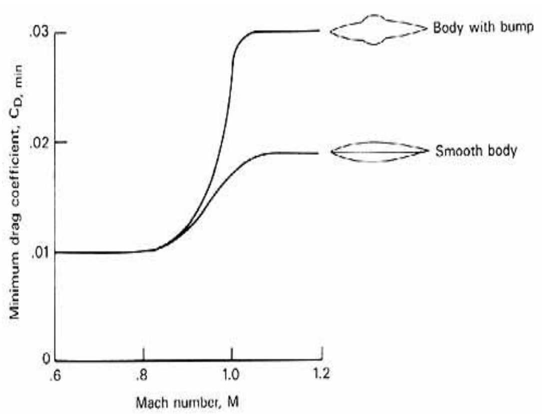
c) Transonic area rule

a) Introduction **(4)**

Area ruled Vs non area ruled aircraft



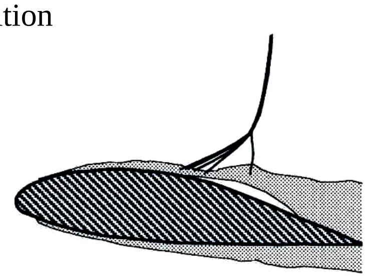
b) Mach number Vs drag curve



Applications

d) Shock induced separation

a) Definition **(4)**

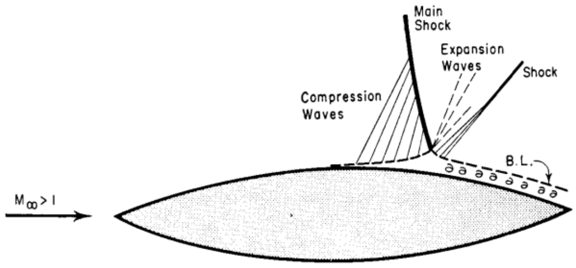


b) Effects of shock induced separation
- External flow
  - Increased aerodynamic drag, loss of lift
  - Aerodynamic heating
  - Increased instabilities such as inlet
- Internal flow
  - Total pressure loss and unsteadiness
  - Loss of flow control performance

c) shock boundary layer interactions
- Weak interaction
- Moderate interaction
- Strong interaction
- Very strong interaction

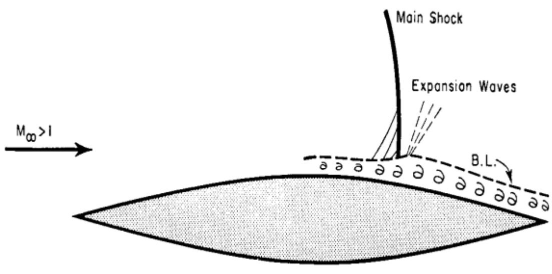
d) Various flows

Laminar



(a)

Turbulent

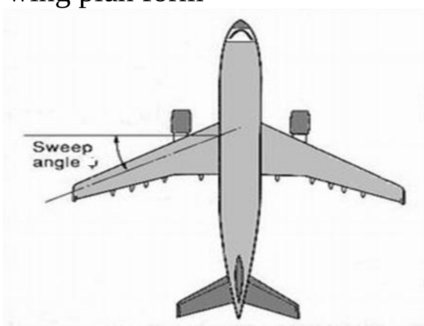


(b)

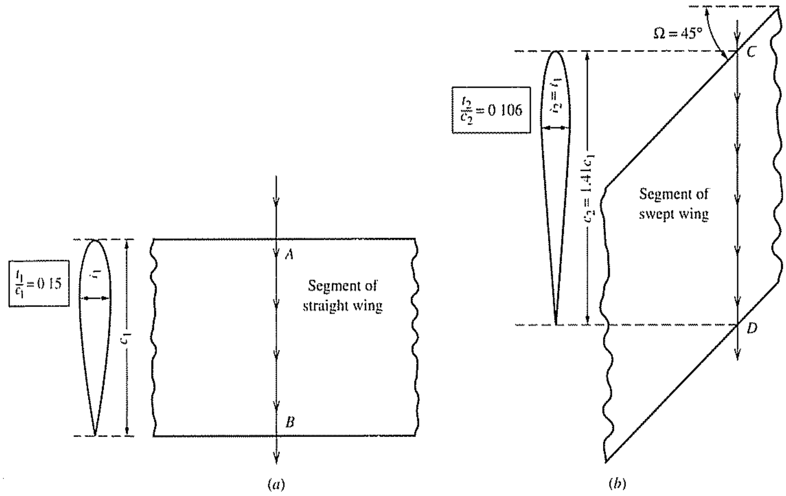
B. Write short notes on **(4*4=16)**

a) Characteristics of swept wings

a) swept wing plan form **(4)**



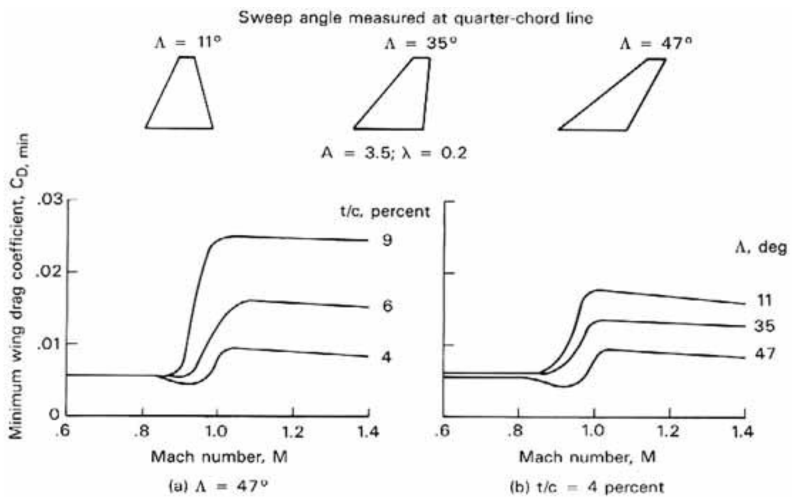
b) Straight wing Vs swept wing



c) Types

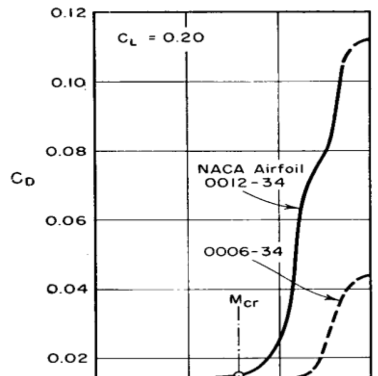
- Forward swept
- Backward swept
- Variable swept

d) Curve

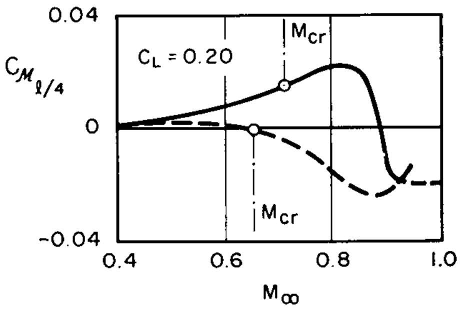


b) Effects of thickness over the performance of wings

a) Drag coefficient VS Mach number **(4)**



b) Moment coefficient VS Mach number

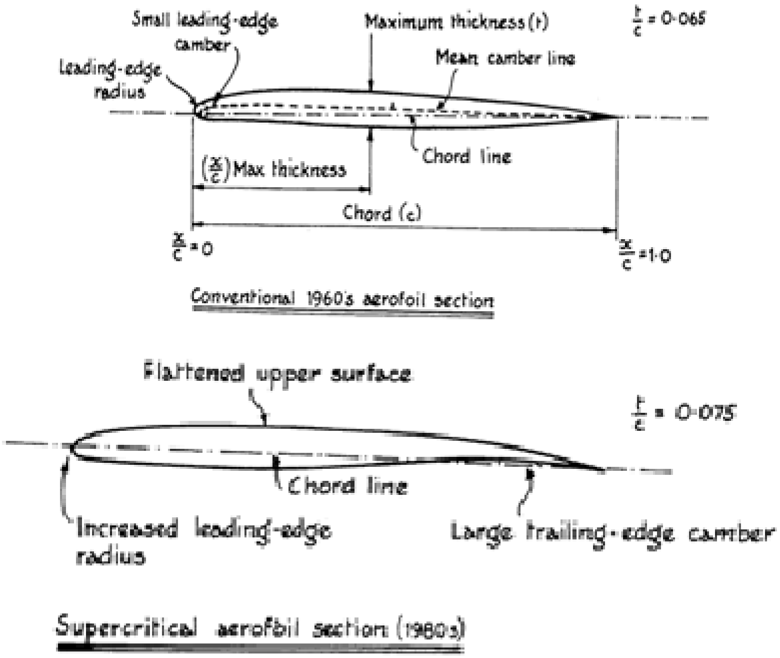


c) Supercritical airfoils

a) Definition with benefits **(4)**

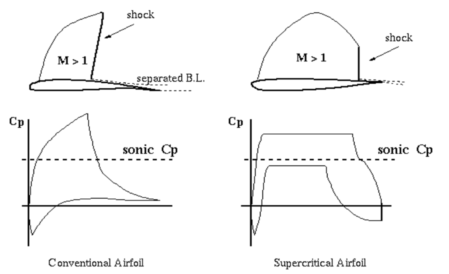
supercritical airfoil

b) Comparison between supercritical and traditional airfoils



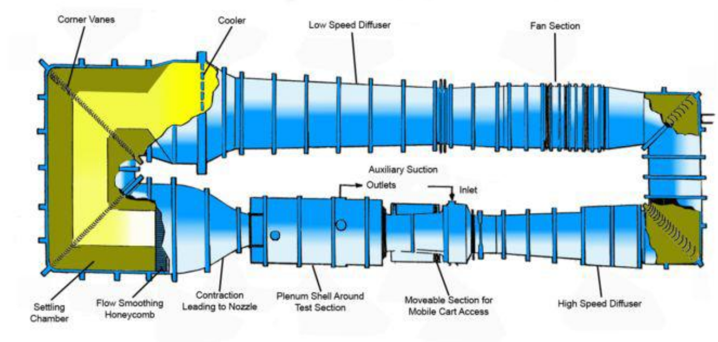
Pressure distribution

Conventional vs. Supercritical Airfoils



d) Transonic wind tunnel **(4)**

a) Construction



b) Working

MODEL TEST II

PART -A

1. Define the term compressibility? Explain the different types of compressibility?

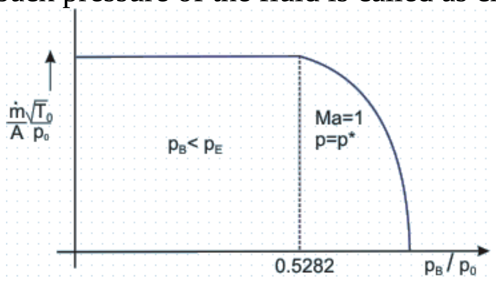
The fractional change in the volume of the fluid element per unit change in pressure is called as compressibility. The property of a substance to be reduced in volume by application of pressure is termed as compressibility. **(1)**

It is of two types

1. Isothermal compressibility
2. Isentropic compressibility **(1)**

2. Explain the phenomenon of choking in a nozzle?

If the mass flow rate in a nozzle remains constant even for further decrease in the back pressure of the fluid is called as choking. **(1)**



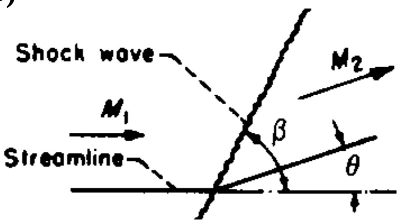
(1)

3. State the importance of Rayleigh Supersonic Pitot formula?

When a pitot static tube is immersed in a supersonic flow, shock waves will be formed in front of the nose of the tube. The pressure measurement will not be accurate. So corrections should be done.
(1)

$$\frac{p_{0,2}}{p_1} = \left(\frac{(\gamma+1)^2 M_1^2}{4\gamma M_1^2 - 2(\gamma-1)}\right)^{\gamma/(\gamma-1)} \frac{1-\gamma+2\gamma M_1^2}{\gamma+1}$$ **(1)**

4. Define Shock angle and Flow deflection angle?
The angle which the shock wave makes with the horizontal line is called as the Shock angle. It is denoted by β. The angle by which the flow is deflected away from the horizontal is called as the Flow deflection angle. It is denoted by θ.
(1.5)



(0.5)

5. What are the effects of friction on the downstream flow when $M_1>1$? **(2)**

Property	$M < 1$	$M > 1$
dp/p	−	+
dt/t	−	+
$d\rho/\rho$	−	+
dP/P	−	−
dV/V	+	−
dM/M	+	−
$d\mathscr{F}/\mathscr{F}$	−	−
ds/c_p	+	+

6. What is meant by expansion hodograph?
The hodograph which shows the expansion characteristics of a Prandtl Meyer flow is called as expansion hodograph. **(1)**
The hodograph characteristics for a uniform steady two dimensional planar isentropic flow are epicycloids which is a curve generated by rolling a circle of radius (b-1)/2 on the circumference of a circle of radius M*=1. **(1)**

7. What is Prandtl-Glauert rule?
Prandtl-Glauert rule states that if we know the incompressible pressure distribution over an airfoil, then the compressible pressure distribution can be found using Prandtl-Glauert rule. **(1)**

$$c_p = \frac{c_{p0}}{\sqrt{1-M^2}}.$$ **(1)**

8. Define wave drag?
When the airfoil is at zero angle of attack, shock wave is formed on the front surface of the airfoil and an expansion wave is formed on the rear surface. So there is

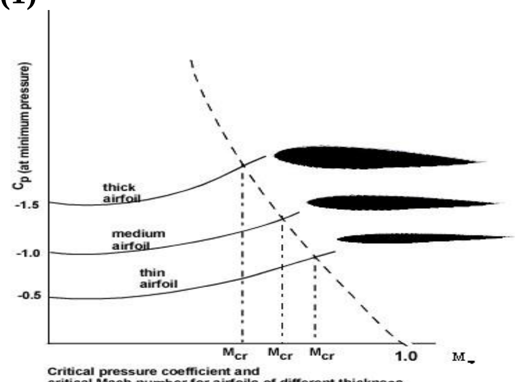
a pressure difference between the two regions. This pressure difference creates a drag force on the airfoil and this drag is called as the wave drag. **(1)**

The drag formed due to the formation of shock and expansion waves in the airfoil even when the airfoil is at zero angle of attack is called as the wave drag. **(1)**

9. What are the effects of shock induced separation?
   - External flow **(1)**
     - Increased aerodynamic drag, loss of lift
     - Aerodynamic heating
     - Increased instabilities such as inlet
   - Internal flow **(1)**
     - Total pressure loss and unsteadiness
     - Loss of flow control performance

10. Why is that airfoil designed for a high critical mach number must have a thin profile?

In thin airfoils, the pressure difference will be a minimum and so the drag can be reduced. Also the thin airfoils can fly at high free stream Mach number. When compared to thick airfoils, the onset of the critical Mach number can be delayed to high mach numbers. **(1)**

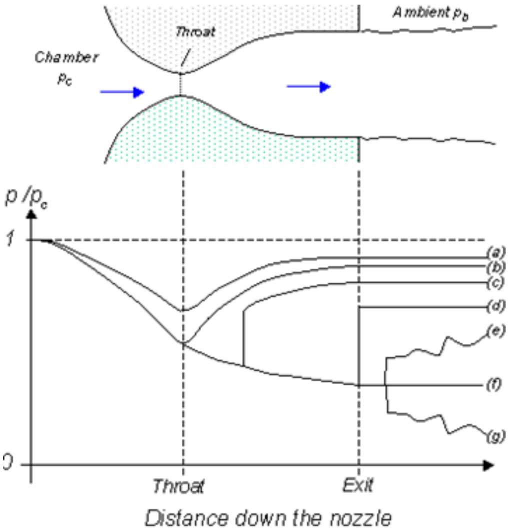


Critical pressure coefficient and critical Mach number for airfoils of different thickness **(1)**

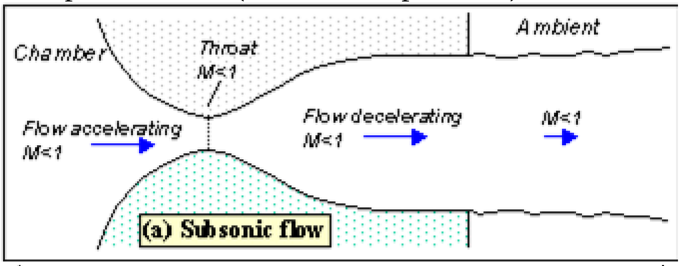
PART-B

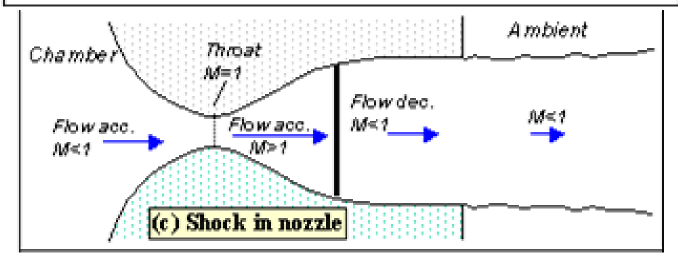
11. A. Explain the term choking in a C-D nozzle and the flow conditions leading to the same. Hence describe with illustrations a) An under expanded b) Correctly expanded c) Over expanded nozzles **(16)**

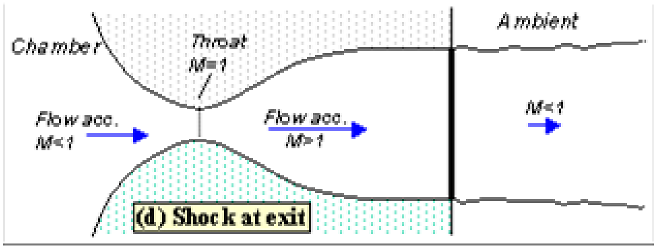
a) CD nozzle introduction and overall performance curve **(4)**



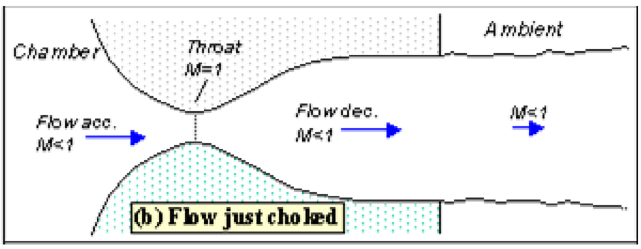
b) Nozzle performance (subsonic, supersonic) **(2)**



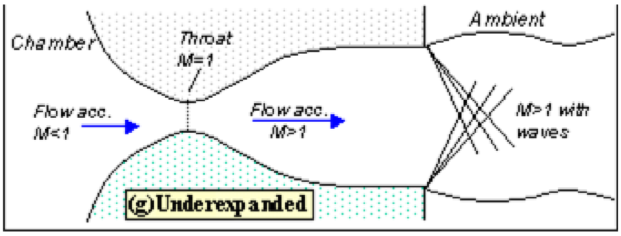




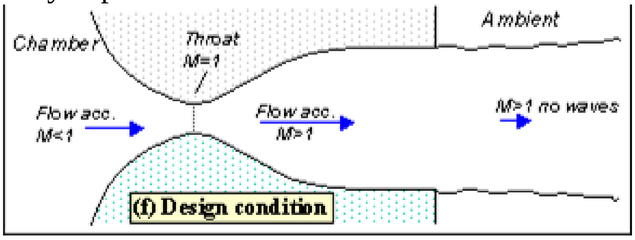
c) Choking condition
(2)



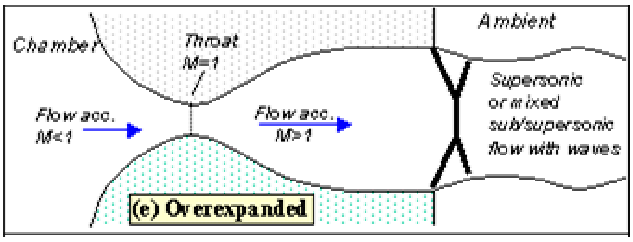
d) Under expanded nozzles
(3)



e) Correctly expanded nozzles **(2)**



f) Over expanded nozzles
(3)



B. (i) A converging- diverging nozzle is connected to a reservoir of pressure and temperature $10 \times 10^5 N/m_2$ and 300K respectively. There are two locations in the nozzle where A/A*=6, one in the converging section and other in the diverging section. At each section, calculate the Mach number, static pressure, static temperature and velocity.**(8)**

GIVEN DATA

$P_0 = 10*10^5$ N/m^2

$T_0 = 300$ K

A/A* = 6

TO FIND **(1)**

Mach number

Static pressure

Static temperature

Velocity

SOLUTION

Case (i) Converging Section

M = 0.10

T = 299.4K

P=993000 N/m^2

a = 346.84 m/s

v = 34.68 m/s **(4)**

Case (ii) Diverging Section

M = 3.38

T = 91.5K

P=15700 N/m^2
a = 191.74 m/s
v = 648.085 m/s **(3)**

RESULT

(ii) An airplane flies at 1800Kmph at an altitude where the pressure is 1/3 of that of sea level value and the temperature is −40 degree centigrade. Derive the necessary equations and calculate the pressure, density and temperature at the wing leading edge which may be assumed as stagnation point **(8)**

GIVEN DATA

v = 1800 kmph = 500 m/s
P = 33.776 * 10^3 N/m^2
T = 233 K
ρ = 0.505 Kg/m^3

TO FIND **(1)**

P_0, T_0, ρ_0

SOLUTION

a = 305.97 m/s
M= 1.63

Equations **(4)**

$$\frac{T_0}{T} = 1 + \frac{\gamma - 1}{2} M^2$$

$$\frac{p_0}{p} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{\gamma/(\gamma-1)}$$

$$\frac{\rho_0}{\rho} = \left(1 + \frac{\gamma - 1}{2} M^2\right)^{1/(\gamma-1)}$$

P_0 = 149627.68 N/m^2
T_0 =356.49 K
ρ_0 = 1.46 Kg/m^3 **(3)**

RESULT

12. A. (i) Air flowing with a mach number of 2.5 with a pressure of 60KPa and temperature of −20°C passes over a wedge which turns the flow through an angle of 4° leading to the generation of an oblique shock wave. The oblique shock wave impinges on a flat wall, which is parallel to the flow upstream of the wedge, and is reflected from it. Find the pressure and velocity behind the reflected shock wave. **(8)**

GIVEN DATA

M1 = 2.5
θ = 4°
P1 = 60*10^{3}N/m^2
T1 = 253 K

DIAGRAM

TO FIND **(1)**

Pressure
Velocity

SOLUTION

$\beta_{12} = 27°$
$M_{1n1} = 1.135$
$P_2 = 80940\ N/m^2$ **(3)**
$M_2 = 2.26$
$\beta_{23} = 29°$
$M_{2n2} = 1.10$
$P_3 = 100770.3 N/m^2$ **(3)**
$M_3 = 2.16$
$\beta_r = 25°$ **(1)**

RESULT

(ii) Derive the Rankine - Hugonoit pressure density relationship for the shocks **(8)**

d) Definition and basic equations **(2)**

e) Derivation

$$e_2 - e_1 = \frac{1}{2}(P_1 + P_2)(\frac{1}{\rho_1} - \frac{1}{\rho_2})$$

(5)

$$= \frac{1}{2}\left(p_1 + p_2\right)\left(v_1 - v_2\right)$$

f) R-H curve **(1)**

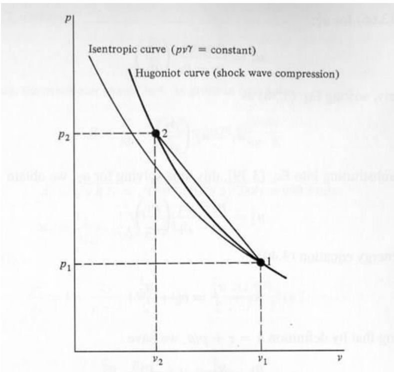
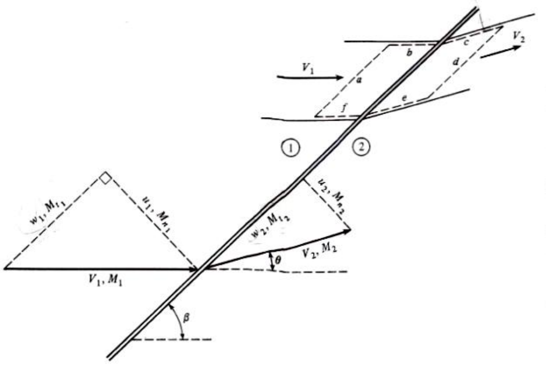


Figure 3.11 | Hugoniot curve; comparison with isentropic compression.

B. Derive a relation connecting flow turning angle, shock angle and free stream Mach number for oblique shock waves

(16)

a) Oblique shock introduction and diagram **(2)**



b) Basic equations **(4)**

iv. Continuity equation

$$\therefore \rho_1 u_1 = \rho_2 u_2$$

v. Momentum equation

$$P_1 + \rho_1 u_1^2 = P_2 + \rho_2 u_2^2$$

vi. Energy equation

$$h_1 + \frac{u_1^2}{2} = h_2 + \frac{u_2^2}{2}$$

c) From velocity triangles **(3)**

$$\therefore \frac{\tan\beta}{\tan(\beta-\theta)} = \frac{u_1}{u_2} = \frac{\rho_2}{\rho_1} = \frac{(\gamma+1)Mn_1^2}{(\gamma-1)Mn_1^2+2} = \frac{(\gamma+1)M_1^2\sin^2\beta}{(\gamma-1)M_1^2\sin^2\beta+2}$$

d) Relation **(3)**

$$\tan\theta = 2\cot\beta\left[\frac{M_1^2\sin^2\beta-1}{M_1^2(\gamma+\cos 2\beta)+2}\right]$$

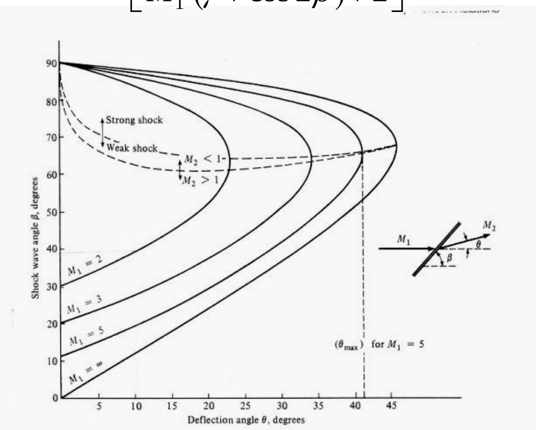


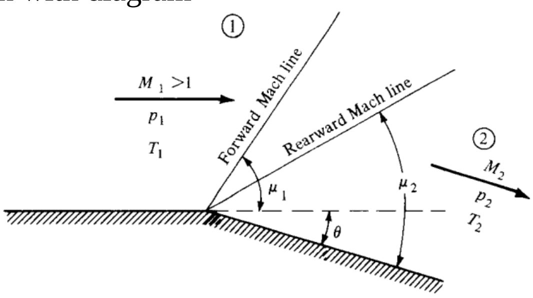
Figure 4.8 | θ-β-M curves. Oblique shock properties. *Important:* See front end pages for a more detailed chart.

e) Inference of the result **(4)**

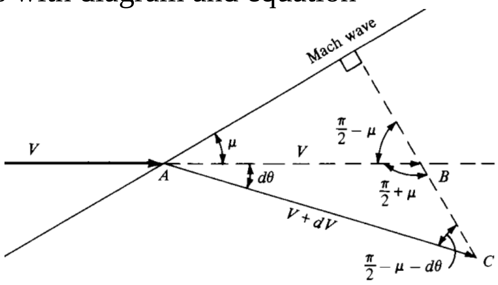
vi. Straight and curved shocks
vii. Attached and detached shocks
viii. Strong and weak shocks
ix. Increasing upstream mach number
x. Increasing deflection angle

13. A. Derive an expression for the Prandtl Meyer function of expansion waves and show that the maximum possible deflection angle is 130.5°
(16)

a) Introduction with diagram **(2)**



b) Law of sines with diagram and equation **(2)**



$$\frac{V+dV}{V}=\frac{\sin(\pi/2+\mu)}{\sin(\pi/2-\mu-d\theta)}$$

c) Flow deflection **(3)**

$$d\theta=\sqrt{M^2-1}\frac{dV}{V}$$

d) Change in velocity **(4)**

$$\frac{dV}{V}=\frac{1}{1+[(\gamma-1)/2]M^2}\frac{dM}{M}$$

e) Prandtlmeyer function **(4)**

$$\nu(M)=\sqrt{\frac{\gamma+1}{\gamma-1}}\tan^{-1}\sqrt{\frac{\gamma-1}{\gamma+1}(M^2-1)}-\tan^{-1}\sqrt{M^2-1}$$

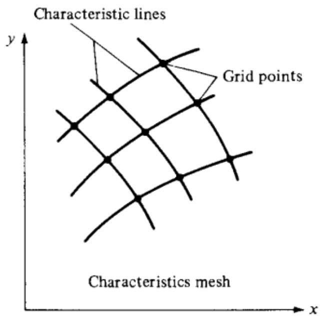
$$\theta=\nu(M_2)-\nu(M_1)$$

f) $v_{max}=\frac{\pi}{2}\left(\sqrt{\frac{\gamma+1}{\gamma-1}}-1\right)$ **(1)**

If γ=1.4, then v_{max} =130.5°

B. Deriving the necessary equation, discuss the method of characteristics and show how it is used for solving nozzle flows
(16)

a) Introduction and definition **(3)**



Procedure

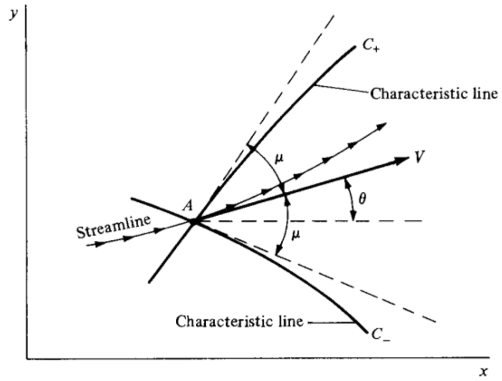
- Find characteristic line
- Convert partial differential equation to ordinary differential equation
- Solve the compatibility equation

b) 2 dimensional supersonic flow till Cramers rule **(3)**

$$\frac{\partial^2\phi}{\partial x\,\partial y} = \frac{\begin{vmatrix} 1-\frac{u^2}{a^2} & 0 & 1-\frac{v^2}{a^2} \\ dx & du & 0 \\ 0 & dv & dy \end{vmatrix}}{\begin{vmatrix} 1-\frac{u^2}{a^2} & -\frac{2uv}{a^2} & 1-\frac{v^2}{a^2} \\ dx & dy & 0 \\ 0 & dx & dy \end{vmatrix}} = \frac{N}{D}$$

c) Slope **(3)**

$$\left(\frac{dy}{dx}\right)_{\text{char}} = \tan(\theta \mp \mu)$$



d) Compatibility equation **(3)**

$$d\theta = \mp\sqrt{M^2-1}\frac{dV}{V}$$

$$\theta + \nu(M) = \text{const} = K_- \qquad \text{(along the } C_- \text{ characteristic)}$$

$$\theta - \nu(M) = \text{const} = K_+ \qquad \text{(along the } C_+ \text{ characteristic)}$$

e) Grid points **(2)**

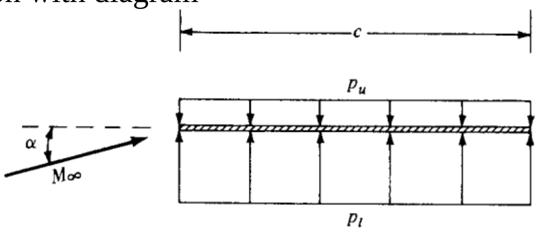
Initial data line, internal points, wall points, shock points

f) Region of influence, domain of dependence **(2)**

14. A. (i) Applying thin aerofoil theory, derive the equation for C_L and C_D as

$C_L = \frac{4\alpha_0}{\sqrt{M_1{}^2-1}}$ and $C_D = \frac{4\alpha_0{}^2}{\sqrt{M_1{}^2-1}}$ **(8)**

a) Introduction with diagram **(2)**



b) Coefficient of pressure **(2)**

$$C_{p,l} = \frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$$
$$C_{p,u} = -\frac{2\alpha}{\sqrt{M_\infty^2 - 1}}$$

c) Normal and axial coefficients **(2)**

$$c_n = \frac{1}{c}\int_0^c (C_{p,l} - C_{p,u})\,dx$$
$$c_a = \frac{1}{c}\int_{\mathrm{LE}}^{\mathrm{TE}} (C_{p,u} - C_{p,l})\,dy$$

d) Lift and drag coefficients **(2)**

$$c_l = \frac{4\alpha}{\sqrt{M_\infty^2 - 1}}$$
$$c_d = \frac{4\alpha^2}{\sqrt{M_\infty^2 - 1}}$$

(ii) Derive the governing differential velocity potential equation of motion **(8)**

a) From continuity equation **(2)**

$$\rho\left(\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2}\right) + \frac{\partial\phi}{\partial x}\frac{\partial\rho}{\partial x} + \frac{\partial\phi}{\partial y}\frac{\partial\rho}{\partial y} = 0$$

b) From Euler's equation **(2)**

$$dp = -\frac{\rho}{2}d\left[\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial y}\right)^2\right]$$

c) Change in density **(2)**

$$\frac{\partial\rho}{\partial x} = -\frac{\rho}{a^2}\left(\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x^2} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial x\partial y}\right)$$
$$\frac{\partial\rho}{\partial y} = -\frac{\rho}{a^2}\left(\frac{\partial\phi}{\partial x}\frac{\partial^2\phi}{\partial x\,\partial y} + \frac{\partial\phi}{\partial y}\frac{\partial^2\phi}{\partial y^2}\right)$$

d) Final equation **(2)**

$$\left[1 - \frac{1}{a^2}\left(\frac{\partial\phi}{\partial x}\right)^2\right]\frac{\partial^2\phi}{\partial x^2} + \left[1 - \frac{1}{a^2}\left(\frac{\partial\phi}{\partial y}\right)^2\right]\frac{\partial^2\phi}{\partial y^2} - \frac{2}{a^2}\left(\frac{\partial\phi}{\partial x}\right)\left(\frac{\partial\phi}{\partial y}\right)\frac{\partial^2\phi}{\partial x\,\partial y} = 0$$

B. (i) Based on small perturbation theory, derive the linearised velocity potential equation for compressible flows
(10)

a) Introduction with diagram **(3)**

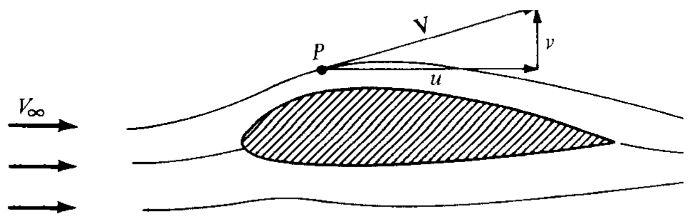


Figure 11.2 Uniform flow and perturbed flow.

Perturbation velocity potential equation

$$\left[a^2 - \left(V_\infty + \frac{\partial \hat{\phi}}{\partial x}\right)^2\right]\frac{\partial^2 \hat{\phi}}{\partial x^2} + \left[a^2 - \left(\frac{\partial \hat{\phi}}{\partial y}\right)^2\right]\frac{\partial^2 \hat{\phi}}{\partial y^2} - 2\left(V_\infty + \frac{\partial \hat{\phi}}{\partial x}\right)\left(\frac{\partial \hat{\phi}}{\partial y}\right)\frac{\partial^2 \hat{\phi}}{\partial x\,\partial y} = 0$$

b) Irrotational isentropic flow **(3)**

$$(1 - M_\infty^2)\frac{\partial \hat{u}}{\partial x} + \frac{\partial \hat{v}}{\partial y} = M_\infty^2\left[(\gamma + 1)\frac{\hat{u}}{V_\infty} + \frac{\gamma + 1}{2}\frac{\hat{u}^2}{V_\infty^2} + \frac{\gamma - 1}{2}\frac{\hat{v}^2}{V_\infty^2}\right]\frac{\partial \hat{u}}{\partial x}$$
$$+ M_\infty^2\left[(\gamma - 1)\frac{\hat{u}}{V_\infty} + \frac{\gamma + 1}{2}\frac{\hat{v}^2}{V_\infty^2} + \frac{\gamma - 1}{2}\frac{\hat{u}^2}{V_\infty^2}\right]\frac{\partial \hat{v}}{\partial y}$$
$$+ M_\infty^2\left[\frac{\hat{v}}{V_\infty}\left(1 + \frac{\hat{u}}{V_\infty}\right)\left(\frac{\partial \hat{u}}{\partial y} + \frac{\partial \hat{v}}{\partial x}\right)\right]$$

Applications and limitations

Linearized velocity potential equation

$$(1 - M_\infty^2)\frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0$$

c) Linearized Pressure Coefficient **(4)**

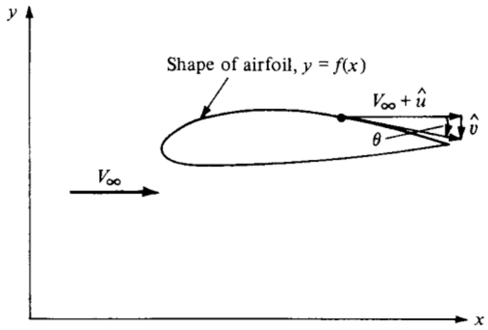
$$C_p = -\frac{2\hat{u}}{V_\infty}$$

Flow tangency condition

$$\frac{\partial \hat{\phi}}{\partial y} = V_\infty \tan\theta$$

(ii) Based on the above equation, establish the prandtlgauert rule **(6)**

a) Introduction with diagram **(1)**



b) Velocity potential equation **(2)**

$$\beta^2 \frac{\partial^2 \hat{\phi}}{\partial x^2} + \frac{\partial^2 \hat{\phi}}{\partial y^2} = 0$$

Laplace equation

$$\frac{\partial^2 \bar{\phi}}{\partial \xi^2} + \frac{\partial^2 \bar{\phi}}{\partial \eta^2} = 0$$

c) Shape of physical and transformed planes **(1)**

$$\frac{df}{dx} = \frac{dq}{d\xi}$$

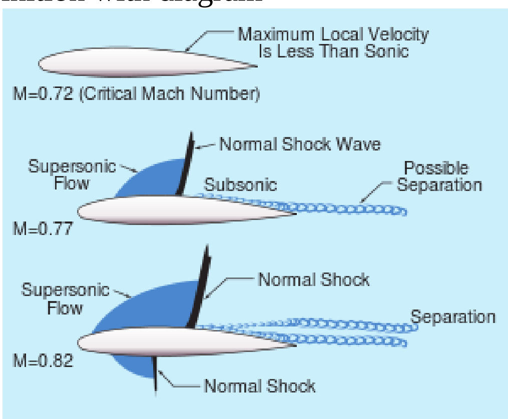
d) Coefficient of pressure **(2)**

$$C_p = \frac{C_{p,0}}{\sqrt{1 - M_\infty^2}}$$

15. A. Discuss the following **(4*4=16)**

a) Critical mach numbers

a) Definition with diagram **(4)**



b) Factors influencing critical Mach number
- Thickness to chord ratio
- Aspect ratio
- Wing sweep

c) Types of critical Mach number
- Upper critical Mach number
- Lower critical Mach number

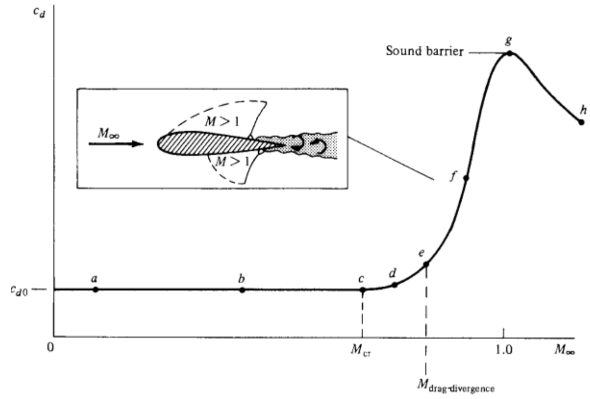
d) Equation

$$c_p = \frac{2}{\gamma M_\infty^2}[(\frac{\frac{\gamma+1}{2}}{1+\frac{\gamma-1}{2}M_{\infty_{cr}}^2})^{\frac{\gamma}{1-\gamma}}-1]$$

b) Lift and drag divergence **(4)**

a. drag divergence definition

b. curve

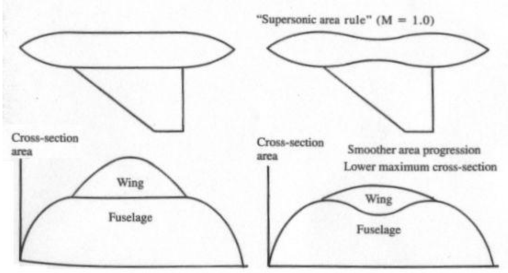


c. Lift divergence

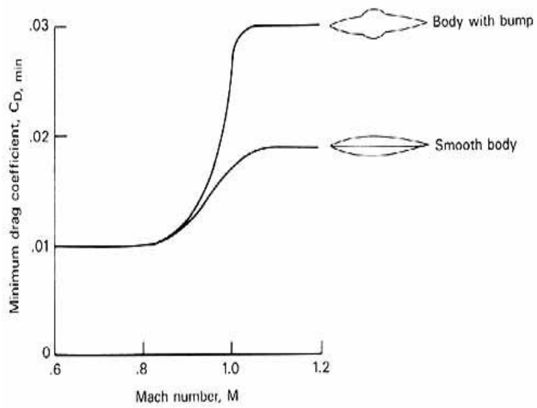
c) Transonic area rule

a) Introduction **(4)**

Area ruled Vs non area ruled aircraft



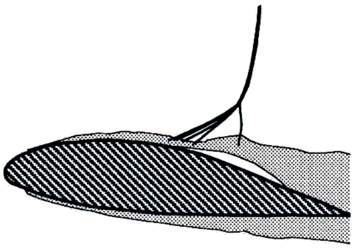
b) Mach number Vs drag curve



Applications

d) Shock induced separation

a) Definition **(4)**

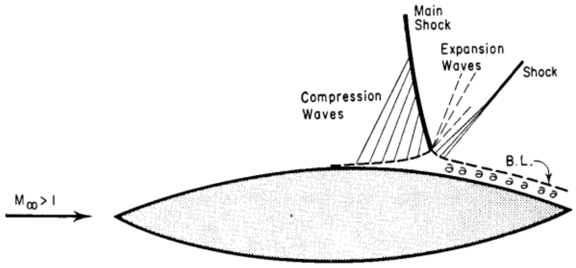


b) Effects of shock induced separation
- External flow
  - Increased aerodynamic drag, loss of lift
  - Aerodynamic heating
  - Increased instabilities such as inlet
- Internal flow
  - Total pressure loss and unsteadiness
  - Loss of flow control performance

c) shock boundary layer interactions
- Weak interaction
- Moderate interaction
- Strong interaction
- Very strong interaction

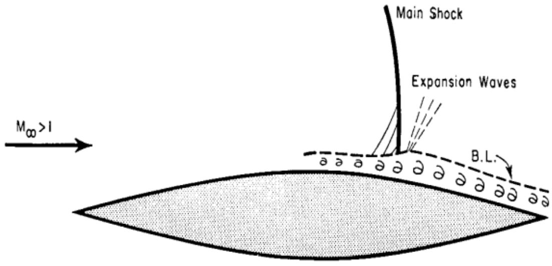
d) Various flows

Laminar



(a)

Turbulent

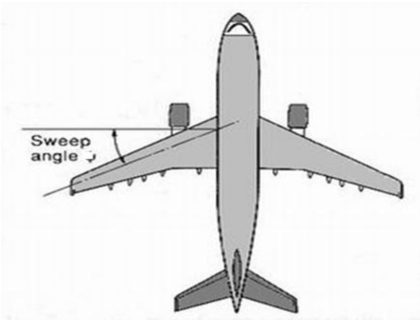


(b)

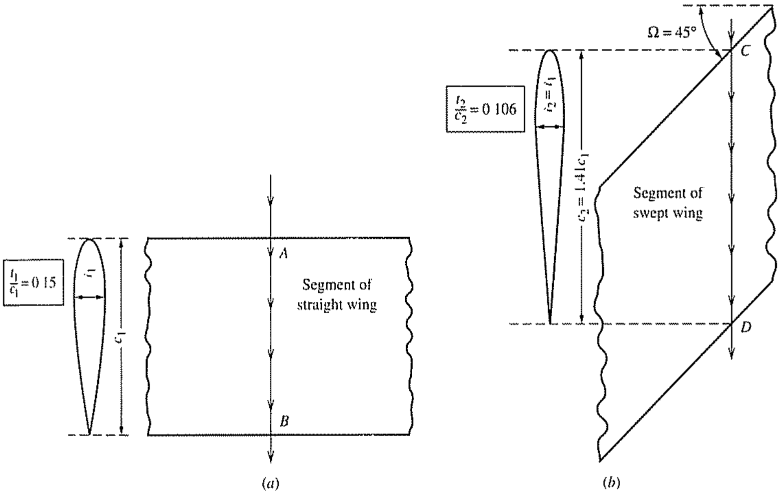
B. Write short notes on **(4*4=16)**

a) Characteristics of swept wings

a) swept wing plan form **(4)**



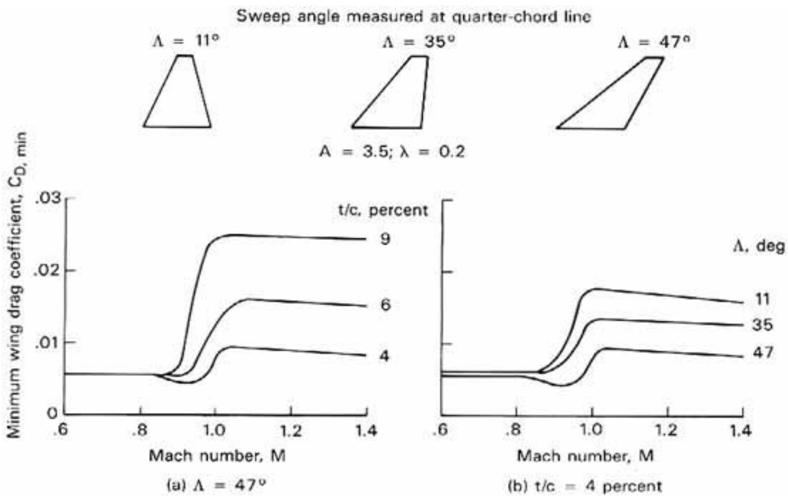
b) Straight wing Vs swept wing



c) Types

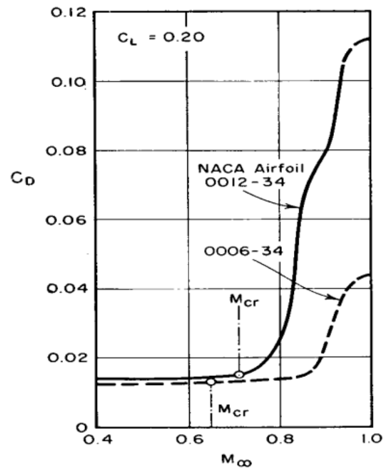
- Forward swept
- Backward swept
- Variable swept

d) Curve

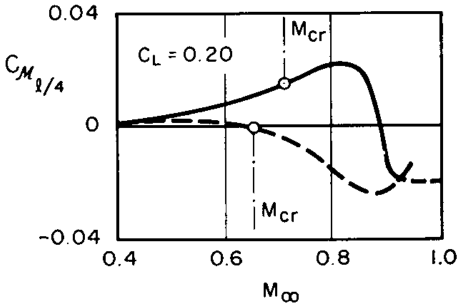


b) Effects of thickness over the performance of wings

a) Drag coefficient VS Mach number **(4)**

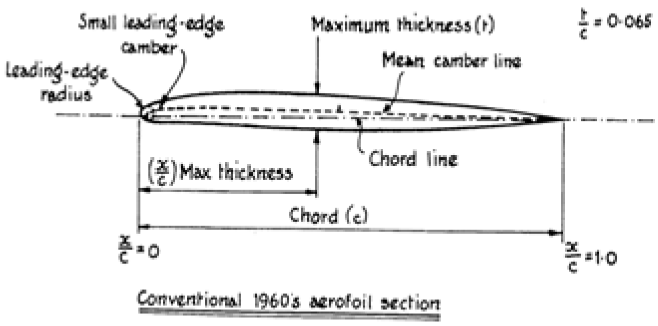


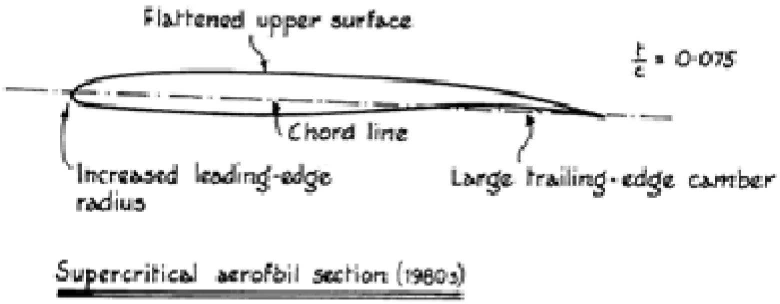
b) Moment coefficient VS Mach number



c) Supercritical airfoils
   a) Definition with benefits **(4)**

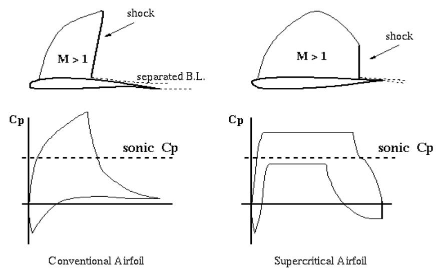
   b) Comparison between supercritical and traditional airfoils





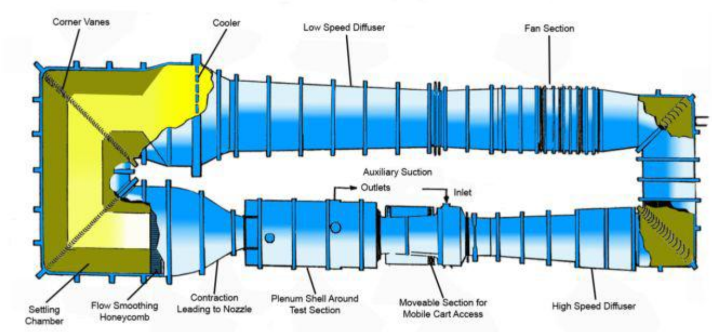
Pressure distribution

Conventional vs. Supercritical Airfoils



d) Transonic wind tunnel **(4)**

c) Construction



d) Working